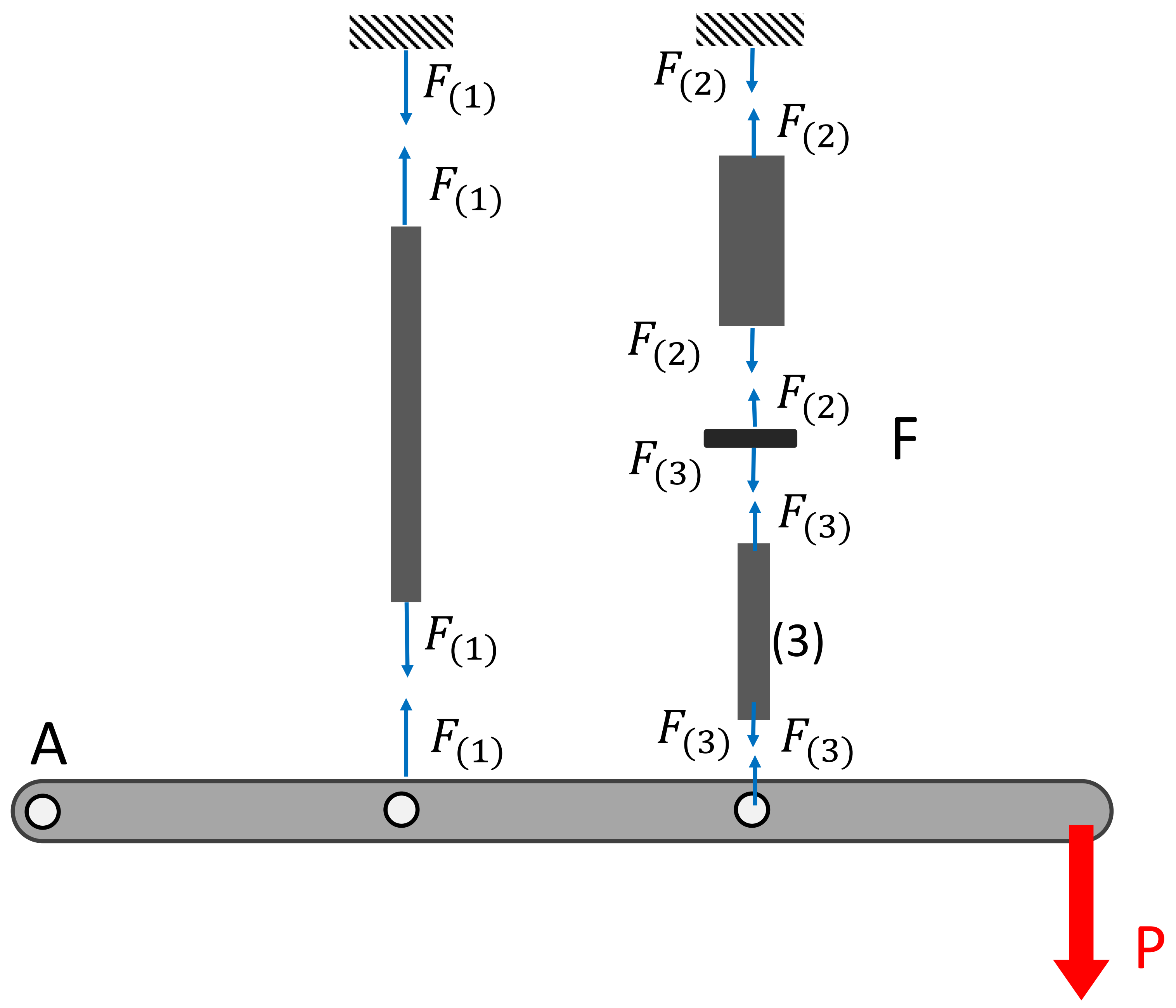
1. Free Body Diagram



2. Force Balances

$$\sum F_y = F_{(2)} - F_{(3)} = 0$$

$$F_{(2)} = F_{(3)} \qquad (1)$$

$$\sum M_A = F_{(1)}L + F_{(3)}(2L) - P(3L) = 0 \qquad (5)$$

2 equations; 3 unknowns
→ indeterminate

3. Force-Elongation

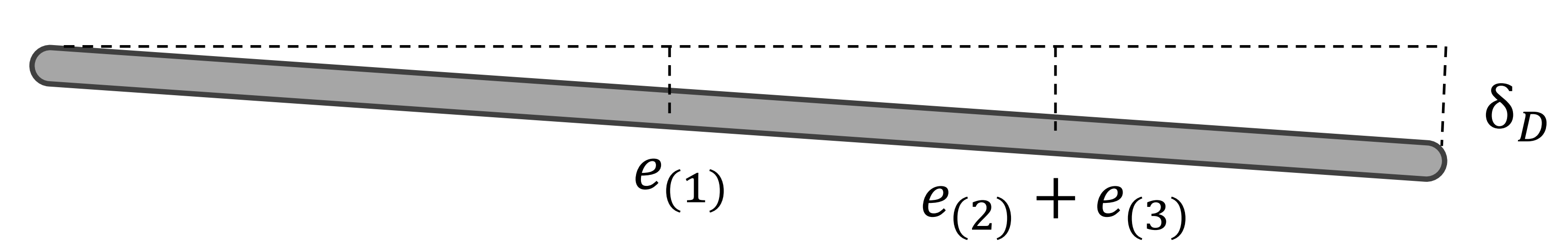
(1) $$e_{(1)} = \frac{F_{(1)}L_{(1)}}{EA_{(1)}} = \frac{F_{(1)}(2L)}{E\pi\left(\frac{d}{2}\right)^2} = \frac{8F_{(1)}}{E\pi d^2}$$

(1) $$e_{(2)} = \frac{F_{(2)}L_{(2)}}{EA_{(2)}} = \frac{F_{(3)}(L)}{E\pi\left(\frac{2d}{2}\right)^2} = \frac{F_{(3)}}{E\pi d^2}$$

(2)

(1) $$e_{(3)} = \frac{F_{(3)}L_{(3)}}{EA_{(3)}} = \frac{F_{(3)}(L)}{E\pi\left(\frac{d}{2}\right)^2} = \frac{4F_{(3)}}{E\pi d^2}$$

4. Compatibility



Similar triangles

$$\frac{e_{(1)}}{L} = \frac{e_{(2)} + e_{(3)}}{2L} \qquad (5)$$

5. Solve

