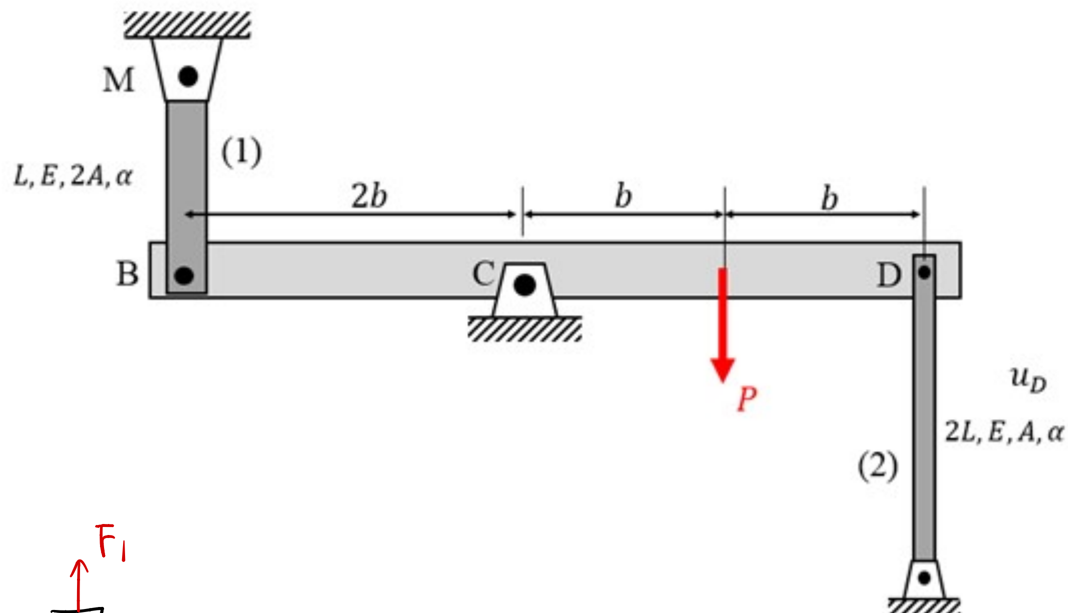
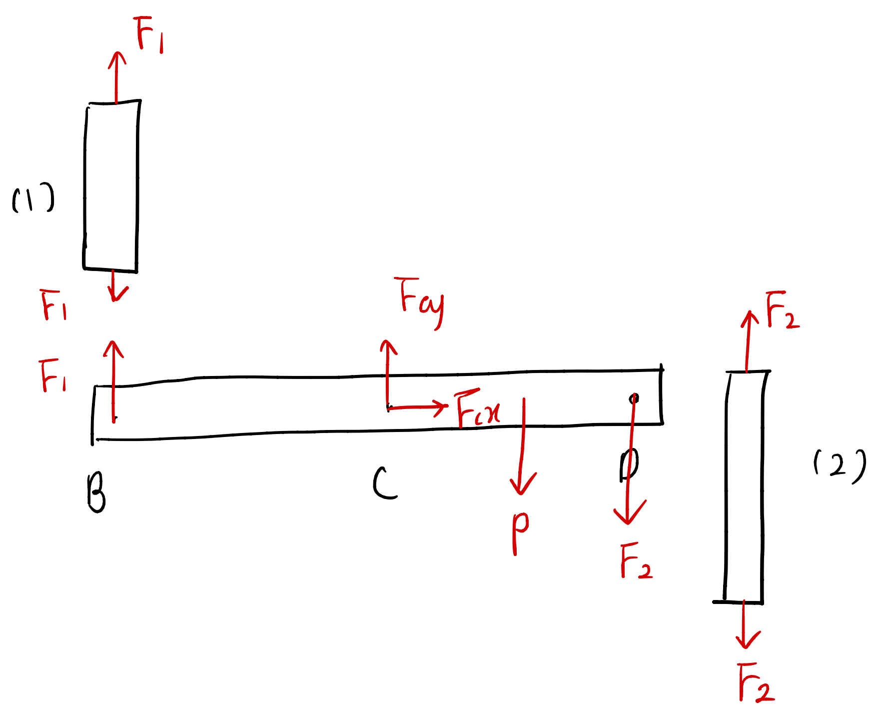
Problem 3.1 ***(10 points)***

