ME323 LECTURE 13

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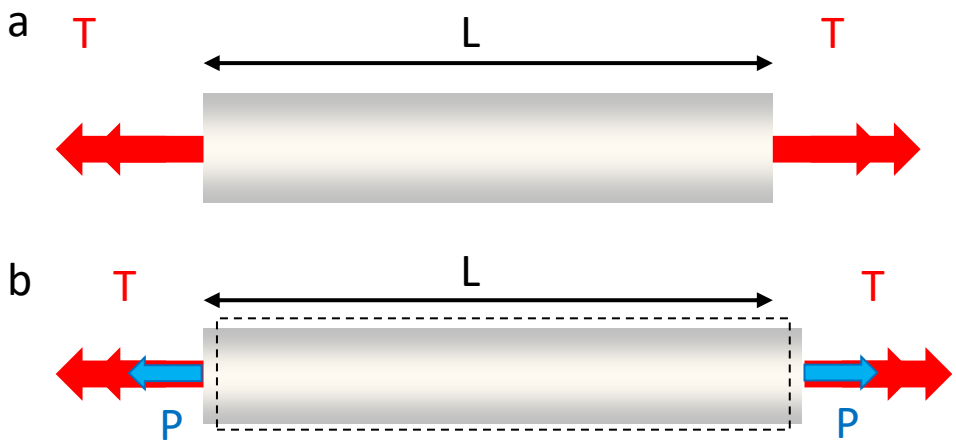
Quiz 2: Axial + Torque

A cylindrical bar of polymer has a shear modulus (G) of 75 MPa, a Young's modulus (E) of 210 MPa, and a Poisson's ratio of 0.40. The length is 0.5 m and the diameter is 10 mm.

a) A torque (T) of 1 N*m is applied to the bar. What is the τ_{max}?

$$\tau = \frac{T\rho}{I_p} = 1 * \frac{0.005}{\left(\frac{\pi}{2}\right)(0.005)^4} = 5.09\ MPa$$

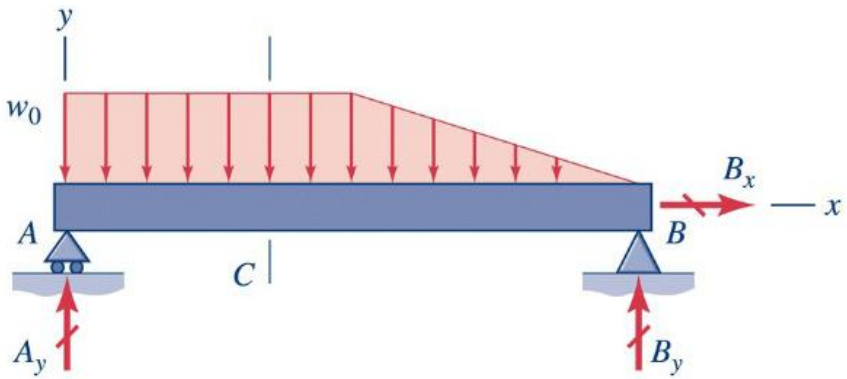


b) A tensile stress of 4 MPa is applied to the bar. What is the new τ_{max}? You can assume that the effect of tensile stress and the effect of torque are independent (linear superposition).

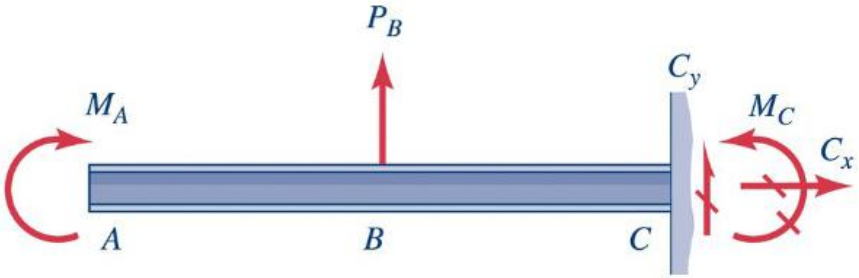
$$\varepsilon_y = \varepsilon_z = \varepsilon_r = \left(\frac{1}{E}\right)\left[\sigma_y - \upsilon(\sigma_x + \sigma_z)\right] + \alpha\Delta T = -0.4 * \frac{4\ MPa}{210\ MPa}$$

$$\varepsilon_r = \frac{\rho - \rho_o}{\rho_o} \qquad \rho = 4.96\ mm \qquad \tau_{max} = 1 * \frac{0.00496}{\left(\frac{\pi}{2}\right)(0.00496)^4} = 5.22\ MPa$$