$$2e_{(1)} = e_{(2)} + e_{(3)}$$

$$2\frac{8F_{(1)}}{E\pi d^2} = \frac{F_{(3)}}{E\pi d^2} + \frac{4F_{(3)}}{E\pi d^2}$$

$$16F_{(1)} = 5F_{(3)}$$

$$F_{(1)} = \left(\frac{5}{16}\right)F_{(3)}$$

$$F_{(1)}L + F_{(3)}(2L) - P(3L) = 0$$

$$\left(\frac{5}{16}\right)F_{(3)} + \left(\frac{32}{16}\right)F_{(3)} = 3P$$

$$F_{(3)} = \left(\frac{3 * 16}{37}\right)P = \left(\frac{48}{37}\right)2000 = 2595\ N$$

(2)

a. Stresses

$$\sigma_{(2)} = \frac{F_{(2)}}{A_{(2)}} = \frac{2595\ N}{\pi\left(\frac{2d}{2}\right)^2} = 2.06\ MPa$$

(2)

$$\sigma_{(3)} = \frac{F_{(3)}}{A_{(3)}} = \frac{2595\ N}{\pi\left(\frac{d}{2}\right)^2} = 8.26\ MPa$$

b. Displacement at D

$$\frac{e_{(1)}}{L} = \frac{\delta_D}{3L} \qquad (3)$$

$$\delta_{(D)} = 3e_{(1)}$$

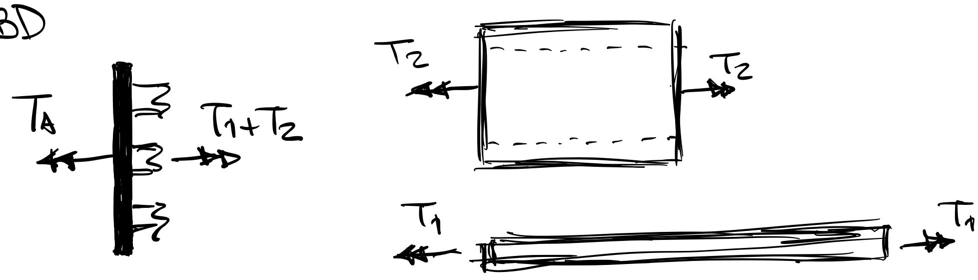
$$F_{(1)} = \left(\frac{5}{16}\right)F_{(3)} = 811\ N$$