Rigid bar BD is pinned to ground at end C. The bar is supported by thin rods (1) and (2), where rod (1) has a Young's modulus of E and a cross-sectional area of 2A and rod (2) has a Young's modulus of E and a cross-sectional area of A. Each rod possesses a coefficient of thermal expansion . A load P acts midway between pins C and D on the bar. Assume that the angle of rotation of the bar as a result of the applied load is small and the temperature of both rods is simultaneously increased by ΔT. Ignore the weight of all components of the structure.

(a) Determine the stress in rods (1) and (2).
(b) Determine the angle of rotation of bar BD.



a)



$$\Sigma M_c = F_1(2b) + F_2(2b) + P(b) = 0$$

$$\Rightarrow F_1 + F_2 = -\frac{P}{2}$$

[FBD + Equilibrium : 3pt]

Force – elongation

$$e_1 = \frac{F_1 L_1}{E_1 A_1} + \alpha \Delta T L_1 = \frac{F_1 L}{2EA} + \alpha \Delta T L$$ 1pt

$$e_2 = \frac{F_2 L_2}{E_2 A_2} + \alpha \Delta T L_2 = \frac{2F_2 L}{EA} + 2\alpha \Delta T L$$ 1pt

Compatibility

From the structure, $\frac{e_1}{2b} = \frac{e_2}{2b} \Rightarrow e_1 = e_2$ 1pt

$$\frac{F_1 L}{2EA} + \alpha \Delta T L = \frac{2F_2 L}{EA} + 2\alpha \Delta T L$$

$$\Rightarrow F_1 - 4F_2 = 2EA\alpha\Delta T$$

$$\Rightarrow F_1 - 4F_2 = F_1 - 4\left(-\frac{P}{2} - F_1\right) = 2EA\alpha\Delta T$$

$$\Rightarrow F_1 = -\frac{2P}{5} + \frac{2}{5}EA\alpha\Delta T \qquad F_2 = -\frac{P}{10} - \frac{2}{5}EA\alpha\Delta T$$

$$\Rightarrow \sigma_1 = \frac{F_1}{A_1} = \frac{F_1}{2A} = -\frac{P}{5A} + \frac{1}{5}E\alpha\Delta T$$

$$\sigma_2 = \frac{F_2}{A_2} = \frac{F_2}{A} = -\frac{P}{10A} - \frac{2}{5}E\alpha\Delta T$$

b) $$\tan\theta = \frac{e_1}{2b} = \frac{e_2}{2b} = \frac{\frac{2F_2 L}{EA} + 2\alpha\Delta T L}{2b} = \frac{-\frac{PL}{10EA} - \frac{2}{5}\alpha\Delta T L + \alpha\Delta T L}{b}$$

$$= -\frac{PL}{10EAb} + \frac{3}{5b}\alpha\Delta T L$$

$$\theta = \arctan\left(-\frac{PL}{10EAb} + \frac{3}{5b}\alpha\Delta T L\right)$$ 2pt

$\theta \ll 1 \Rightarrow \tan\theta \approx \sin\theta \approx \theta$

Problem 3.2 ***(10 points)*** A rod is made up of elements (1), (2) and (3), as shown below. Element (2) is hollow with outer and inner diameters of 2d and d, respectively, whereas elements (1) and (3) are solid with diameters of d and 3d, respectively. Elements (1) and (2) are joined by rigid connector D, elements (2) and (3) are joined by rigid connector C, and elements (1) and (3) are connected to ground at ends H and B, respectively. The modulus of elasticity for all three elements is E. The weights of connectors D and C are 2W and W, respectively, whereas the weights of the rod elements (1), (2) and (3) are to be considered negligible.