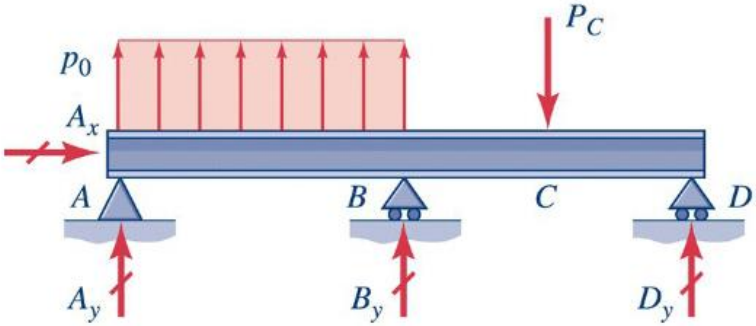
Types of Beam Supports



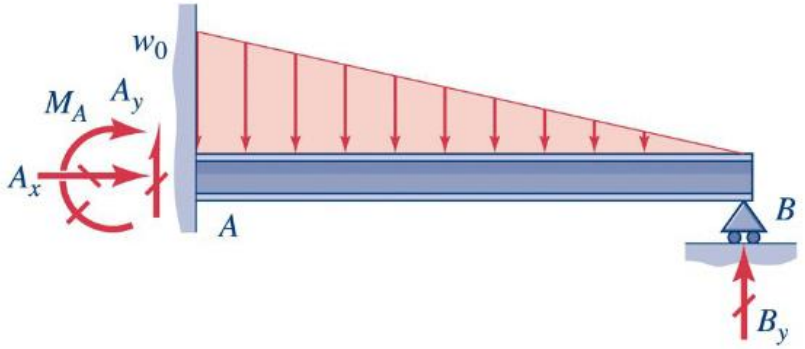
(*a*) A simply supported beam with distributed load.



(*b*) A cantilever beam with a couple at *A* and a concentrated force at *B*.

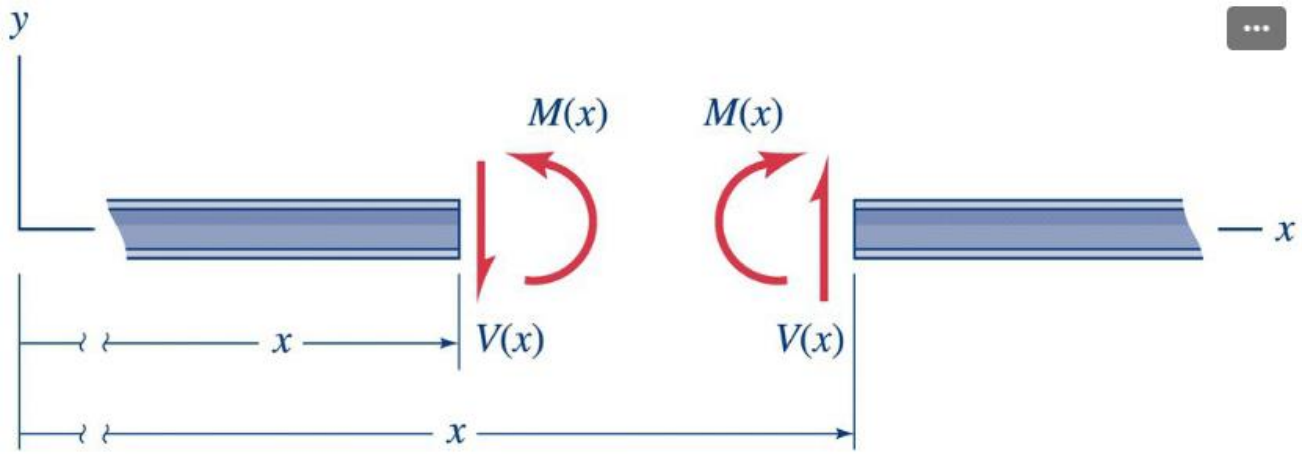


(*c*) A two-span continuous beam.

