

Useful Equations

Bending deformation:

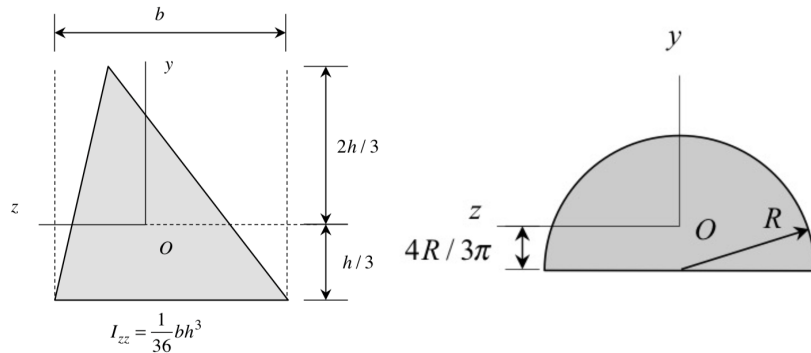
$$\frac{dV}{dx} = w(x) \quad \frac{dM}{dx} = V(x) \quad M = EIv'' \quad \Delta V = P \quad \Delta M = -M_0$$

$$\sigma(x, y) = \frac{-Ey}{\rho} = \frac{-M_{zz}y}{I_{zz}} \quad I_{zz} = \frac{bh^3}{12} \text{ (rectangle)}, \quad I_{zz} = \frac{\pi r^4}{4} \text{ (circle)}$$

$$\tau(x, y) = \frac{VQ}{I_{zz}t} = \frac{VA^*y^*}{I_{zz}t}$$

$$\tau_{\max} = \frac{3V}{2A} \text{ (rectangle)},$$

$$\tau_{\max} = \frac{4V}{3A} \text{ (circle)}$$



Strain energy density:

$$\bar{u} = \frac{1}{2} [\sigma_x(\epsilon_x - \alpha\Delta T) + \sigma_y(\epsilon_y - \alpha\Delta T) + \sigma_z(\epsilon_z - \alpha\Delta T) + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}]$$

Energy methods:

$$U = \frac{1}{2} \int_0^L \frac{F^2(x)}{EA} dx \quad U = \frac{1}{2} \int_0^L \frac{f_s V^2(x)}{GA} dx \quad U = \frac{1}{2} \int_0^L \frac{M^2(x)}{EI} dx \quad U = \frac{1}{2} \int_0^L \frac{T^2(x)}{GI_p} dx$$

Work-energy principle: $U = W$

Castigliano's 2nd theorem:

$$\delta_{P_i} = \frac{\partial U}{\partial P_i} \quad \theta_{M_i} = \frac{\partial U}{\partial M_i} \quad \phi_{T_i} = \frac{\partial U}{\partial T_i}$$

$$\delta_{P_i} = \int_0^L \frac{M(x)}{EI} \frac{\partial M(x)}{\partial P_i} dx + \int_0^L \frac{F(x)}{EA} \frac{\partial F(x)}{\partial P_i} dx + \int_0^L \frac{T(x)}{GI_p} \frac{\partial T(x)}{\partial P_i} dx + \int_0^L \frac{f_s V(x)}{AG} \frac{\partial V(x)}{\partial P_i} dx$$

$$\theta_{M_i} = \int_0^L \frac{M(x)}{EI} \frac{\partial M(x)}{\partial M_i} dx + \int_0^L \frac{F(x)}{EA} \frac{\partial F(x)}{\partial M_i} dx + \int_0^L \frac{T(x)}{GI_p} \frac{\partial T(x)}{\partial M_i} dx + \int_0^L \frac{f_s V(x)}{AG} \frac{\partial V(x)}{\partial M_i} dx$$

$$\phi_{T_i} = \int_0^L \frac{M(x)}{EI} \frac{\partial M(x)}{\partial T_i} dx + \int_0^L \frac{F(x)}{EA} \frac{\partial F(x)}{\partial T_i} dx + \int_0^L \frac{T(x)}{GI_p} \frac{\partial T(x)}{\partial T_i} dx + \int_0^L \frac{f_s V(x)}{AG} \frac{\partial V(x)}{\partial T_i} dx$$

$f_s = 6/5$ (rectangular cross section), $f_s = 10/9$ (circular cross section)