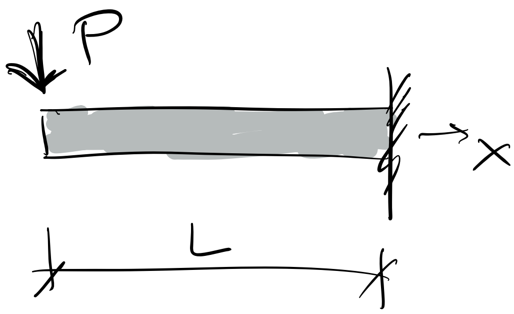
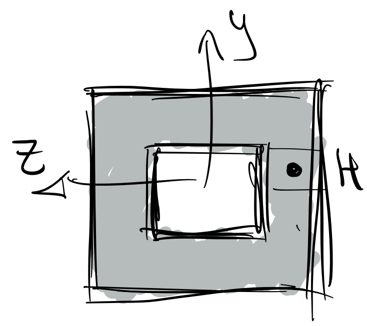
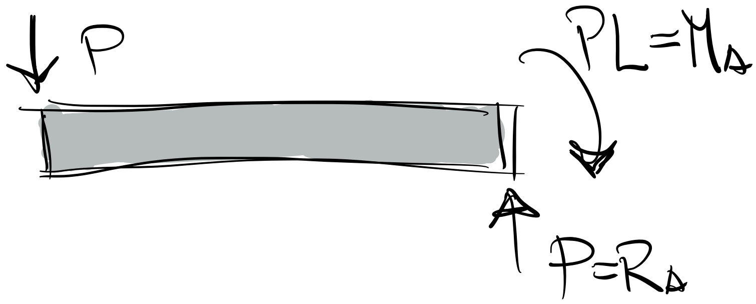




FBD & Equilibrium



$\sum F_y = 0 = R_A - P$

$(\sum M) = 0 = PL - M_A$

Bending moment ⇒ normal stress

$\sigma_x > 0$

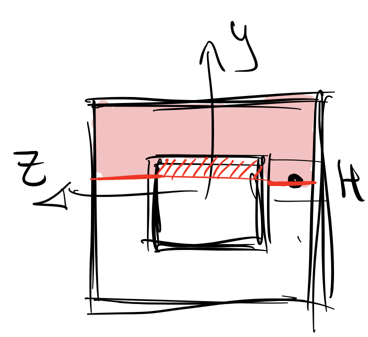
$$I_{zz} = \frac{(2b)^4}{12} - \frac{b^4}{12} = \frac{15}{12} b^4 = \frac{5}{4} b^4$$

$\sigma > 0$ (tension) @ H ⇒ $\boxed{\sigma = \frac{(PL)(b/4)}{I_{zz}} = \frac{PL}{5b^3}}$

Shear force ⇒ shear stress

$\tau = \frac{VQ}{I_{zz} t}$ ⇒ Need $Q(y=b/4)$

and $t(y=b/4)$

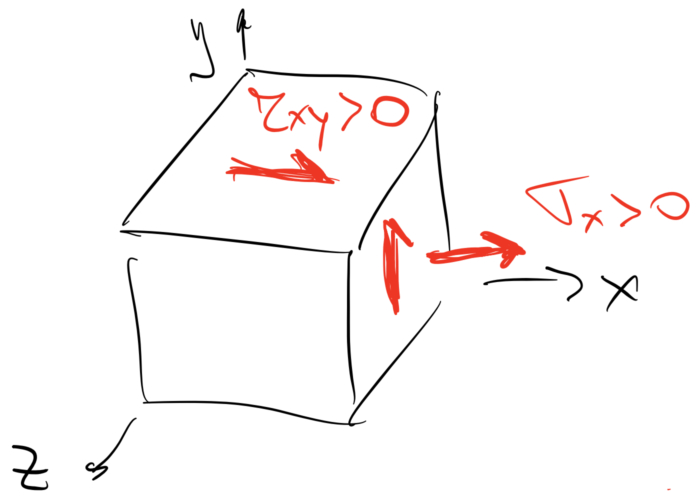


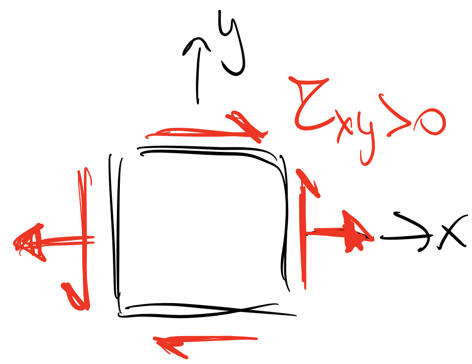
$$Q = \underbrace{\left(2b \times \frac{3}{4}b\right)}_{A_1^*}\underbrace{\left(\frac{3}{8}b + \frac{1}{4}b\right)}_{y_1^*} - \underbrace{\left(b \times \frac{1}{4}b\right)}_{A_2^*}\underbrace{\left(\frac{1}{8}b + \frac{1}{4}b\right)}_{y_2^*} = \frac{27b^3}{32}$$

