ME 323 Mechanics of Materials

Homework 12

December 5, 2019

Problem 12.1 (10 points). The state of stress acting at a critical point on the seat frame of an automobile during a crash is shown in Fig 12.1

a) Determine the three principal stresses, the absolute maximum shear stress $\tau_{max,abs}$, and the von-Mises stress σ_M.

b) If the frame was made of specialized material (considered as ductile material) having yield strength $S_y = 300$ ksi, determine the factor of safety according to

   1. The maximum shear stress theory.
   2. The maximum distortional energy theory.

c) If the frame was made of an experimental light weight composite material which behaves close to a brittle material whose ultimate strength in tension $S_{UT} = 200$ ksi, and whose ultimate strength in compression $S_{UC} = 500$ ksi, determine the factor of safety according to

   1. The maximum normal stress theory (for both tensile and compressive loadings).
   2. Mohr's theory of failure.

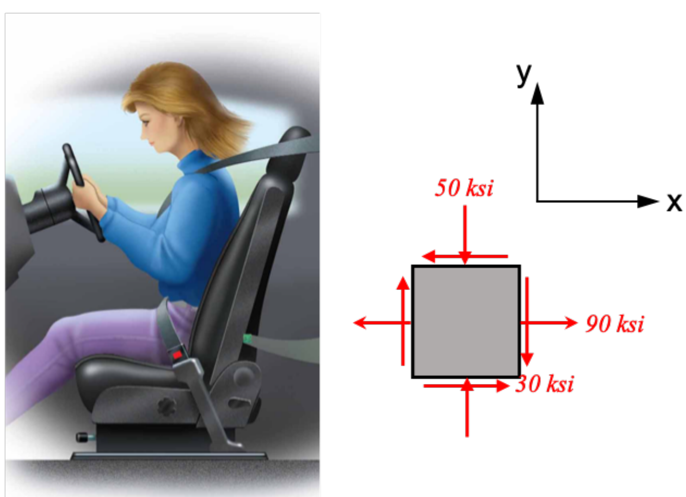


Fig. 12.1

Proof. Part a): Given,

$$\sigma_x = 90 \text{ ksi}; \sigma_y = -50 \text{ ksi}; \tau_{xy} = -30 \text{ ksi}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{90 - 50}{2} = 20 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{90 + 50}{2}\right)^2 + 30^2} = 76.16 \text{ ksi}$$

The principal stresses are,

$$\sigma_{P_1} = \sigma_{avg} + R = 96.16 \text{ ksi}; \sigma_{P_2} = \sigma_{avg} - R = -56.16 \text{ ksi}$$

$$\sigma_{P_3} = 0$$

Since $\sigma_{P_1} > 0; \sigma_{P_2} < 0$. The $\tau_{max,abs}$ can be obtained by,

$$\tau_{max,abs} = \frac{\sigma_{P_1} - \sigma_{P_2}}{2} = 76.16 \text{ ksi}$$

calculating Von-Mises stress,

$$\sigma_M = \sqrt{\sigma_{P_1}^2 - \sigma_{P_1}\sigma_{P_2} + \sigma_{P_2}^2} = 133.42 \text{ ksi}$$

Part b): Given, yield strength $S_y = 300$ ksi. The factor of safety FS according to:

1) MSST,

$$FS = \frac{S_y}{2 * \tau_{max,abs}} = \frac{300}{2 * 76.16} = 1.97$$

2) MDET,

$$FS = \frac{S_y}{\sigma_M} = \frac{300}{133.42} = 2.25$$

Part c): Given, $S_{UT} = 200$ ksi; $S_{UC} = 500$ ksi. The factor of safety FS according to:

1) MNST,

Since $\sigma_{P_1} > 0; \sigma_{P_2} < 0$, the tensile factor of safety is,

$$FS_T = \frac{S_{UT}}{\sigma_{P_1}} = \frac{200}{96.16} = 2.08$$

compressive factor of safety is,

$$FS_C = \frac{S_{UC}}{|\sigma_{P_2}|} = \frac{500}{56.16} = 8.9$$

$$\therefore FS = FS_T (< FS_C)$$

2) Mohr's theory,

$$\frac{\sigma_{P_1}}{S_{UT}} - \frac{\sigma_{P_2}}{S_{UC}} = \frac{96.16}{200} - \frac{-56.16}{500} \simeq 0.59$$

$$FS = \frac{1}{0.59} \simeq 1.69$$

□

Problem 12.2 (10 points). The A-36 steel pipe (yield strength, $S_y = 250$ MPa) has outer and inner diameters of $30mm$ and $20mm$, respectively. If the factor of safety guarding against yielding at point A (as shown in Fig. 12.2 (b)) is 1.5,

a) Determine the maximum allowable force P according to the maximum-shear-stress theory.

b) Determine the maximum allowable force P according to the maximum-distortional-energy theory.