(2)

$$3e_{(1)} = 3\frac{8F_{(1)}}{E\pi d^2} = \frac{24(811)}{E\pi d^2} = -1.55\ mm$$

- Assume all members under positive torque

- FBD



- Equilibrium $\oplus \rightarrow \quad \Sigma T = 0 = T_1 + T_2 - T_A \Rightarrow$ Statically Indeterminate

- Torque-twist behavior

$$\phi_1 = \frac{T_1 L}{G_1 I_{p1}} \qquad \phi_2 = \frac{T_2 L/2}{G_2 I_{p2}}$$

$$I_{p1} = \frac{\pi (d/2)^4}{2} = \frac{\pi d^4}{32}$$

$$I_{p2} = \frac{\pi\left[(3d/2)^4 - (2d/2)^4\right]}{2} = \frac{65}{32}\pi d^4$$

- Compatibility conditions

$$\phi_1 = \phi_A - \phi_C \quad ; \quad \phi_2 = \phi_A - \phi_B \quad ; \quad \phi_C = \phi_B = 0$$

$$\Rightarrow \phi_1 = \phi_2$$

if $G_1 = 130\,G$, $G_2 = G$

- Solve $\{T_1, T_2\}$

$$T_1 + T_2 - T_A = 0$$

$$\frac{T_1 L}{G_1 \pi d^4} \times 32 = \frac{T_2 L}{G_2 \pi d^4} \times \frac{16}{65} \Rightarrow \frac{T_1}{G_1} = \frac{T_2}{G_2} \times \frac{1}{130}$$

$$\Rightarrow T_2 \frac{G_1}{G_2} \times \frac{1}{130} + T_2 = T_A$$

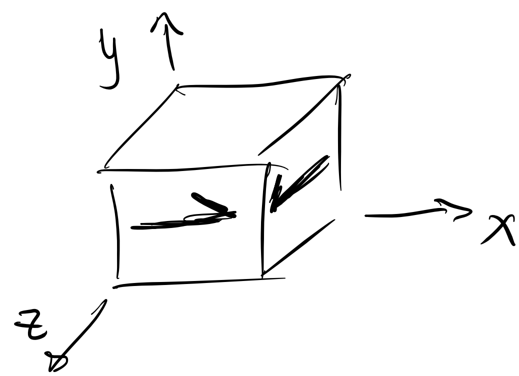
$\Rightarrow \quad 2T_2 = T_A \Rightarrow T_2 = T_A/2 \qquad \Rightarrow T_1 = T_A/2$

- Stress in shaft (1) and (2).

$$\tau = \frac{3d/2 \cdot T_A/2}{65\pi d^4/32} = \frac{3 \times 32}{4 \times 65} \frac{T_A}{\pi d^3} = \frac{24}{65} \frac{T_A}{\pi d^3}$$

$$\tau = \frac{d/2 \; T_A/2}{\pi d^4/32} = \frac{8\,T_A}{\pi d^3}$$ → outer radius of shaft (1).