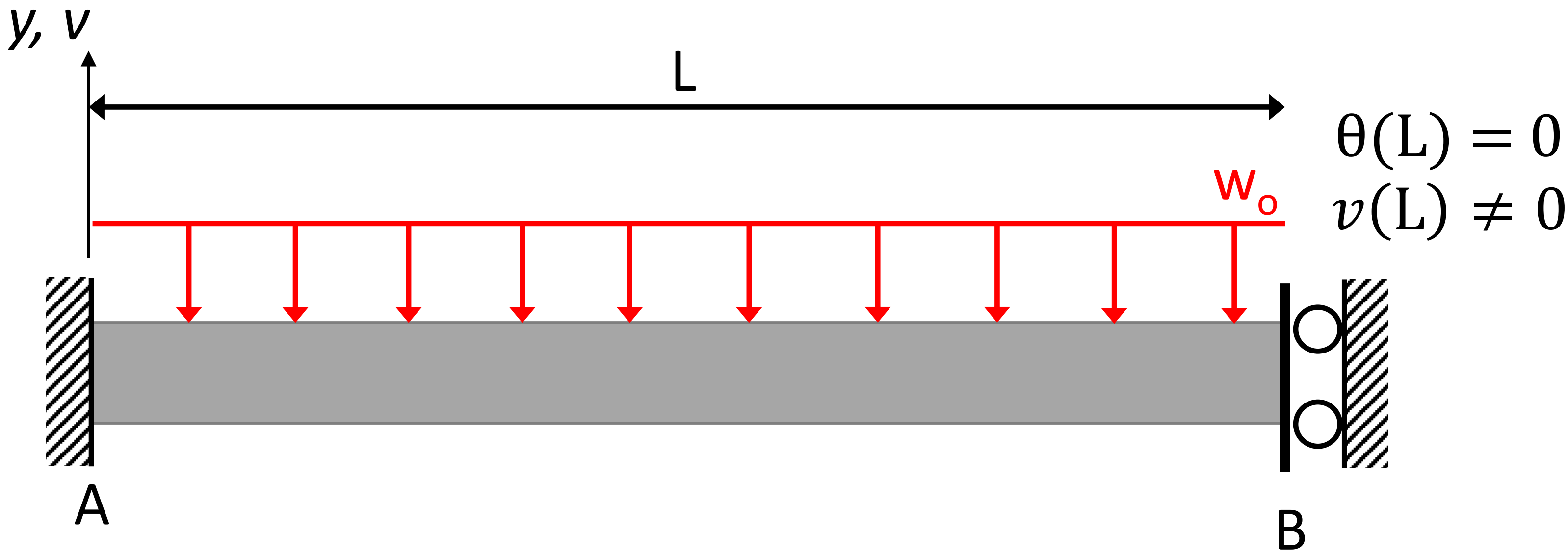
$$t = \frac{b}{2} + \frac{b}{2} = b$$

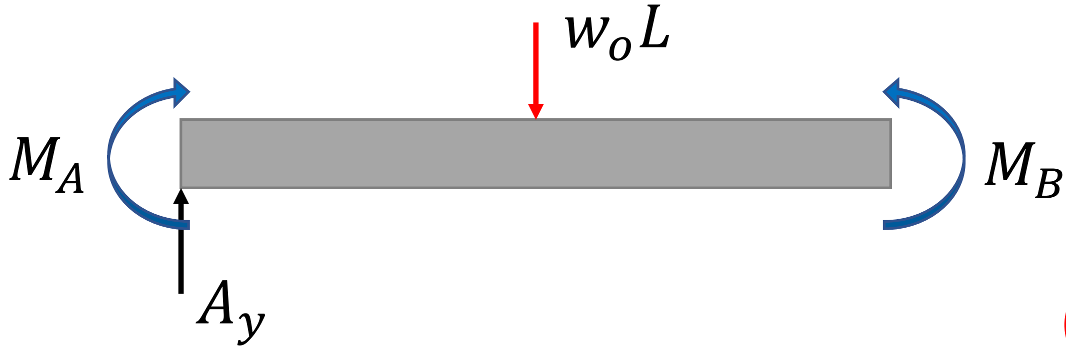
$$\Rightarrow \boxed{\tau = \frac{P\,27b^3/32}{5b^4/4 \times b} = \frac{27P}{40b^2}} \longrightarrow \tau_{xy} > 0$$

(same direction as shear force)









(2)