(a) Determine the stress in element (3) resulting only from the weights of the connectors (refer to Fig A).

(b) Suppose that an axial load P is applied to connector D in a way that the magnitude of the stress in element (3) is reduced (refer to Fig B). Determine the load value for P such that the magnitude of the compressive stress in (3) reduced by 50 percent from that found in (a).

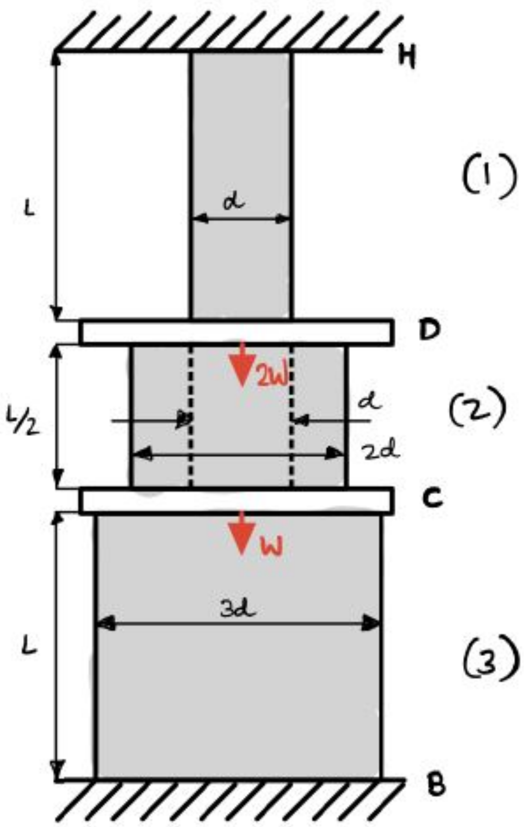


Fig A

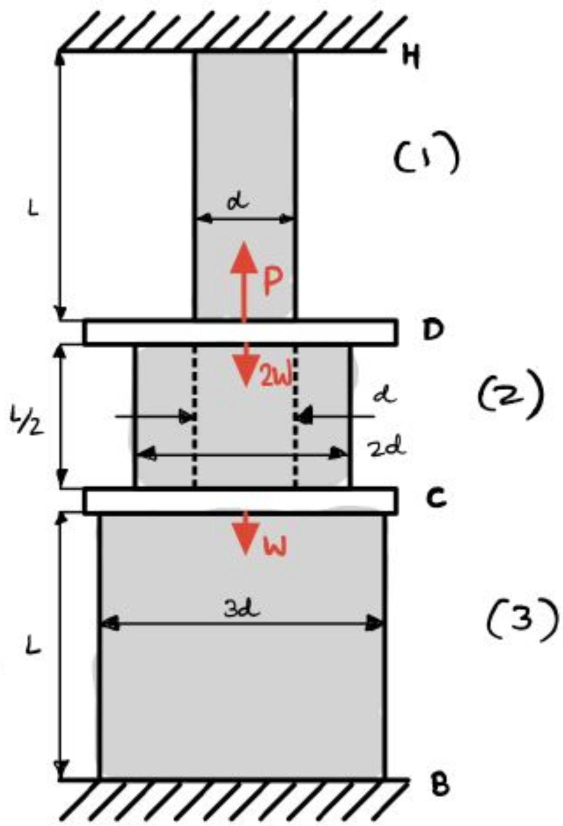
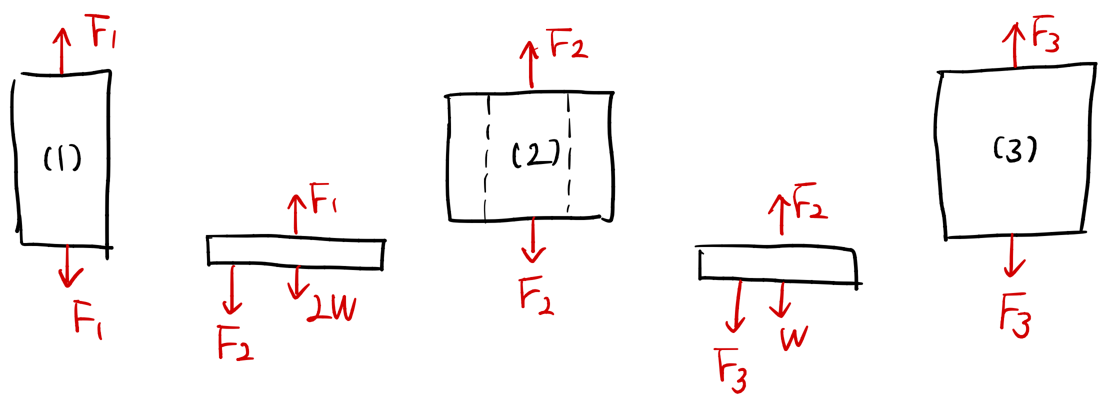