Hint: Maximum shear stress for a disk cross-section is $\frac{2V}{A}$

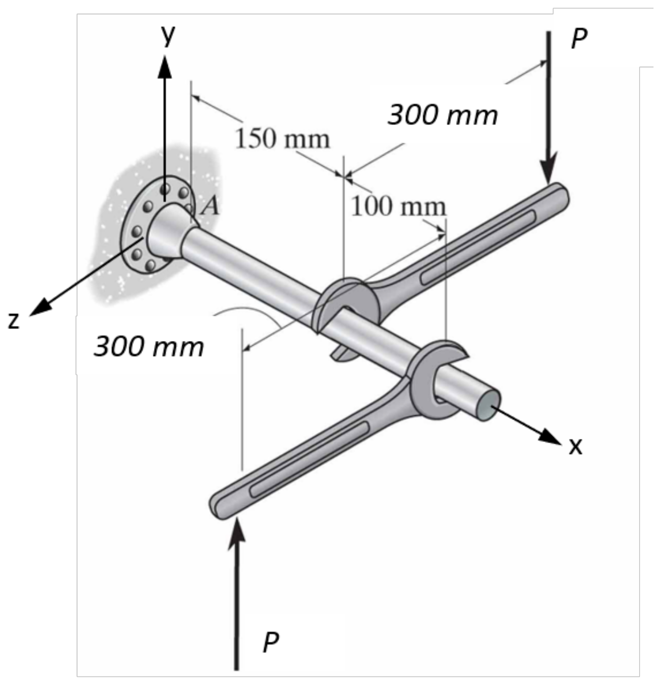


Fig. 12.2 (a)

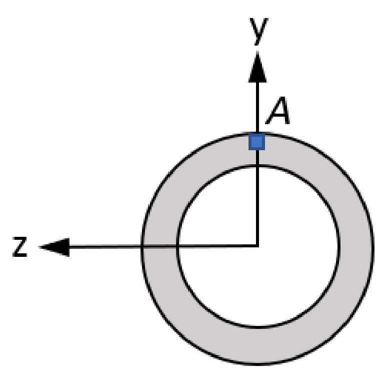
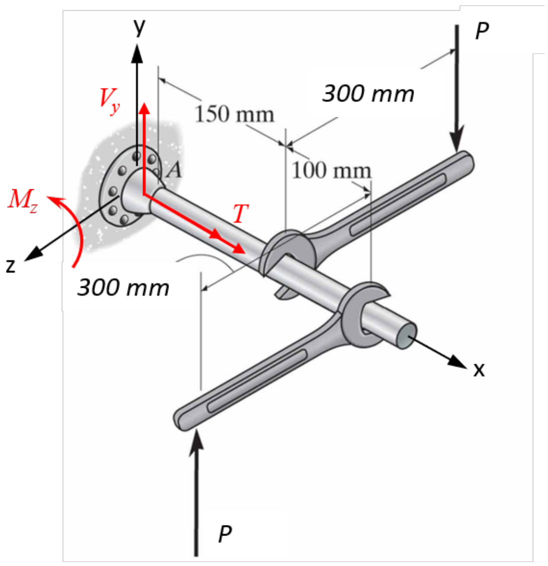


Fig. 12.2 (b)

Proof. FBD:



Equilibrium equations:

$$\Sigma F_y : V_y + P - P = 0 \implies V_y = 0$$

$$\Sigma T_x : T - P(0.3) - P(0.3) = 0$$

$$\implies T = 0.6P$$

$$\Sigma M_z : M_z + P(0.15) - P(0.25) = 0$$

$$\implies M_z = 0.1P$$

polar moment of inertia,

$$I_P = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(0.03^4 - 0.02^4) = 6.381 \times 10^{-8} \text{ m}^4$$

second area moment of inertia,

$$I_z = \frac{\pi}{64}(d_o^4 - d_i^4) = 3.191 \times 10^{-8} \text{ m}^4$$

flexural stress at point A,

$$\sigma_A = \frac{-M_z y_A}{I_z} = \frac{-0.1P \times 0.015}{3.191 \times 10^{-8}} = (-47P) \text{ kPa}$$

shear stress at point A,

$$\tau_A = \tau_{xz} = \frac{T\rho}{I_P} = \frac{0.6P \times 0.015}{6.381 \times 10^{-8}} = (141.04P) \text{ kPa}$$

state of stress at A:

$$\sigma_x = \sigma_A = (-47P) \text{ kPa}; \sigma_y = \sigma_z = 0$$

$$\tau_A = (141.04P) \text{ kPa}; \tau_{xy} = \tau_{yz} = 0$$

the principal stresses are,

$$\sigma_{P_1,P_2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = -23.5P \pm 142.98P$$

$$\sigma_{P_1} = (119.48P) \text{ kPa}; \sigma_{P_2} = (-166.48P) \text{ kPa}$$

a) According to MSS theory,

$$\tau_{max,abs} = \frac{\sigma_{P_1} - \sigma_{P_2}}{2} = (142.98P) \text{ kPa}$$

$$FS = \frac{S_y}{2 * \tau_{max,abs}} \implies 1.5 = \frac{250 \times 10^6}{2 * 142.98P \times 10^3} \implies P = 582.83 \text{ N}$$

b) According to the MDE theory,

$$\sigma_M = \sqrt{\sigma_{P_1}^2 - \sigma_{P_1}\sigma_{P_2} + \sigma_{P_2}^2} = (248.76P) \text{ kPa}$$

$$FS = \frac{S_y}{\sigma_M} \implies 1.5 = \frac{250 \times 10^6}{248.76P \times 10^3} \implies P = 670 \text{ N}$$

□

Problem 12.3 (10 points). An elbow with a circular cross section of diameter $20cm$ is fixed to a wall at the origin of the coordinate system. At the centroid of cross section C, a load equal to $80kN$ is applied in the positive x-direction and a load equal to $30kN$ is applied in the negative z-direction.

a) Determine the six components of the stresses induced at the locations M and N on the cross section D (located at $x = 0.5m$ from A). Draw separate three-dimensional stress elements for M and N. Indicate both the direction and magnitude of the non-zero stresses.

b) Suppose the elbow is made of a ductile material with yield strength $\sigma_y = 600$ MPa, calculate the factor of safety according to the maximum shear stress theory at both the locations M and N.

Hint: Maximum shear stress for a disk cross-section is $\frac{4V}{3A}$

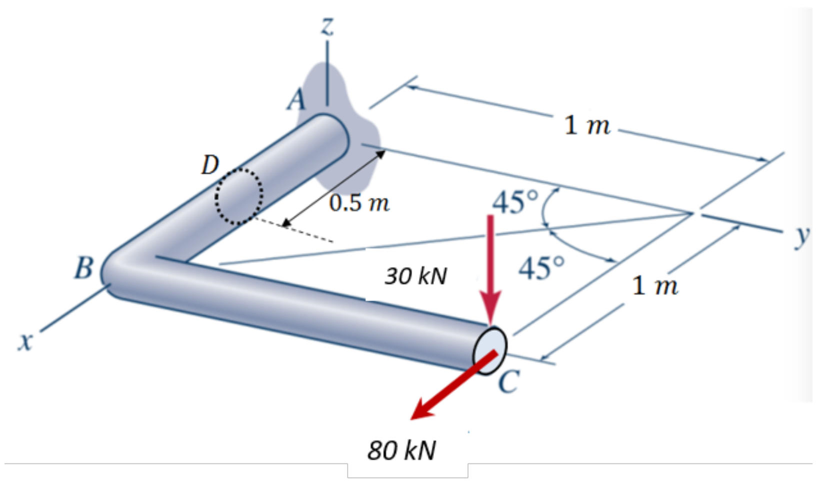


Fig. 12.3 (a)