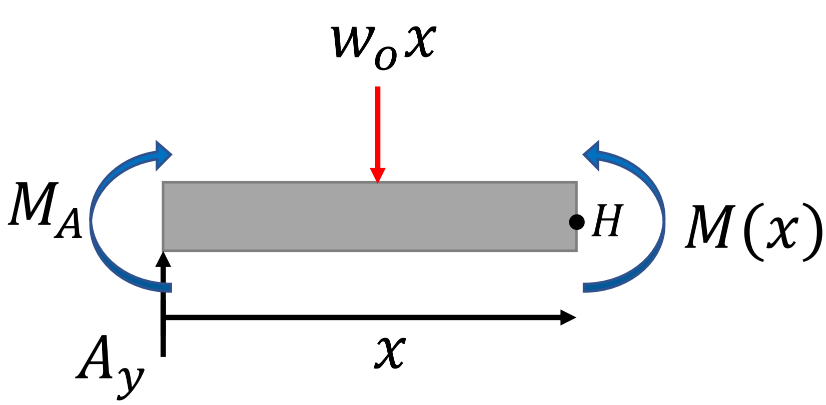
$$\sum F_y = 0 = A_y - w_o L$$

$A_y = w_o L$ (1)

$$\left(\sum M\right)_A = 0 = -M_A - w_o L\left(\frac{L}{2}\right) + M_B$$

$M_A = M_B - w_o L\left(\frac{L}{2}\right)$ (1)

$\Longrightarrow$ indeterminate



(3)

$$\left(\sum M\right)_H = 0 = -M_A - A_y x + w_o x\left(\frac{x}{2}\right) + M(x)$$

$$M(x) = M_A + A_y x - \left(\frac{1}{2}\right) w_o x^2$$

(3)

$$\theta(x) = \theta(0) + \left(\frac{1}{EI}\right)\int_0^x M(x)dx$$ (3)

$$\theta(x) = \theta(0) + \left(\frac{1}{EI}\right)\int_0^x M_A + w_o Lx - \left(\frac{1}{2}\right) w_o x^2 dx$$

$$\theta(x) = \theta(0) + \left(\frac{1}{EI}\right)\left[M_A x + \left(\frac{1}{2}\right) w_o L x^2 - \left(\frac{1}{6}\right) w_o x^3\right]$$

$$\theta(x) = \cancelto{0}{\theta(0)} + \left(\frac{1}{EI}\right)\left[M_A x + \left(\frac{1}{2}\right) w_o L x^2 - \left(\frac{1}{6}\right) w_o x^3\right]$$ (1)

$$\theta(L) = \left(\frac{1}{EI}\right)\left[M_A L + \left(\frac{1}{2}\right) w_o L^3 - \left(\frac{1}{6}\right) w_o L^3\right] = 0$$ (1)

$$M_A = -\left(\frac{1}{3}\right) w_o L^2$$ (1)

$$A_y = w_o L$$ (1)

$$M_B = M_A + w_o L\left(\frac{L}{2}\right) \qquad M_B = \left(\frac{1}{6}\right) w_o L^2$$ (1)

$$\theta(x) = \left(\frac{1}{EI}\right)\left[M_A x + \left(\frac{1}{2}\right) w_o L x^2 - \left(\frac{1}{6}\right) w_o x^3\right]$$

$$\theta(x) = \left(\frac{1}{EI}\right)\left[-\left(\frac{1}{3}\right) w_o x L^2 + \left(\frac{1}{2}\right) w_o x^2 L - \left(\frac{1}{6}\right) w_o x^3\right]$$

(3)

$$\mathrm{v}(x) = v(0) + \left(\frac{1}{EI}\right)\int_0^x -\left(\frac{1}{3}\right) w_o x L^2 + \left(\frac{1}{2}\right) w_o x^2 L - \left(\frac{1}{6}\right) w_o x^3 \, dx$$

$$\mathrm{v}(x) = v(0) + \left(\frac{1}{EI}\right)\left[-\left(\frac{1}{6}\right) w_o x^2 L^2 + \left(\frac{1}{6}\right) w_o x^3 L - \left(\frac{1}{24}\right) w_o x^4\right]$$

$$\mathrm{v}(x) = v(0) + \left(\frac{1}{EI}\right)\left[-\left(\frac{1}{6}\right) w_o x^2 L^2 + \left(\frac{1}{6}\right) w_o x^3 L - \left(\frac{1}{24}\right) w_o x^4\right]$$

(1)

$$\mathrm{v}(x) = \cancelto{0}{v(0)} + \left(\frac{1}{EI}\right)\left[-\left(\frac{1}{6}\right) w_o x^2 L^2 + \left(\frac{1}{6}\right) w_o x^3 L - \left(\frac{1}{24}\right) w_o x^4\right]$$

(2)

$$\mathrm{v}(x) = \left(\frac{1}{EI}\right)\left[-\left(\frac{1}{6}\right) w_o x^2 L^2 + \left(\frac{1}{6}\right) w_o x^3 L - \left(\frac{1}{24}\right) w_o x^4\right]$$

$$\mathrm{v}(L) = \left(\frac{1}{EI}\right)\left[-\left(\frac{1}{6}\right) w_o L^4 + \left(\frac{1}{6}\right) w_o L^4 - \left(\frac{1}{24}\right) w_o L^4\right]$$

$$\mathrm{v}(L) = \left(\frac{1}{EI}\right)\left[-\left(\frac{1}{24}\right) w_o L^4\right]$$