Fig B

FBD



a)

At D:

$+\uparrow \Sigma F_y = 0$: $F_1 - F_2 - 2W = 0 \rightarrow (1)$

At C:

3pt: FBD & equilibrium

$+\uparrow \Sigma F_y = 0$: $F_2 - W - F_3 = 0 \rightarrow (2)$

$F_2 = F_3 + W$ $\therefore F_1 = F_3 + 3W$

$F_1 = F_2 + 2W$

$\Rightarrow$ (2) eqns, (3) variables $\Rightarrow$ it is indeterminate

<u>Compatibility condition</u>

$$e_1 = \frac{F_1 L_1}{A_1 E_1} \quad e_2 = \frac{F_2 L_2}{A_2 E_2} \quad e_3 = \frac{F_3 L_3}{A_3 E_3}$$ 1pt

$$\cancel{u_B}^{0} = \cancel{u_H}^{0} + e_1 + e_2 + e_3$$

$$0 = e_1 + e_2 + e_3$$

$$0 = \frac{F_1 L_1}{A_1 E} + \frac{F_2 L_2}{A_2 E} + \frac{F_3 L_3}{A_3 E} \rightarrow (3)$$ 1pt

$$\Rightarrow F_3 = -\frac{57W}{23} \sim -2.4782W$$

$$\sigma_3 = \frac{F_3}{A_3} = \frac{-\frac{57W}{23}}{(9\pi d^2/4)}$$ (compression) 2pt

b) $+\uparrow \Sigma F_D = 0$: $F_1 - F_2 - 2W + P = 0$

$F_1 = F_2 + 2W - P = F_3 + 3W - P$

1pt

compatibility $\Rightarrow$ $e_1 + e_2 + e_3 = 0$

$\Rightarrow F_3 = -\frac{57W}{23} + \frac{18}{23}P$

$$\Delta_3 = \frac{F_3}{A_3} = \frac{-\frac{57W}{23} + \frac{18}{23}P}{\frac{9\pi \cancel{d^2}}{\cancel{4}}} = \frac{1}{2}\left(\frac{-\frac{57W}{23}}{\frac{9\pi \cancel{d^2}}{\cancel{4}}}\right)$$