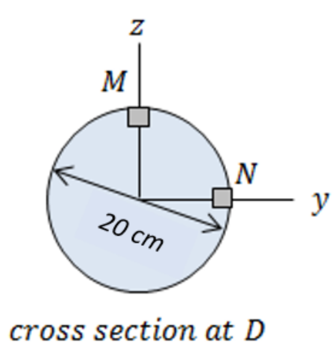


Fig. 12.3 (b)

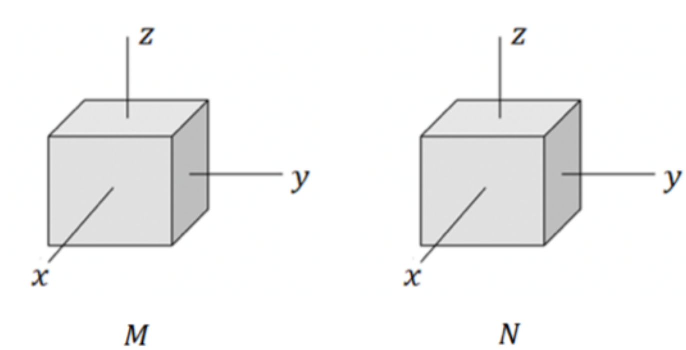
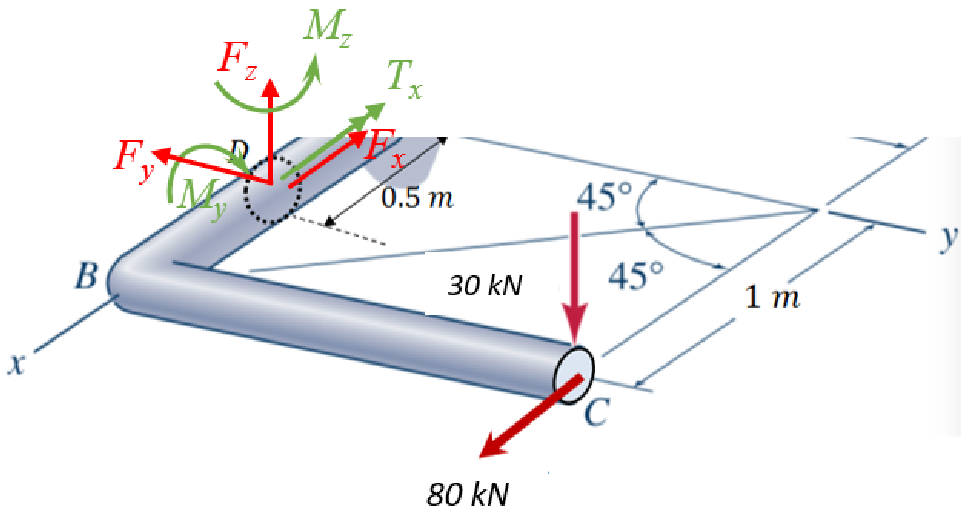


Fig. 12.3 (c)

Proof. FBD:



Equilibrium equations:

$$\Sigma F_x : -F_x + 80 = 0 \implies F_x = 80 \text{ kN}$$

$$\Sigma F_z : F_z - 30 = 0 \implies F_z = 30 \text{ kN}$$

$$\Sigma T_x : -T_x - 30(1) = 0 \implies T_x = -30 \text{ kNm}$$

$$\Sigma M_y : -M_y + 30(0.5) = 0 \implies M_y = 15 \text{ kNm}$$

$$\Sigma M_z : M_z - 80(1) = 0 \implies M_z = 80 \text{ kNm}$$

area of cross-section,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}0.2^2 = 0.0314 \text{ m}^2$$