- Stress @ M $\quad \tau_{xz} > 0$

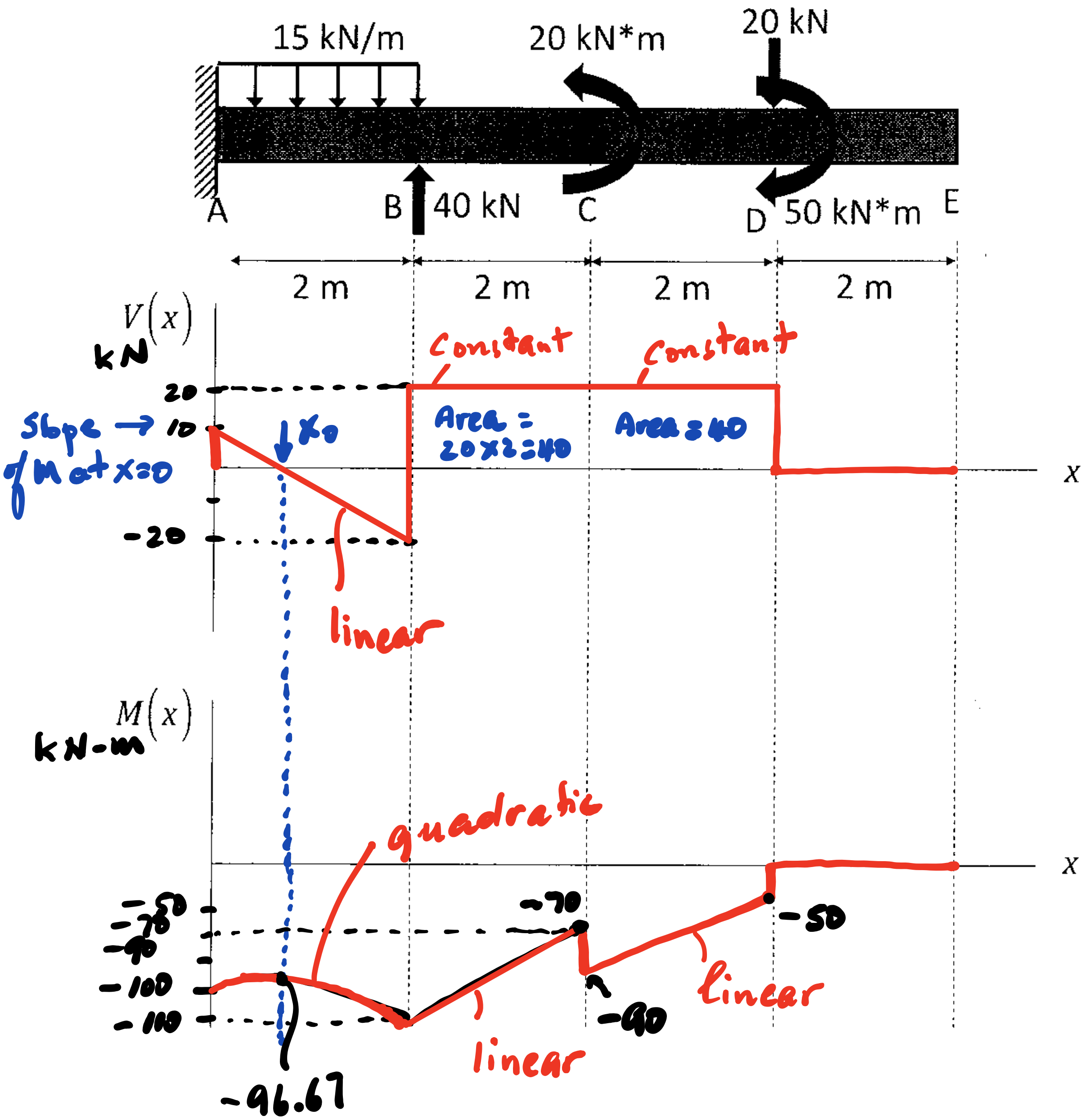


ME 323 – Mechanics of Materials
Examination #1
October 6th, 2021

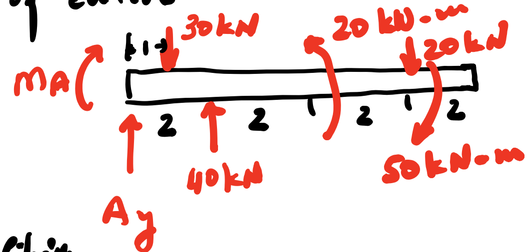
PURDUE UNIVERSITY | Mechanical Engineering

Name (Print) SOLUTION
(Last) (First)

PROBLEM # 3 (cont.)



FBD of entire beam:



Equilibrium

$\Sigma F_y = A_y + 40 - 30 - 20 = 0$ $\quad \boxed{A_y = 10\text{ kN}}$

$\Sigma M_A = -M_A - 30(1) + 40(2) + 20 - 20(6) - 50 = 0$

$\boxed{M_A = -100\text{ kN-m}}$

For $0 < x < 2$

$V(x) = V(0) + \int_0^x p(x)\,dx$

$= 10 - 15x$

$V(x_0) = 0 = 10 - 15x_0$ $\quad \boxed{x_0 = 0.667\text{ m}}$

$M(x) = M(0) + \int_0^x (10 - 15x)\,dx$

$= -100 + 10x - \frac{15x^2}{2}$

$M(x_0) = M(0.667) = -100 + 6.667 - 3.337$

$= \boxed{-96.67\text{ kN-m}}$

$M(2) = -100 + 20 - 30 = \boxed{-110\text{ kN-m}}$

For $2 < x < 4$

$V(2)^- = 10 - 15(2) = -20\text{ kN}$

$V(2)^+ = -20 + 40 = 20\text{ kN}$

$V(4)^- = 20\text{ kN}$

$M(4)^- = -110 + 40 = -70\text{ kN}\cdot\text{m}$

$M(4)^+ = -70 \underbrace{-20}_{M_c > 0} = -90\text{ kN-m}$

For $4 < x < 6$

$$V(4) = 20\text{ kN}$$

$$V(6)^- = 20\text{ kN}$$

$$V(6)^+ = 20 - 20 = 0$$

$$M(6)^- = -90 + 40 = -50\text{ kN-m}$$

$$M(6)^+ = -50 \underbrace{+50}_{M_D < 0} = 0$$

SOLUTION

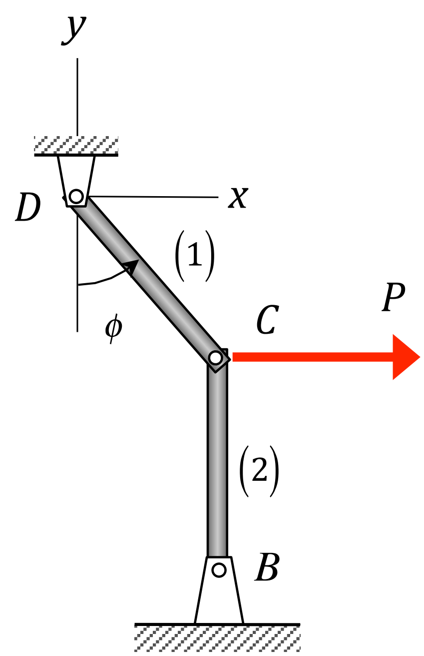
Problem No. 4

Part 4A

Consider the truss shown that is made up of identical elements (1) and (2), with each element have a Young's modulus of E, a length L and cross-sectional area of A. A horizontal load P acting to the right of joint C, element (2) is vertical and $\phi < 45°$.

Let e_1 and e_2 represent the elongations of elements (1) and (2), respectively, and F_1 and F_2 be the corresponding loads (forces) carried by the elements.

Circle the correct responses below:



2 points:

a) $|F_1| < P$

b) $|F_1| = P$

c) $|F_1| > P$

2 points:

a) $|F_2| < P$

b) $|F_2| = P$

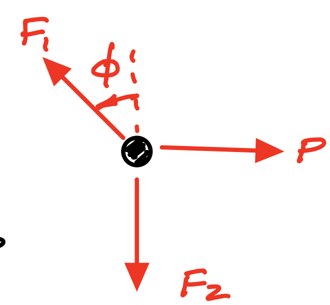
c) $|F_2| > P$

2 points:

a) $|e_2| < |e_1|$

b) $|e_2| = |e_1|$

c) $|e_2| > |e_1|$



- $\sum F_x = P - F_1 \sin\phi = 0$

  $\hookrightarrow \quad F_1 = \dfrac{P}{\sin\phi} > P$

- $\sum F_y = F_1 \cos\phi - F_2 = 0$

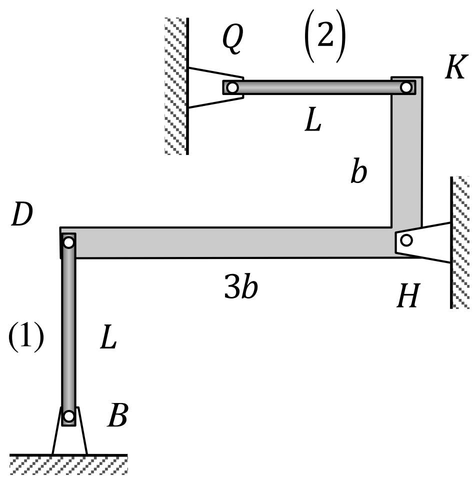
  $\hookrightarrow \quad F_2 = F_1 \cos\phi$