$\Rightarrow \frac{18}{23}P = \frac{57}{46}W$

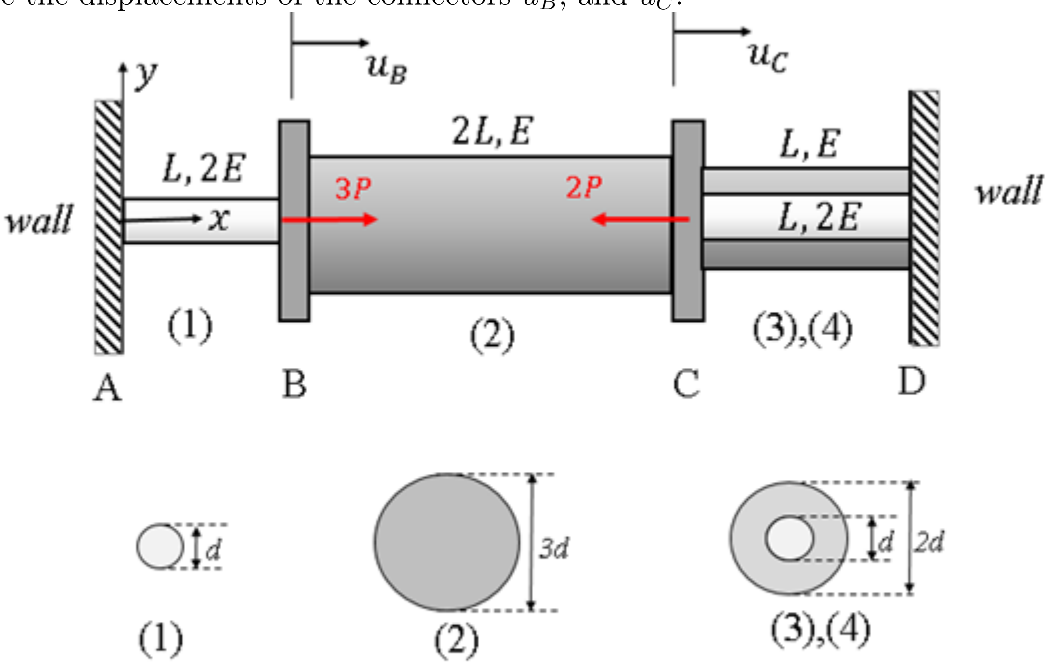
$\therefore P = \frac{19}{12}W \sim 1.5833W$