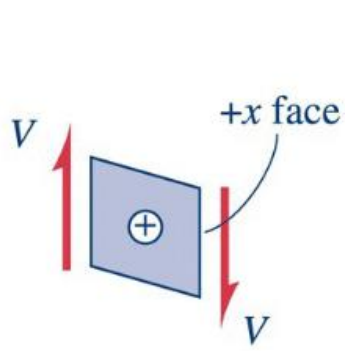


(*d*) A propped cantilever beam.

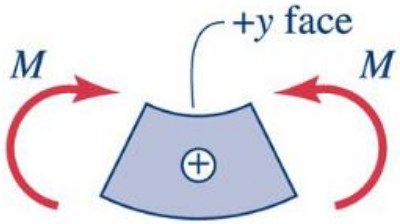
Sign Conventions for Resultants



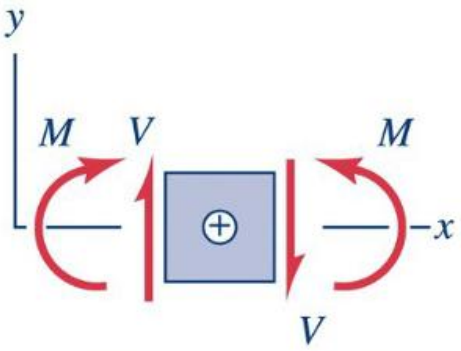
(*a*) Positive *V* and *M* on section "*x*."



(*b*) Positive shear.



(*c*) Positive moment.

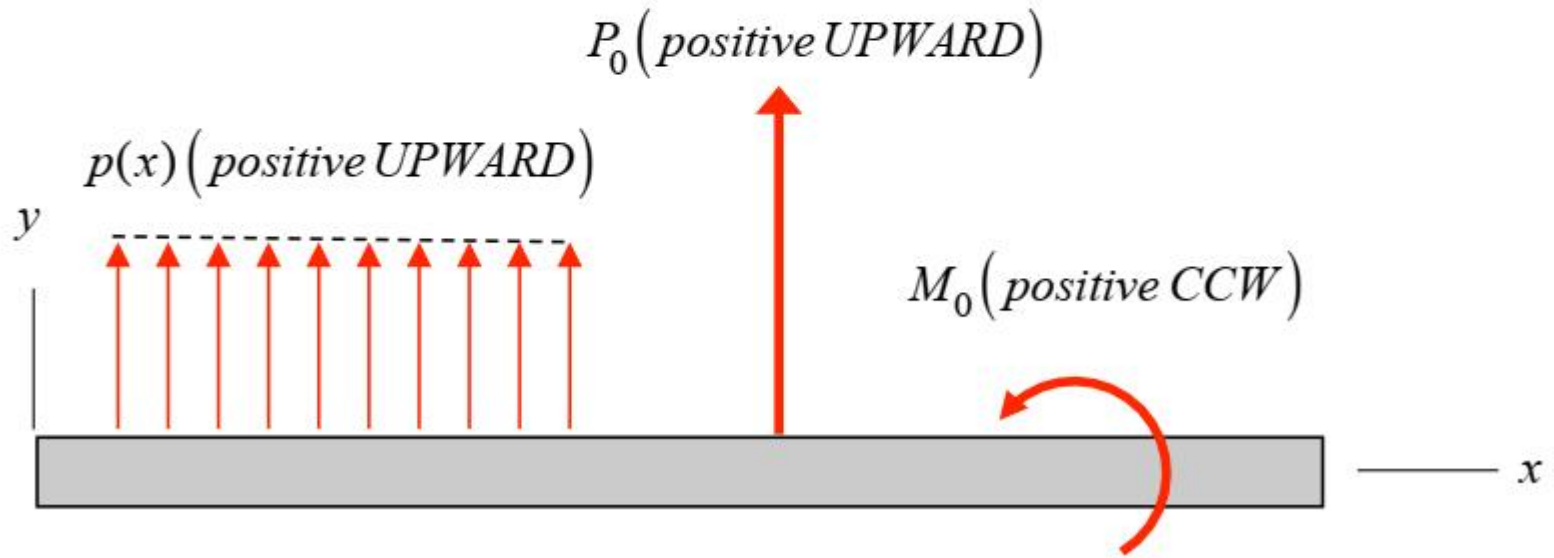


(*d*) Positive *V* and *M*.