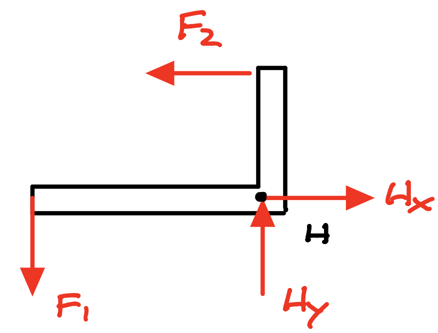
  $= P \cot\phi > P$

- $\dfrac{e_2 EA}{L} = \dfrac{e_1 EA}{L} \cos\phi \Rightarrow e_2 = e_1 \cos\phi < e_1$

Part 4B

The rigid, L-shaped bar DHK is pinned to ground at H, and identical elastic links (1) and (2) are connected between D and B, and between Q and K, respectively. Links (1) and (2) are vertical and horizontal, respectively. The temperature of link (2) is raised by an amount of ΔT, whereas the temperature of link (1) is held constant. Let ε_1 and ε_2 be the axial strains in (1) and (2), respectively, and σ_1 and σ_2 be the axial stresses in (1) and (2), respectively.





Circle the correct responses below:

2 points:

a) $|\sigma_1| > |\sigma_2|$

b) $|\sigma_1| = |\sigma_2|$

c) $|\sigma_1| < |\sigma_2|$

2 points:

a) σ_1 and ε_1 have the same signs

b) σ_1 and ε_1 are both zero

c) σ_1 and ε_1 have different signs

2 points:

a) σ_2 and ε_2 have the same signs

b) σ_2 and ε_2 are both zero

c) σ_2 and ε_2 have different signs

- $\sum M_H = F_2(b) + F_1(3b) = 0$

  ↳ $|F_2| = 3|F_1| \Rightarrow |\sigma_2| = 3|\sigma_1|$

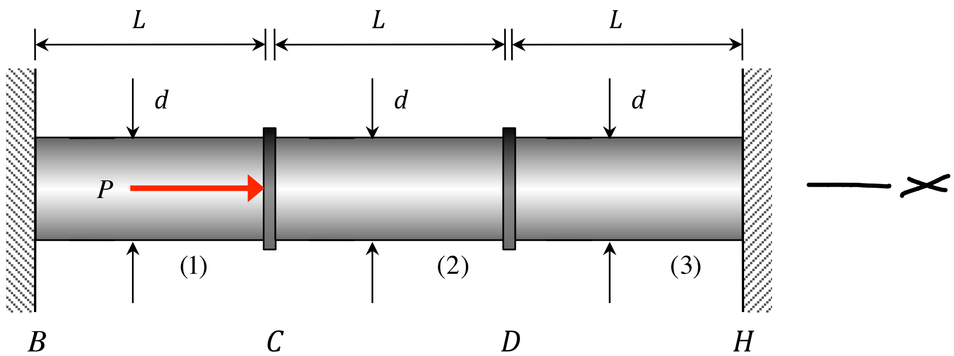
- $\varepsilon_1 = \dfrac{\sigma_1}{E}$

- $\varepsilon_2 = \dfrac{\sigma_2}{E} + \alpha \Delta T$

  As (2) is heated, $\varepsilon_2 > 0$. With $\varepsilon_2 > 0$, (2) becomes compressed $\Rightarrow \sigma_2 < 0$

Part 4C

A rod is made up of solid, circular cross-sectioned elements (1), (2) and (3), with (1) and (2) joined with a rigid connector C, and (2) and (3) joined by rigid connector D. All three elements are made of the same type of steel, having a Young's modulus of E_{steel}. A load P acts in the axial direction on connector C. Let F_1, F_2 and F_3 be the axial load (force) carried by, and σ_1, σ_2 and σ_3 be the axial stresses in elements (1), (2) and (3), respectively.