(1)

ME 323 – Mechanics of Materials
Examination #1
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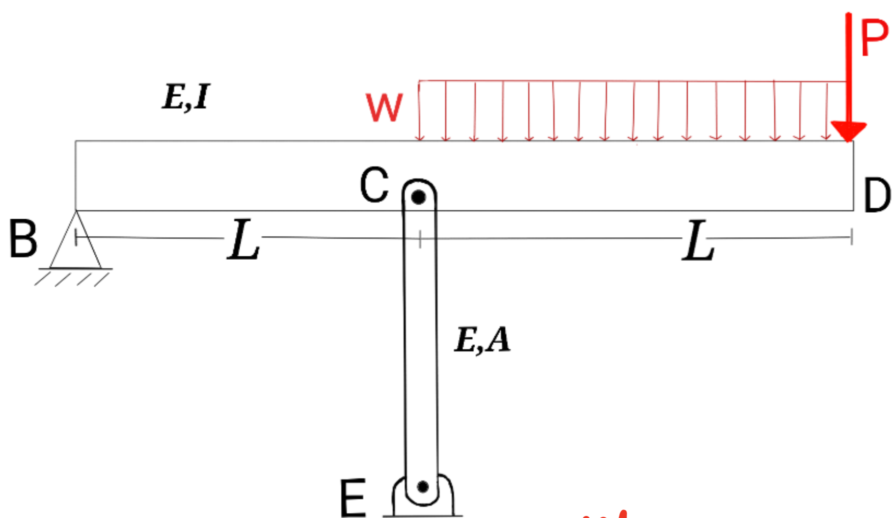
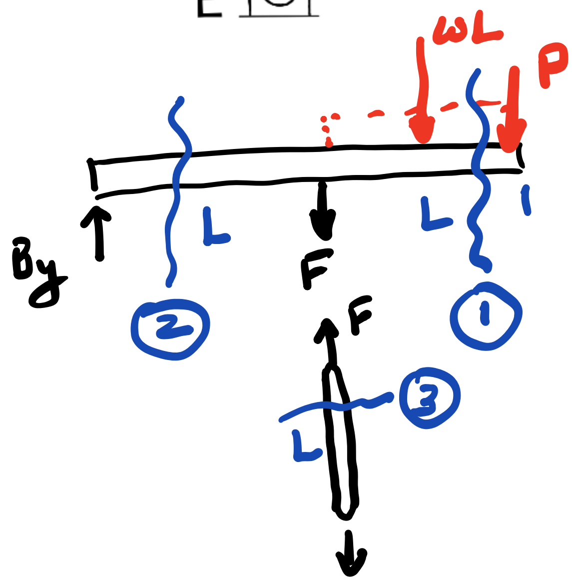
PROBLEM # 3 (25 points)

A beam BCD of length 2L is pinned at B and is connected to a member CE at a distance L from B. The length of CE is L. The beam and the member are all made of the same material with Young's modulus, E. The beam is uniform with a second moment of the area of I, while the cross-sectional area of CE is A. The beam is subjected to a uniform distributed load of w between C and D and a concentrated force of $\boldsymbol{P}$.

Use Castigliano's second theorem to calculate the vertical deflection of the beam at D. **Neglect the strain energy due to shear forces**.



FBD



Name (Print) ________________________________
(Last) (First)

PROBLEM # 3 (cont.)

Equilibrium eqn's

$$\Sigma M_B = -FL - wL\left(\frac{3L}{2}\right) - P(2L) = 0$$

$$\boxed{F = -2P - \frac{3}{2}wL} \qquad (1)$$

$$\Sigma F_y = B_y - F - wL - P = 0$$

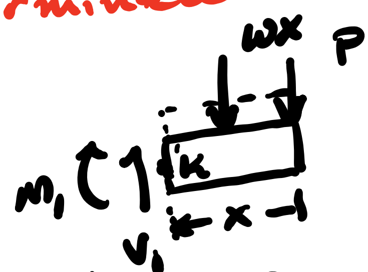
$$B_y = F + wL + P = -2P - \frac{3}{2}wL + wL + P$$

$$\boxed{B_y = -P - \frac{wL}{2}} \qquad (2)$$

Statically determinate

3 sections:

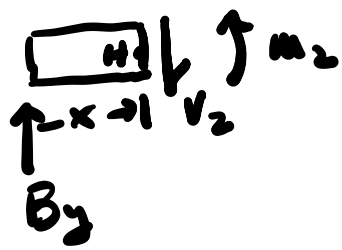
Section CD:



$$\Sigma M_K = -m_1 - wx\left(\frac{x}{2}\right) - Px = 0$$

$$\boxed{m_1(x) = -Px - \frac{wx^2}{2}}$$

Section BC



$$\Sigma M_H = m_2 - B_y x = 0$$

$$m_2 = B_y x$$

Replace B_y from (2)

$$\boxed{m_2(x) = \left(-P - \frac{wL}{2}\right)x}$$

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PROBLEM # 3 (cont.)