2pt

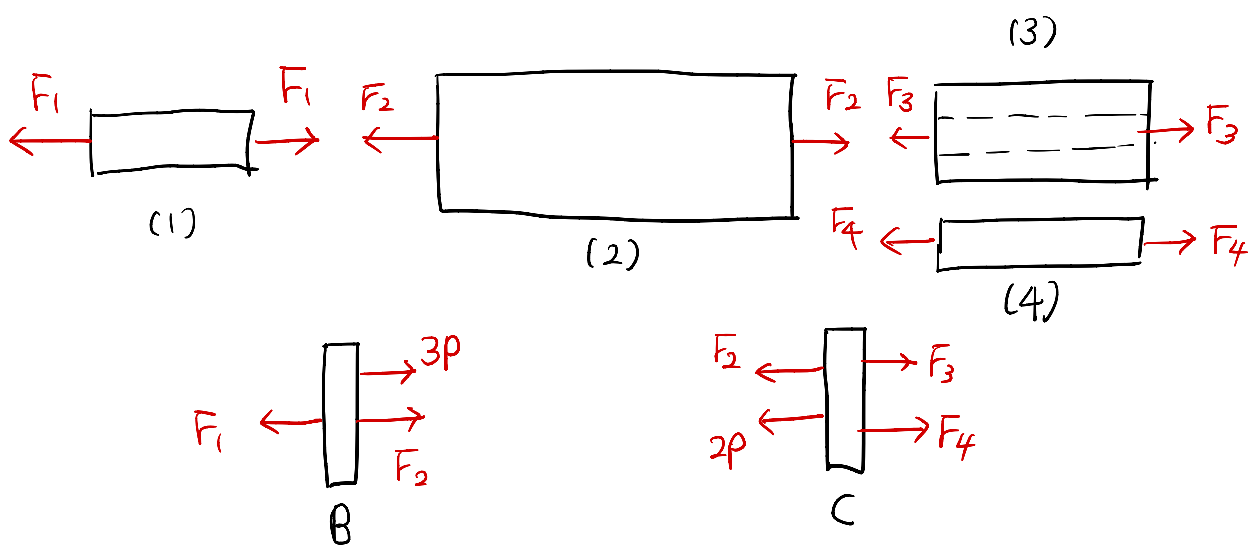
Problem 3.3 ***(10 points)***

A composite rod is fixed to the wall at ends A and D, and is made of rods (1), (2), (3) and (4) joined by rigid connectors B and C. Rod (1) of length L has a solid circular cross section, and is made of a material of Young's modulus $2E$. Rod (2) of length $2L$ has a solid circular cross section and is composed of a material of Young's modulus E. Rod (3) of length L is an outer shell placed concentrically with Rod (4), which is an inner core of length L. The outer shell (3) is composed of a material of Young's modulus E and the inner core (4) is composed of a material of Young's modulus $2E$. Each rod possesses a coefficient of thermal expansion α. The cross sections of the 4 rods are shown in figure. In addition, the temperature of Rod (1) is increased by ΔT, while the temperature of other rods is kept constant. Axial loads $3P$, and $2P$ are applied to rigid connectors at B and C.

(a) Calculate the axial stresses experienced by rods (1), (2), (3), and (4).
(b) Calculate the displacements of the connectors u_B, and u_C.



a)



$$\Sigma F_B = F_2 - F_1 + 3P = 0$$

2pt: FBD, equilibrium

$$\Sigma F_C = F_3 + F_4 - F_2 - 2P = 0$$

$$A_1 = A_4 = \frac{1}{4}\pi d^2 \qquad A_2 = \frac{1}{4}\pi(3d)^2 = \frac{9}{4}\pi d^2$$

$$A_3 = \frac{1}{4}\pi\left[(2d)^2 - d^2\right] = \frac{3}{4}\pi d^2$$

$$e_1 = \frac{F_1 L}{2EA_1} + \alpha\Delta T L = \frac{2F_1 L}{E\pi d^2} + \alpha\Delta T L$$ 0.5pt

$$e_2 = \frac{F_2(2L)}{EA_2} = \frac{8F_2 L}{9E\pi d^2}$$ 0.5pt

$$e_3 = \frac{F_3(L)}{EA_3} = \frac{4F_3 L}{3E\pi d^2}$$ 0.5pt

$$e_4 = \frac{F_4(L)}{2EA_4} = \frac{2F_4 L}{E\pi d^2}$$ 0.5pt

$$u_D = u_A + e_1 + e_2 + e_3 = 0 \quad \because \text{Fixed } A \,\&\, D$$

$$e_3 = e_4$$

$$\Rightarrow e_1 + e_2 + e_3 = \frac{2F_1 L}{E\pi d^2} + \alpha\Delta T L + \frac{8F_2 L}{9E\pi d^2} + \frac{4F_3 L}{3E\pi d^2} = 0$$

$$\Rightarrow 27F_1 + 12F_2 + 18F_3 = -\frac{27}{2}\alpha\Delta T E\pi d^2$$

$$e_3 = e_4 \Rightarrow \frac{4F_3 L}{3E\pi d^2} = \frac{2F_4 L}{E\pi d^2} \Rightarrow 2F_3 = 3F_4$$

$$\Rightarrow F_1 = \frac{178}{83}P - \frac{45}{166}\alpha \Delta T E \pi d^2$$

$$F_2 = \frac{-171}{83}P - \frac{45}{166}\alpha \Delta T E \pi d^2$$

$$F_3 = \frac{-3}{83}P - \frac{27}{166}\alpha \Delta T E \pi d^2$$

$$F_4 = \frac{-2}{83}P - \frac{9}{83}\alpha \Delta T E \pi d^2$$

$$\sigma_i = \frac{F_i}{A_i}$$

$$\Rightarrow \sigma_1 = \frac{312P}{83\pi d^2} - \frac{90}{83}\alpha \Delta T E \qquad 0.5\text{pt}$$

$$\sigma_2 = \frac{-76P}{83\pi d^2} - \frac{10}{83}\alpha \Delta T E \qquad 0.5\text{pt}$$

$$\sigma_3 = \frac{-4P}{83\pi d^2} - \frac{10}{83}\alpha \Delta T E \qquad 0.5\text{pt}$$

$$\sigma_4 = \frac{-8P}{83\pi d^2} - \frac{36}{83}\alpha \Delta T E \qquad 0.5\text{pt}$$