Sign Conventions for External Loads

- Positive *EXTERNAL* *distributed loads* p(x) and *EXTERNAL* *concentrated loads* P_0 act in the "+" y-direction:

- Positive *EXTERNAL* *couples* are in the "+" z-direction (CCW by the right hand rule):



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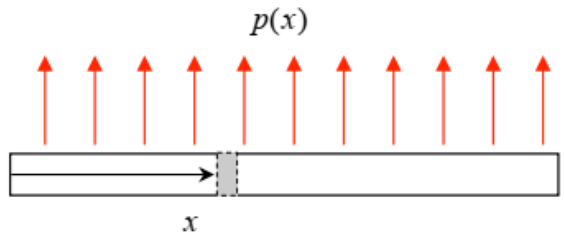
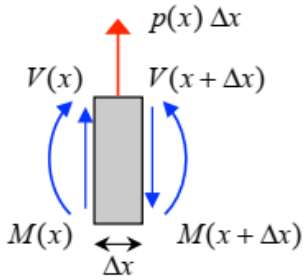
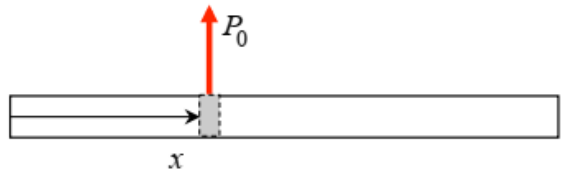
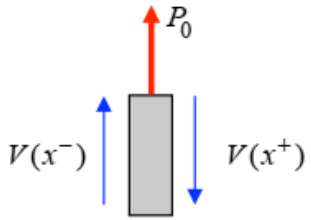
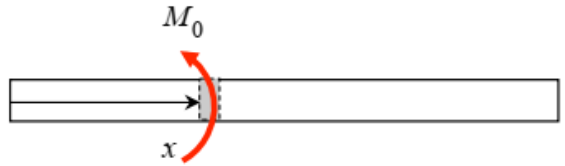
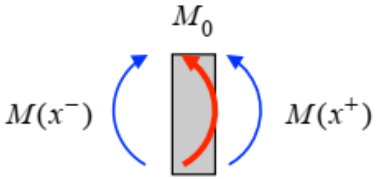
Key Relationships

applied loading	*FBD*	*key relationship(s)*
		$\frac{dV}{dx} = p(x)$ $\frac{dM}{dx} = V(x)$
		$V\left(x^+\right) = V\left(x^-\right) + P_0$
		$M\left(x^+\right) = M\left(x^-\right) - M_0$

Explaining in Words

Geometric meaning of the equilibrium relationships for beams

- $\frac{dV}{dx} = p(x)$

  The slope of the shear force diagram at any location x equals the value of the distributed external loading p at that location.

- $V(x_2) = V(x_1) + \int_{x_1}^{x_2} p(\xi)\, d\xi$ (integral form of the above)

  The shear force at point x_2 is equal to the shear force at x_1 plus the area under the external loading curve between these two points.

- $\frac{dM}{dx} = V(x)$

  The slope of the bending moment diagram at any location x equals the value of the shear force at that location.

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Explaining in Words

- $M(x_2) = M(x_1) + \int_{x_1}^{x_2} V(\xi)\, d\xi$ (integral form of the above)

  The bending moment at point x_2 is equal to the bending moment at x_1 plus the area under the external loading curve between these two points.

- $V(x^+) = V(x^-) + P_0$

  The shear force diagram has an *upward* step jump at location x where an external point force is applied. The value of the shear force jump *increase* equals the value of the external point force.

- $M(x^+) = M(x^-) - M_0$

  The bending moment diagram has a *downward* step jump at location x where an external point moment is applied. The value of the bending moment jump *decrease* equals value of the external point moment.