Member CE is loaded <u>axially</u> with constant force F

<u>Strain energy</u>: $U = U_1 + U_2 + U_3$

$$U_1 = \frac{1}{2EI}\int_0^L m_1^2\,dx$$

$$U_2 = \frac{1}{2EI}\int_0^L m_2^2\,dx$$

$$U_3 = \frac{1}{2EA}\int_0^L F^2\,dy$$

Castigliano's 2nd theorem: Deflection at D:

$$v_D = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^L m_1 \frac{\partial m_1}{\partial P}\,dx$$

$$+ \frac{1}{EI}\int_0^L m_2 \frac{\partial m_2}{\partial P}\,dx$$

$$+ \frac{1}{EA}\int_0^L F \frac{\partial F}{\partial P}\,dy$$

$$\frac{\partial m_1}{\partial P} = -x \quad \frac{\partial m_2}{\partial P} = -x \quad \frac{\partial F}{\partial P} = -2$$

$$v_D = \frac{1}{EI}\int_0^L \left(-Px - \frac{wx^2}{2}\right)(-x)\,dx$$
$$+ \frac{1}{EI}\int_0^L \left(-P - \frac{wL}{2}\right)x(-x)\,dx$$
$$+ \frac{1}{EA}\int_0^L \left(-2P - \frac{3}{2}wL\right)(-2)\,dy$$

$$v_D = \frac{1}{EI}\left[P\frac{x^3}{3} + \frac{wx^4}{8}\right]_0^L$$
$$+ \frac{1}{EI}\left[P\frac{x^3}{3} + \frac{wLx^3}{6}\right]_0^L$$
$$+ \frac{1}{EA}\left[(4P + 3wL)y\right]_0^L$$
$$= \frac{1}{EI}\left[P\frac{L^3}{3} + \frac{wL^4}{8} + \frac{PL^3}{3} + \frac{wL^4}{6}\right]$$
$$+ \frac{L}{EA}\left[4P + 3wL\right]$$

$$\boxed{v_D = \frac{1}{EI}\left[\frac{2PL^3}{3} + \frac{7}{24}wL^4\right] + \frac{L}{EA}\left[4P + 3L\right]}$$

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Examination #2
November 3rd, 2021

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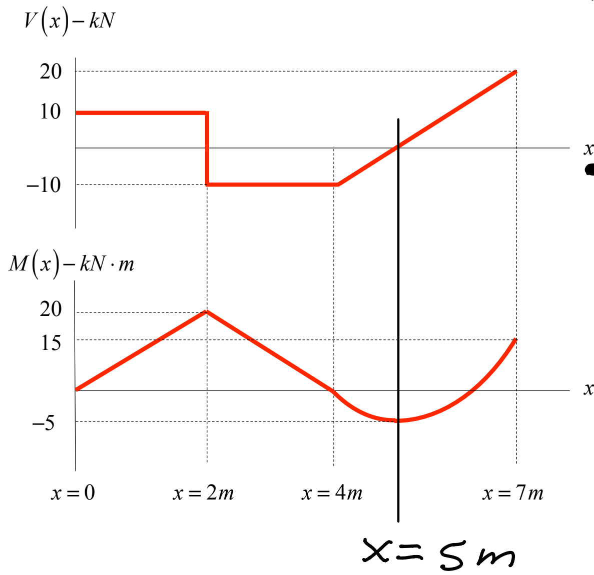
PROBLEM # 4 (25 points total – partial credit will not be granted)

Part 4A



• At $x = 2m$ & $y = \frac{h}{3}$:

$$\sigma = -\frac{(20)(h/3)}{I} = -\frac{20}{3}\frac{h}{I} \ (C)$$

• At $x = 2m$ & $y = -2h/3$:

$$\sigma = -\frac{(20)(-2h/3)}{I} = \frac{40}{3}\frac{h}{I} \ (T)$$

• At $x = 5m$ & $y = \frac{h}{3}$:

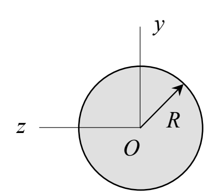
$$\sigma = -\frac{(-5)\frac{h}{3}}{I} = \frac{5}{3}\frac{h}{I} \ (T)$$

• At $x = 5m$ & $y = -\frac{2h}{3}$:

$$\sigma = -\frac{(-5)(-\frac{2h}{3})}{I} = -\frac{10}{3}\frac{h}{I} \ (C)$$

The shear force/bending moment diagrams for a loaded beam are shown above.