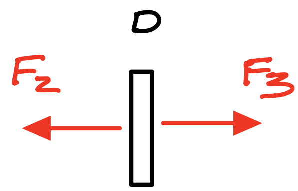
Circle the correct responses below:

2 points:

a) $|F_2| > |F_3|$

b) $|F_2| = |F_3|$

c) $|F_2| < |F_3|$



$$\sum F_x = -F_2 + F_3 = 0$$

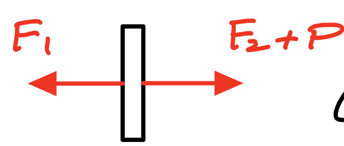
$$\hookrightarrow F_2 = F_3$$

2 points:

a) $|F_1| > |F_2|$

b) $|F_1| = |F_2|$

c) $|F_1| < |F_2|$



$$e_1 + e_2 + e_3 = 0$$

$$\hookrightarrow \frac{F_1 \cancel{L}}{\cancel{EA}} + \frac{F_2 \cancel{L}}{\cancel{EA}} + \frac{F_3 \cancel{L}}{\cancel{EA}} = 0$$

$$\hookrightarrow F_1 = -F_2 - F_3 = -2F_2$$

2 points: Suppose the material of element (3) is changed to aluminum having a Young's modulus $E_{aluminum}$, where $E_{steel} > E_{aluminum}$. With this change in material:

a) $|\sigma_1|$ is increased

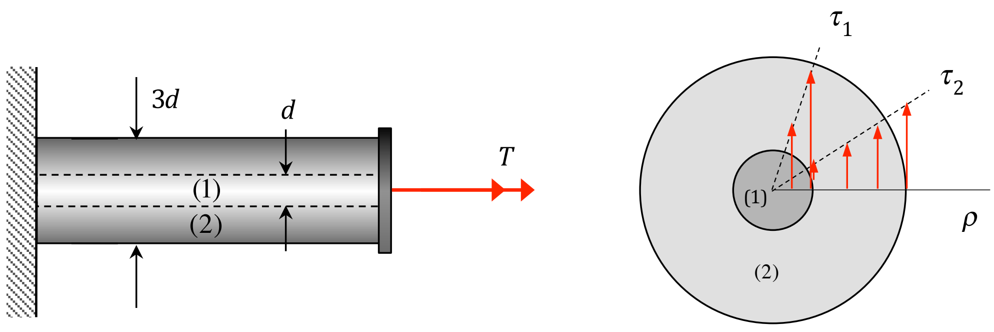
b) $|\sigma_1|$ is unchanged

c) $|\sigma_1|$ is decreased

As E_3 is decreased, (3) becomes less stiff $\Rightarrow |F_1|$ increases $\Rightarrow |\sigma_1|$ increases

Part 4D

A composite shaft is made up a tubular shell (1) and a core (2), where the shear moduli of (1) and (2) are G_1 and G_2, respectively. Let τ_1 and τ_2 represent the shear stress on the shaft cross-section for (1) and (2), respectively.



enlarged view of cross-section

Circle the correct responses below:

2 points: For the shear stress distribution on the shaft cross-section shown above:

a) $G_1 > G_2$

b) $G_1 = G_2$

c) $G_1 < G_2$

Since slope of τ_1 vs. ρ is greater than slope of τ_2 vs. ρ, $G_1 > G_2$

2 points: For a different set of materials for the shell and core, it is known that $G_2 = 3G_1$. At what location ρ (the radial distance from the shaft center) does the maximum magnitude of shear stress $|\tau|_{max}$ in the shaft cross-section occur?

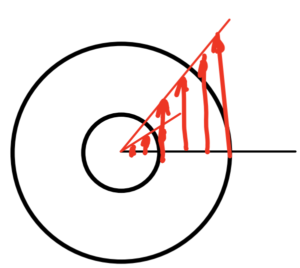
a) $\rho < d/2$

b) $\rho = d/2$

c) $d/2 < \rho < 3d/2$

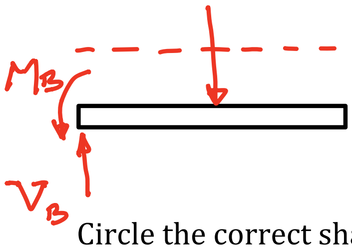
d) $\rho = 3d/2$

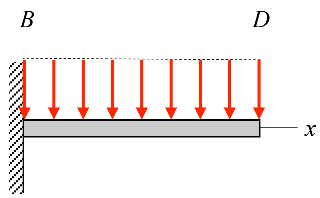
With $G_2 > G_1$, the max. magnitude shear stress occurs on outer surface