Three methods to analyze beams

c) *Bending-moment and shear-force diagrams*

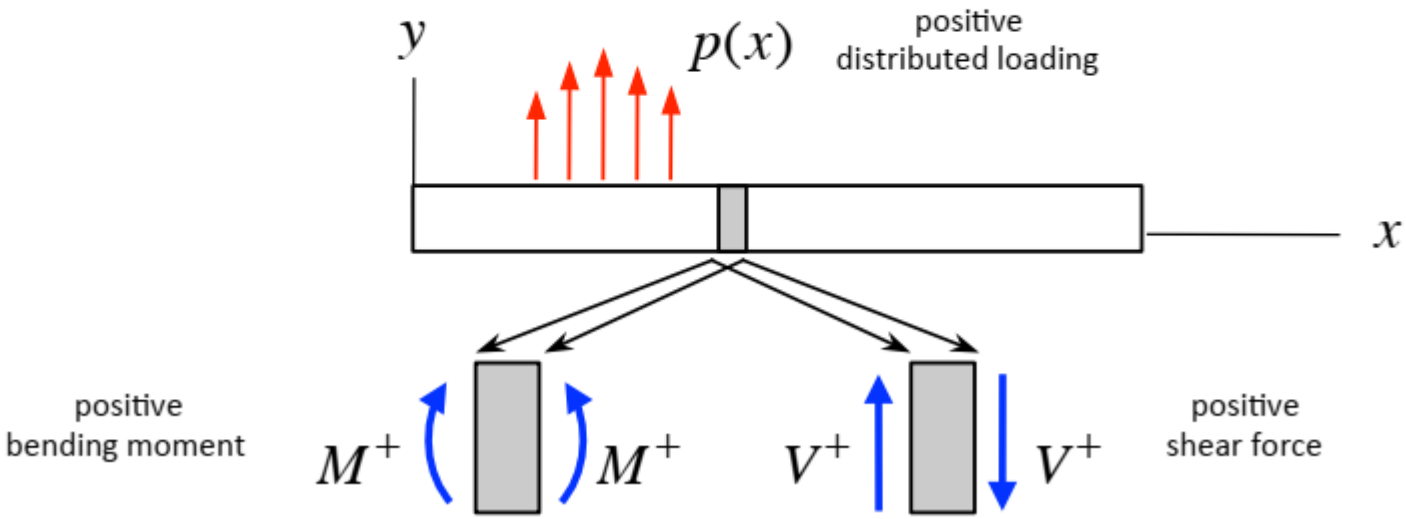
Three methods for determining the internal shear force and bending moment resultants:

- Using free body diagrams with cut sections, as demonstrated in the earlier examples of this section of notes.
- Using equilibrium relationships among applied loads, shear force and bending moments derived earlier and summarized above (integration and discontinuities).
- Using a graphical method based on the integration and discontinuity equations from the equilibrium method. The description of this method follows.

Intro to Graphical Method

Graphical method for constructing shear force and bending moment diagrams

Sign conventions:



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Intro to Graphical Method

Basic relationships (as derived via equilibrium relations):

$$\frac{dV}{dx} = p(x) \qquad \Rightarrow \qquad V_2 = V_1 + \int_{x_1}^{x_2} p(x)\,dx$$

$$\frac{dM}{dx} = V(x) \qquad \Rightarrow \qquad M_2 = M_1 + \int_{x_1}^{x_2} V(x)\,dx$$

Concentrated shear force V_0 applied at location x:

$V(x^+) = V(x^-) + V_0$ (jump UP in shear force)

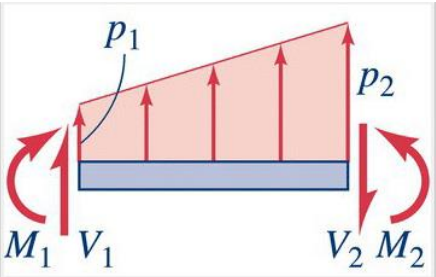
Concentrated moment M_0 applied at location x:

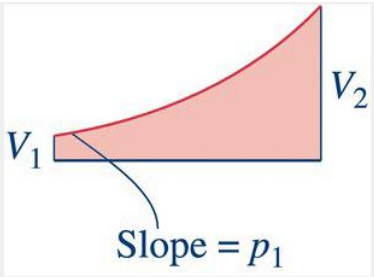
$M(x^+) = M(x^-) - M_0$ (jump DOWN in moment)

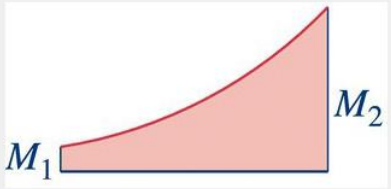
Graphical Method – V Equations

$\frac{dV}{dx} = p(x)$

(Eq. 5.2)