4.A.1 - 2 points: If the beam has the *circular* cross-section shown to the right, identify the *xy*-components of the location of a point on the beam having the *maximum magnitude shear stress*.

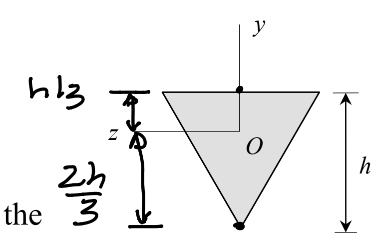


O is the centroid of the cross-section

$x =$ 7m and $y =$ 0

4.A.2 - 2 points: If the beam has the *triangular* cross-section shown to the right, identify the *xy*-components of the location of a point on the beam having the *maximum compressive normal stress*.

$x =$ 2m and $y = \frac{h}{3}$



O is the centroid of the cross-section

4.A.3 - 2 points: If the beam has the *triangular* cross-section shown to the right, identify the *xy*-components of the location of a point on the beam having the *maximum tensile normal stress*.

$x =$ 2m and $y = -\frac{2h}{3}$

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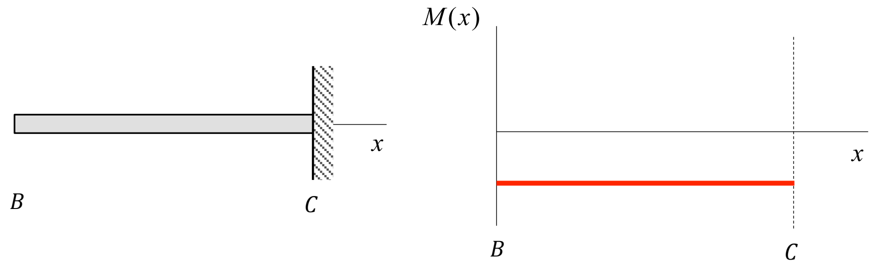
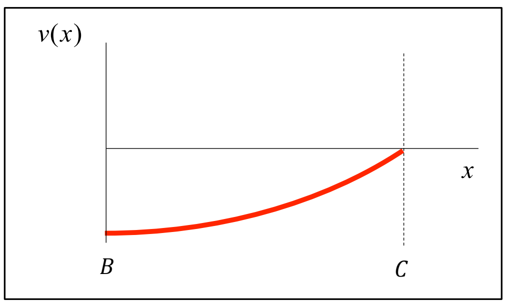
Part 4B - 3 points

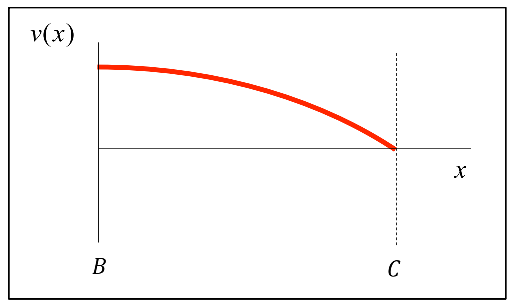


The bending moment $M(x)$ in a cantilevered beam is shown above. (Note that the actual loading on the beam is not known.) Circle the plot of $v(x)$ below that most closely represents the deflection of this cantilevered beam.

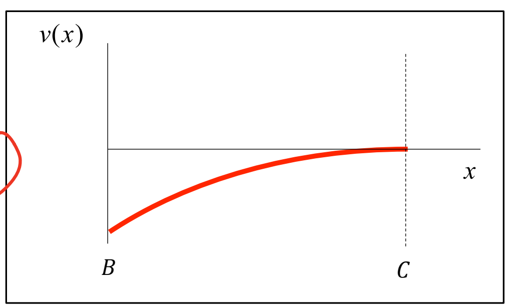
(a)



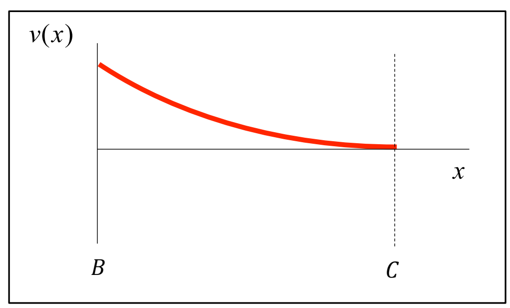
(b)



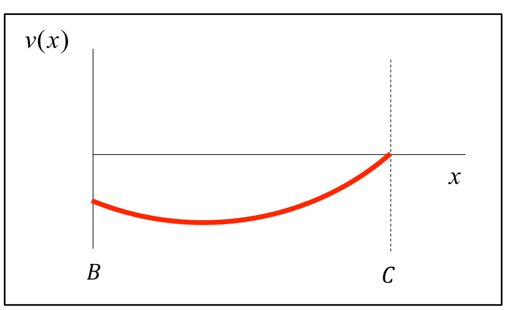
(c)