second area moment of inertia,

$$I_z = I_y = \frac{\pi}{64}d^2 = 7.85 \times 10^{-5} \text{ m}^4$$

polar moment of inertia,

$$I_P = \frac{\pi}{32}d^4 = 1.57 \times 10^{-4} \text{ m}^4$$

Part a): The following forces and stresses are on +x face,

Load at D:	Stress at M:	Stress at N:
$F_x = 80$ kN (Normal)	$\sigma_x = \frac{F_x}{A} = 2.548$ MPa	$\sigma_x = \frac{F_x}{A} = 2.548$ MPa
$F_y = 0$	$\tau_{xy} = 0$	$\tau_{xy} = 0$
$F_z = 30$ kN (Shear)	$\tau_{xz} = 0$	$\tau_{xz} = -\left\|\frac{4F_z}{3A}\right\| = 1.274$ MPa
$T_x = -30$ kNm (Torque)	$\tau_{xy} = \left\|\frac{T_x R}{I_P}\right\| = 19.108$ MPa	$\tau_{xz} = -\left\|\frac{T_x R}{I_P}\right\| = 19.108$ MPa
$M_y = 15$ kNm (Bending)	$\sigma_x = \left\|\frac{M_y R}{I_z}\right\| = 19.108$ MPa	$\sigma_x = 0$
$M_z = 80$ kN (Bending)	$\sigma_x = 0$	$\sigma_x = \left\|\frac{M_z R}{I_z}\right\| = 101.91$ MPa

For M:

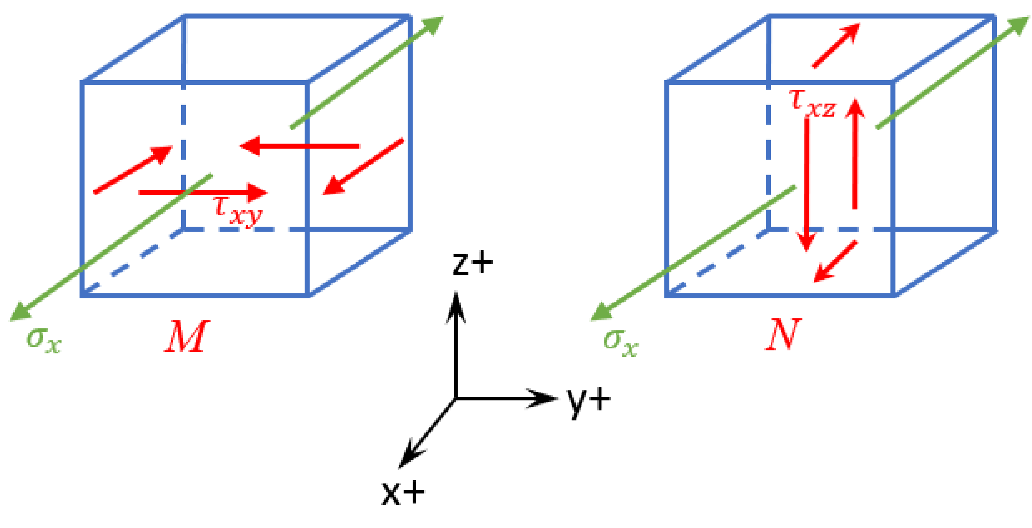
$$\sigma_x = 2.548 + 19.108 = 21.656 \text{ MPa}$$

$$\tau_{xy} = 19.108 \text{ MPa}$$

For N:

$$\sigma_x = 2.548 + 101.91 = 104.458 \text{ MPa}$$

$$\tau_{xz} = -1.274 - 19.108 = -20.382 \text{ MPa}$$



Part b): At M,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 10.828 \text{ MPa}; R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 21.96 \text{ MPa}$$

$$\sigma_{P_1} = \sigma_{avg} + R = 32.79 \text{ MPa}; \sigma_{P_2} = \sigma_{avg} - R = -11.132 \text{ MPa}$$

$$\tau_{max,abs} = \frac{\sigma_{P_1} - \sigma_{P_2}}{2} = 21.961 \text{ MPa}$$

$$\therefore FS_M = \frac{S_y}{2 * \tau_{max,abs}} = 13.66$$

At N,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = 52.229 \text{ MPa}; R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 56.065 \text{ MPa}$$

$$\sigma_{P_1} = \sigma_{avg} + R = 108.29 \text{ MPa}; \sigma_{P_2} = \sigma_{avg} - R = -3.836 \text{ MPa}$$

$$\tau_{max,abs} = \frac{\sigma_{P_1} - \sigma_{P_2}}{2} = 56.063 \text{ MPa}$$

$$\therefore FS_N = \frac{S_y}{2 * \tau_{max,abs}} = 5.36$$

Hence,

$$FS = FS_N (< FS_M) = 5.36$$

□

Problem 12.4 (10 points). The diameter of the solid L2-steel (a low-alloy special purpose steel) rod AB (it is in pinned-pinned configuration) is 50 mm. The elastic modulus E and yield stress σ_Y of the material are 150 GPa and 680 MPa, respectively.

a) Verify that the Euler buckling equation is valid for this setup. Determine the mass of C that the rod AB can support without buckling.

b) Determine the mass of C that the rod AB can support without yielding.