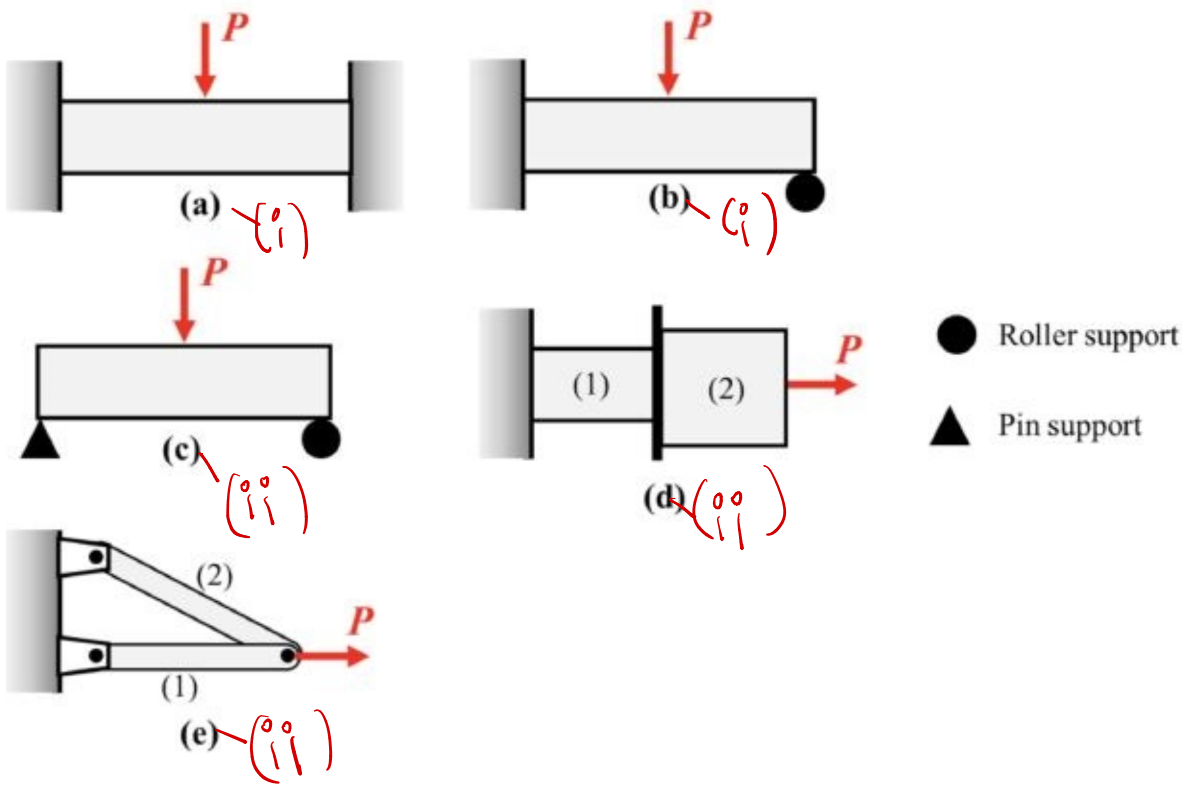
b) $$u_B = e_1 = \frac{2F_1 L}{E\pi d^2} + \alpha \Delta T L = \frac{156}{83}\frac{PL}{E\pi d^2} + \frac{38}{83}\alpha \Delta T L \qquad 1\text{pt}$$