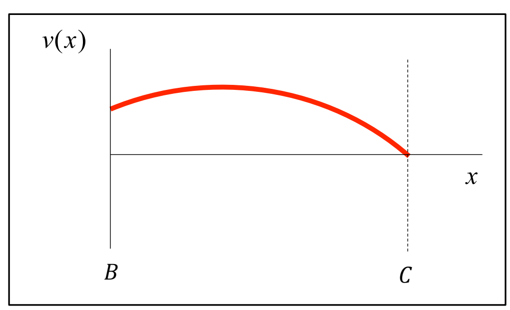
(d)



(e)



(f)



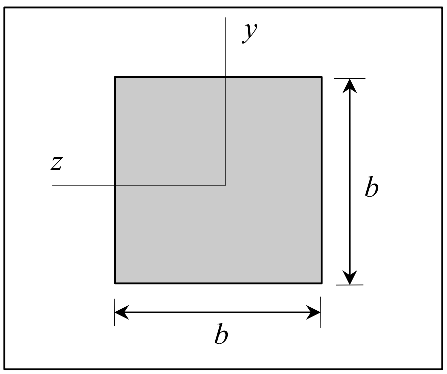
$$\left.\begin{array}{l} V_c = 0 \\ \theta_c = 0 \end{array}\right\} \quad \left\{ \frac{d\theta}{dx} = \frac{1}{EI} M < 0 \right.$$ over entire length of beam

$\Rightarrow$ (c)

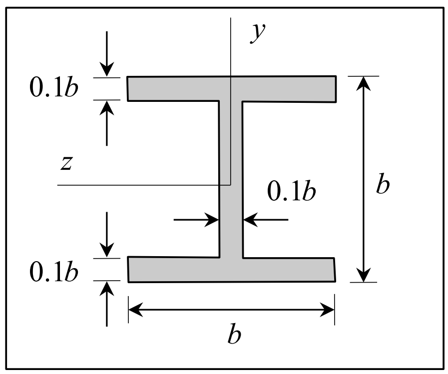
Part 4C

A cantilevered beam is loaded by a concentrated couple M_0 at end C. Consider three possible cross-sections for this beam shown below:

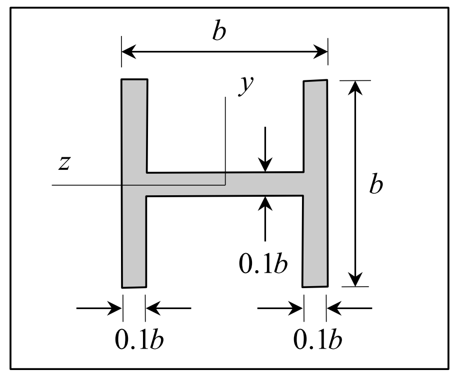
cross-section (a)



cross-section (b)



cross-section (c)



$I_a > I_b > I_c$ & $|\sigma| = \frac{|M||y|}{I}$

4.C.1 - 2 points: Which cross-section above produces the *largest maximum magnitude normal stress* in the beam? Circle your response below.

a) Cross-section (a)
b) Cross-section (b)
c) Cross-section (c)
d) All three cross-sections produce the same maximum magnitude normal stress.

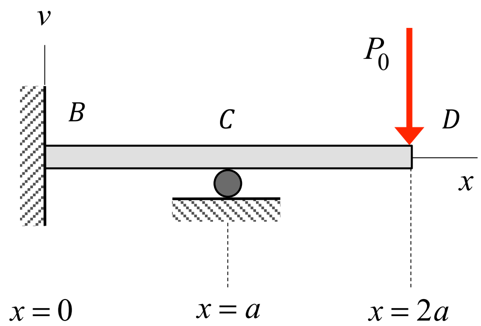
I_c is smallest $\Rightarrow |\sigma| =$ largest

4.C.2 - 2 points: Which cross-section above produces the *smallest maximum normal stress* in the beam? Circle your response below.

a) Cross-section (a)
b) Cross-section (b)
c) Cross-section (c)
d) All three cross-sections produce the same maximum magnitude normal stress.

I_a is largest $\Rightarrow |\sigma| =$ smallest

Part 4D



A propped-cantilevered beam is loaded with a point load P_0 at end D. Let $V(x)$, $M(x)$ and $v(x)$ represent the shear force, bending moment and deflection, respectively, along the length of the beam.

4.D.1 - 2 points: Which of the following are *zero* at $x = 0$? Circle your responses.

$$V(0)\overset{?}{=}0; \quad M(0)\overset{?}{=}0; \quad \boxed{\frac{dv}{dx}(0)\overset{?}{=}0;} \quad \boxed{v(0)\overset{?}{=}0;}$$

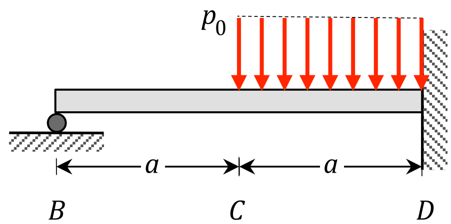
4.D.2 - 2 points: Which of the following are *zero* at $x = a$? Circle your responses.

$$V(a)\overset{?}{=}0; \quad M(a)\overset{?}{=}0; \quad \frac{dv}{dx}(a)\overset{?}{=}0; \quad \boxed{v(a)\overset{?}{=}0;}$$

4.D.3 - 2 points: Which of the following are *zero* at $x = 2a$? Circle your responses.