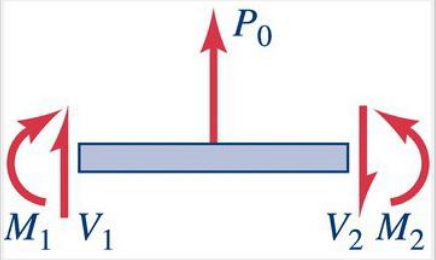


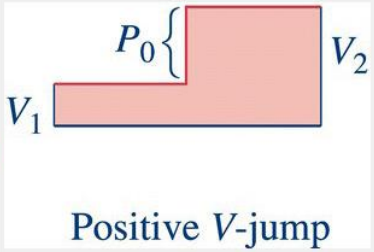


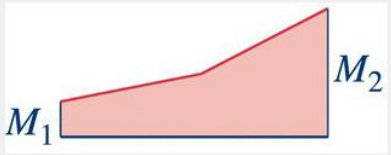
2. Jump in shear equals value of concentrated load

$\Delta V = P_0$

(Eq. 5.4)

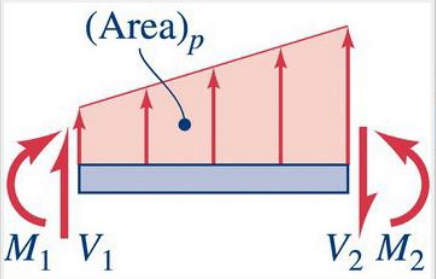


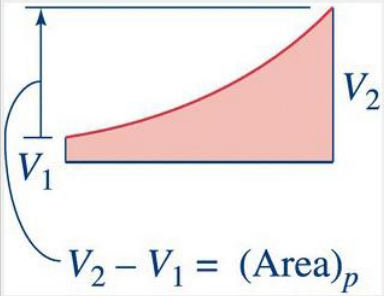




3. Change in shear equals area under distributed-load diagram

$V_2 - V_1 = \int_{x_1}^{x_2} p(x)dx$



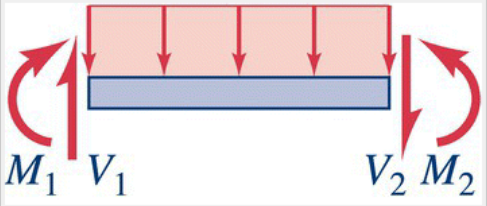


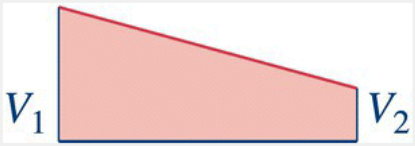
Graphical Method – M Equations

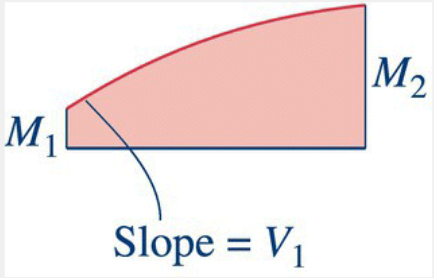
4. Slope of moment diagram equals value of shear

$$\frac{dM}{dx} = V(x)$$

(Eq. 5.3)



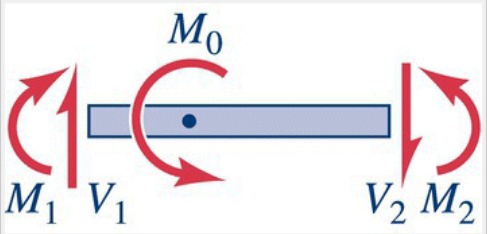


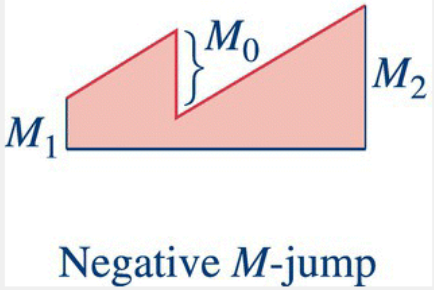


5. Jump in moment equals—(value of concentrated couple)

$$\Delta M = -M_0$$

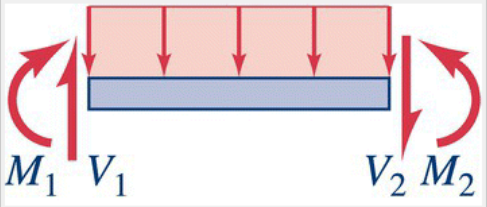
(Eq. 5.5)