$$u_C = u_B + e_2 = u_B + \frac{8F_2 L}{9E\pi d^2} = \frac{4}{83}\frac{PL}{E\pi d^2} + \frac{18}{83}\alpha \Delta T L \qquad 1\text{pt}$$

$$u_D = u_C + e_3 = 0 \qquad \text{check!}$$

Problem 3.4 ***(2.5 + 2.5 +2.5 points)***

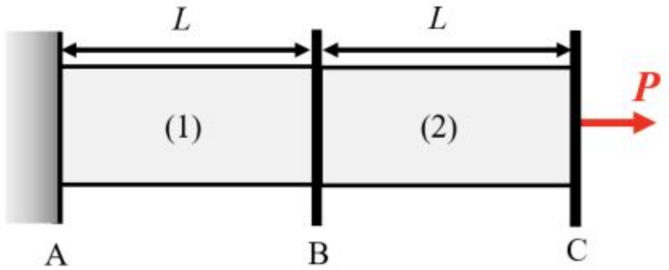
1. Match the following six structures (a)-(e) with correct option given in i-iii.



   i. Statically indeterminate structure
   ii. Statically determinate structure
   iii. Insufficient information

Number of variables vs number of equation

2. The rod consists of elements (1) and (2) and rigid connectors B and C. Both elements have length L and cross-sectional area A. Element (1) has Young's modulus E_1, and element (2) has Young's modulus E_2, with $E_1 > E_2$

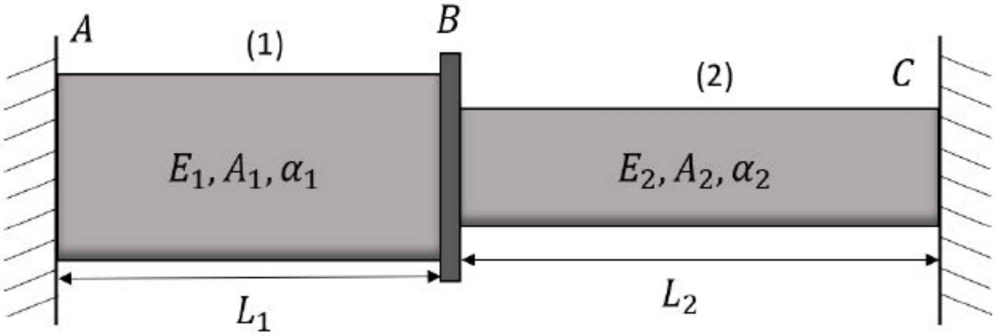


Let F_1 and F_2 represent the axial forces in members (1) and (2). Choose the correct option.

(a) $F_1 > F_2$
(b) $F_1 = F_2$
(c) $F_1 < F_2$
(d) Insufficient information

Reaction force is the same throughout it's length. Deformation might vary

3. Consider a bar made of two sections fixed at both ends. For section (1) let the length be L_1, area A_1, Young's modulus E_1 and coefficient of thermal expansion α_1. The corresponding values for section (2) are L_2, A_2, E_2 and α_2. It is known that $L_1 < L_2$, $E_1 > E_2$, $A_1 > A_2$ and $\alpha_1 > \alpha_2$. The bar is free of stress at temperature T_1. Let σ_1 and σ_2 represent the axial stresses in section 1 and 2, respectively, after the rise in temperature. The temperature is raised from T_1 to T_2 ($T_2 > T_1$).



If δ_1 is the change in length of section 1 and δ_2 is the change in length of section 2, which of the following statements is true?

(a) $\delta_1 = \delta_2 = 0$
(b) $\delta_1 + \delta_2 = 0$
(c) $\delta_1 = \delta_2 \neq 0$
(d) $\delta_1 = \frac{E_1}{E_2}\delta_2$

$\delta_C = \delta_A + \delta_L + \delta_2$

$\delta_C = \delta_A = 0$

(Elongation at the support should be zero)