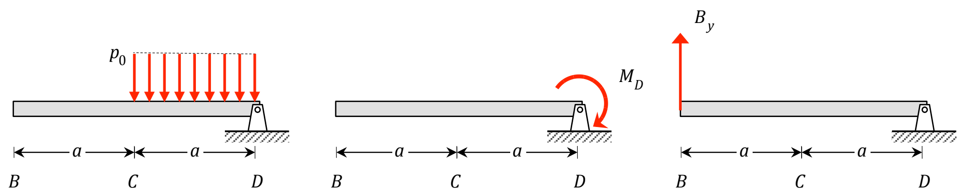
$$V(2a)\overset{?}{=}0; \quad \boxed{M(2a)\overset{?}{=}0;} \quad \frac{dv}{dx}(2a)\overset{?}{=}0; \quad v(2a)\overset{?}{=}0;$$

Part 4E



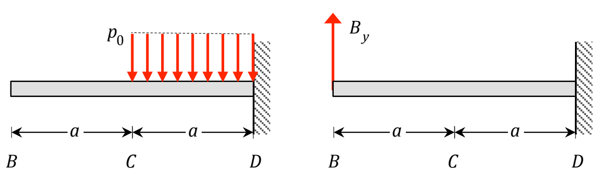
It is desired to determine the reactions acting on the indetermintate beam above using the principle of superposition.

4.E.1 - 1 point: *TRUE* or *FALSE*, the following set of three beam problems can be used with superposition to solve the problem above.

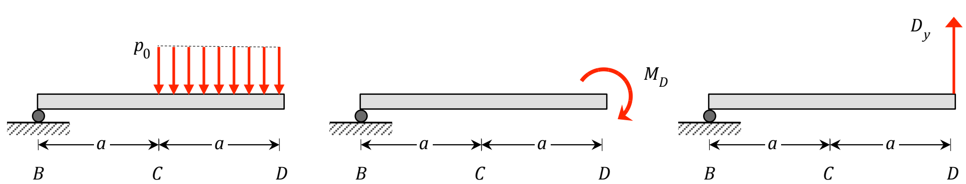


Each problem must be in equilibrium

4.E.2 - 1 point: *TRUE* or *FALSE*, the following set of two beam problems can be used with superposition to solve the problem above.

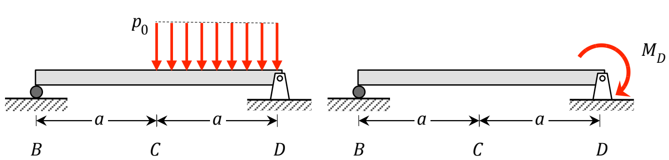


4.E.3 - 1 point: *TRUE* or *FALSE*, the following set of three beam problems can be used with superposition to solve the problem above.

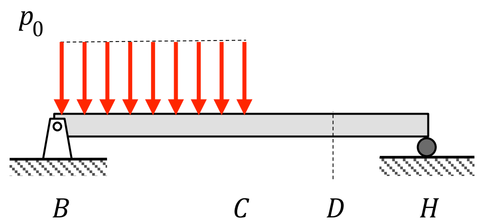


Each problem must be in equilibrium

4.E.4 - 1 point: *TRUE* or *FALSE*, the following set of two beam problems can be used with superposition to solve the problem above.



Part 4F – 2 point

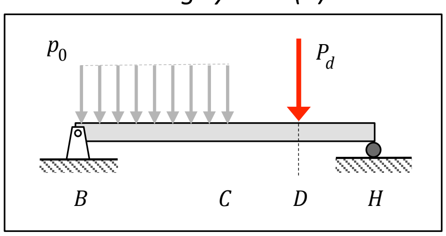


It is desired to use Castigliano's method to determine the vertical displacement of the beam at location D as a result of the applied loading shown. With this approach, a strain energy function for the beam needs to be developed. Which of the loading systems below is needed for establishing this strain energy?

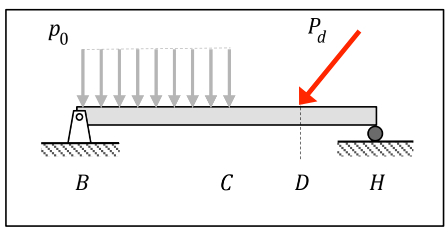
a) loading system (a)
b) loading system (b)
c) loading system (c)
d) the original loading system above

Dummy force must be in the direction of desired displacement.

loading system (a)



loading system (b)



loading system (c)