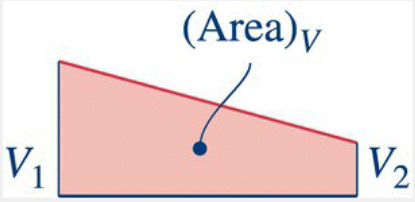


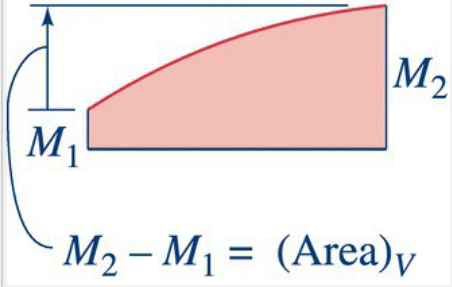


6. Change in moment equals area under shear diagram

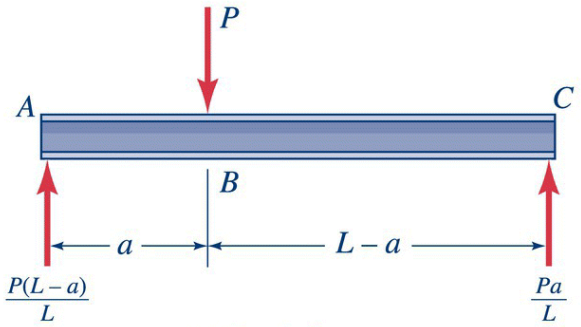
$$M_2 - M_1 = \int_{x_1}^{x_2} V(x)dx$$



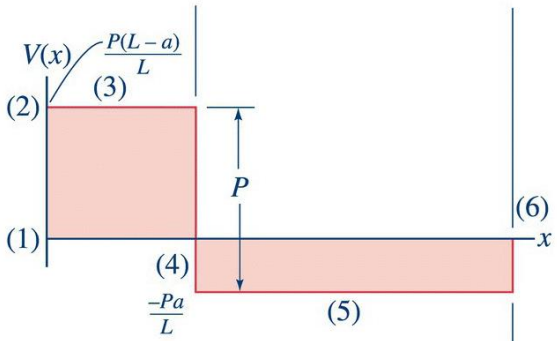




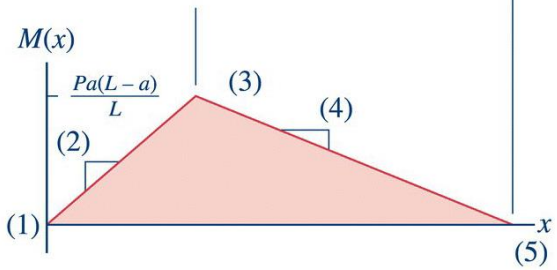
Examples of Shear and Moment Diagrams



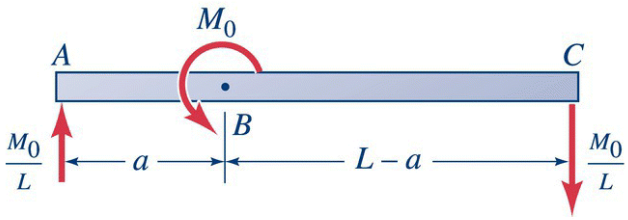
(a) Load diagram.



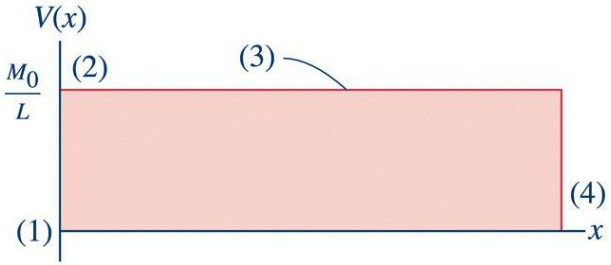
(b) Shear diagram.