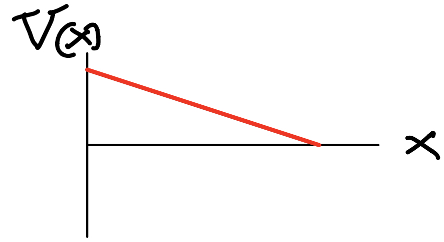


Part 4E – 2 points

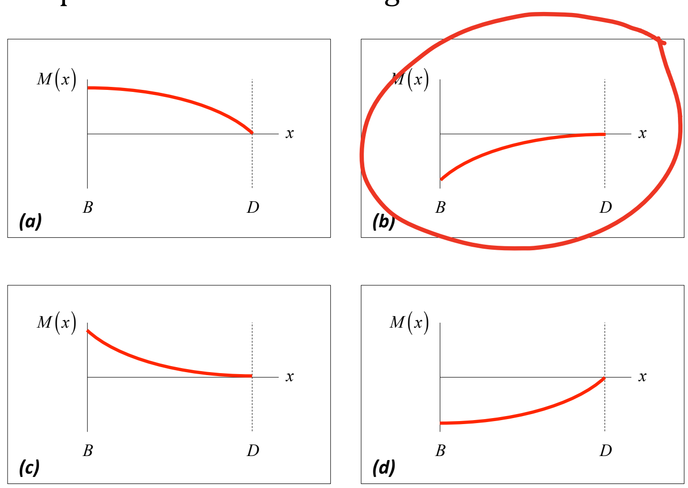
Consider the cantilevered beam that is experiencing a line load (force/length) that is constant over the length of the beam.







Circle the correct shape below for the bending moment distribution along the beam:



$V_D = 0 = \left(\frac{dM}{dx}\right)_D$

$M_D = 0$

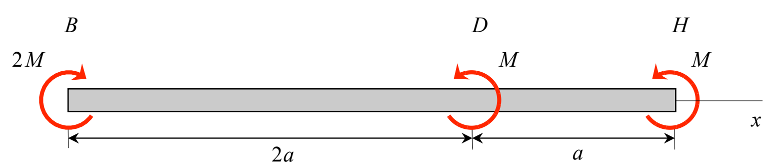
$M_B < 0$

$V_B > 0 \Rightarrow$

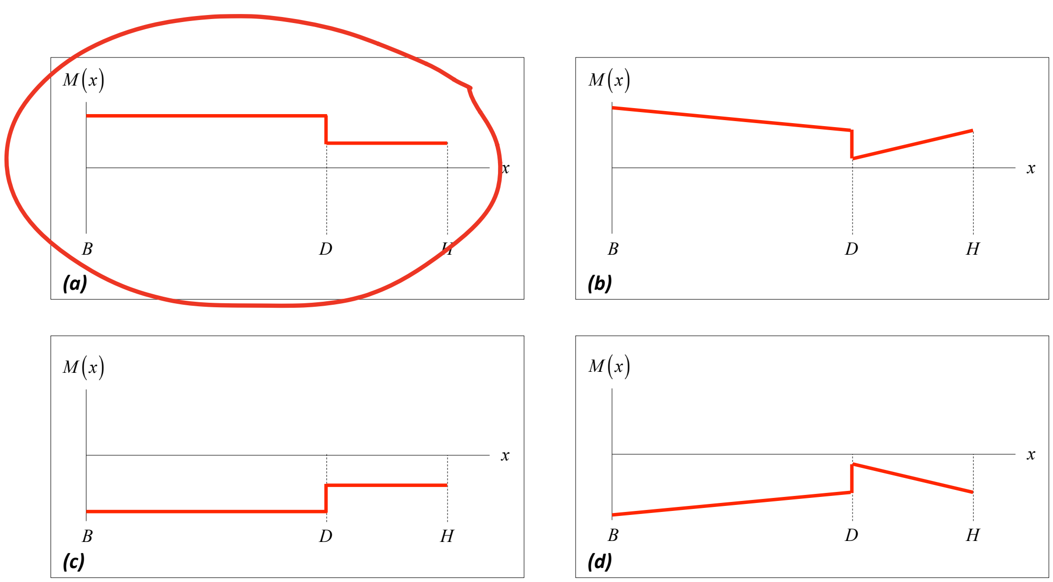
$\left(\frac{dM}{dx}\right)_B > 0$

Part 4F – 1 point

Consider the beam below that is acted upon by three bending couples.



Circle the correct shape below for the bending moment distribution along the beam:



- Jump down in M at D
- M = const across BD and DH