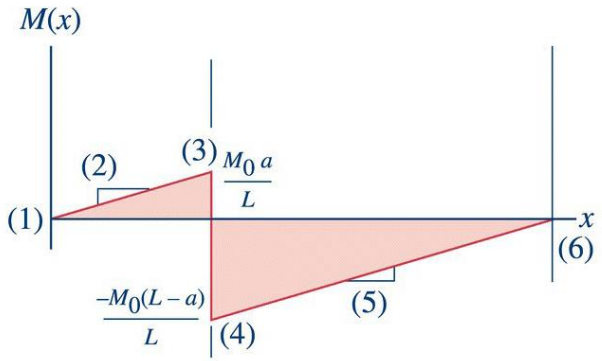
(c) Moment diagram.



(a) Load diagram.



(b) Shear diagram.

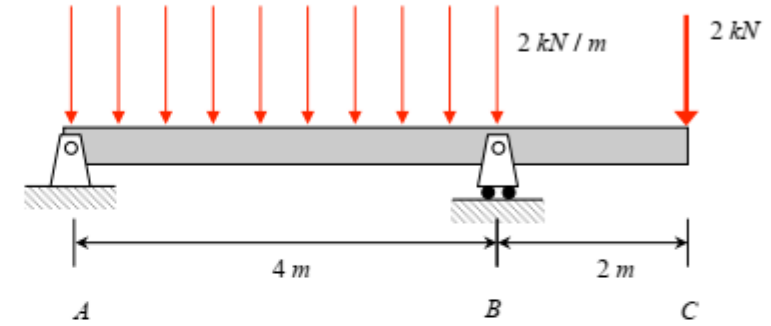


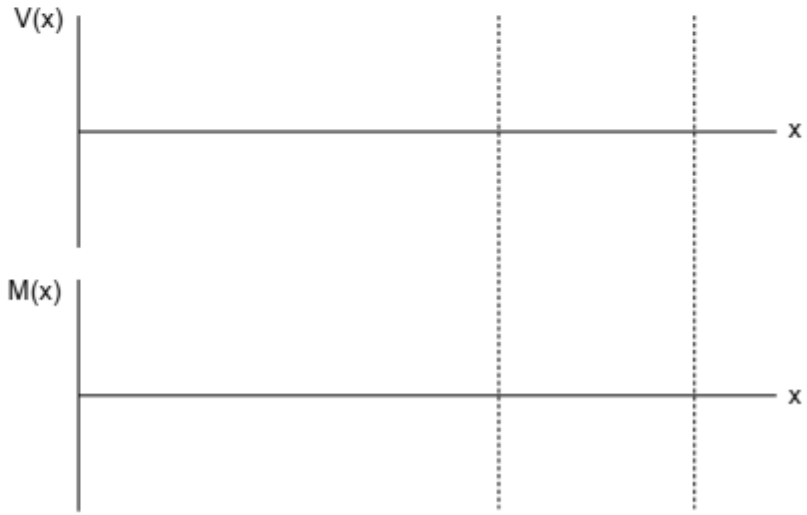
(c) Moment diagram.

Lecturebook Example 9.4

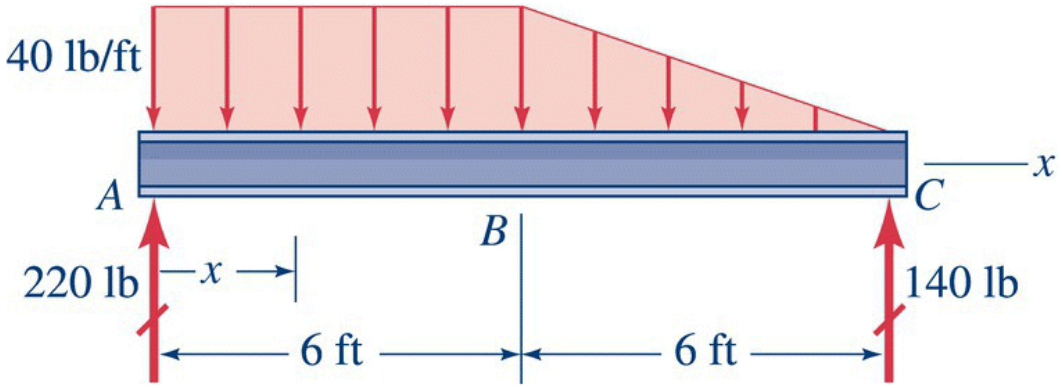
Example 9.4

Draw the shear force and bending moment diagrams in the plot axes below for the loaded beam shown.





Textbook Example 5.4



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