Use a factor of safety equal to 1.5 and $g = 9.81$ m/s^2.

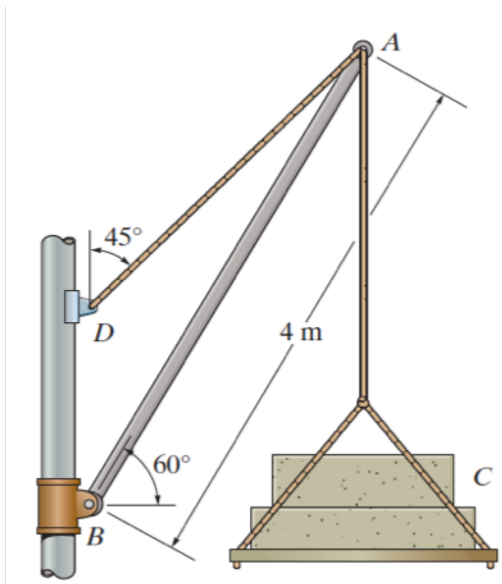
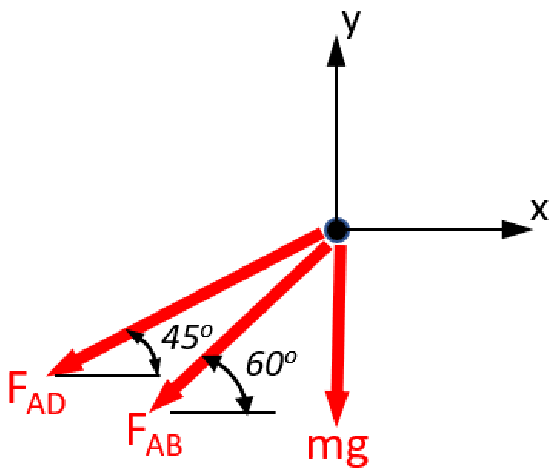


Fig. 12.4

Proof. FBD:



Equilibrium equations:

$$\Sigma F_x = -F_{AB}cos(60) - F_{AD}cos(45) = 0$$

$$\implies F_{AB} = -\sqrt{2}F_{AD}$$

$$\Sigma F_x = -F_{AB}sin(60) - F_{AD}sin(45) - mg = 0$$

$$\implies \sqrt{2}F_{AD}\frac{\sqrt{3}}{2} - \frac{F_{AD}}{\sqrt{2}} = mg$$

$$\therefore F_{AD} = 1.932mg$$

$$F_{AB} = -26.8m \text{ (compressive force)}$$

Part a): For pin - pin column, $L_{eff} = L = 4$ m

$$r = \sqrt{\frac{I}{A}}; A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{64}d^4 = 3.068 \times 10^{-7}m^4 \implies r = 0.0125 \text{ m}$$

verifying the Euler-buckling criteria,

$$\left(\frac{L_{eff}}{r}\right) = 319.71; \sqrt{\frac{\pi^2 E}{0.5\sigma_y}} = \left(\frac{L_{eff}}{r}\right)_c = 66$$

$$\therefore \left(\frac{L_{eff}}{r}\right) > \left(\frac{L_{eff}}{r}\right)_c$$

Hence, Euler theory is valid for this situation. So, to find out the critical load,

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2} = 28.387kN = |F_{AB}| \times FS = 26.8m \times 1.5$$

$$\implies m = 706.14 \text{ kg}$$

Part b) If the column was to yield without buckling then the mass C could be,

$$\sigma_{comp.} = \frac{F_{AB}}{A} = \frac{25.8m}{1.96 \times 10^{-3}} = \frac{S_y}{FS} = \frac{680 \times 10^6}{1.5}$$

$$\implies m = 33,154 \text{ kg}$$

this shows that buckling is a very important phenomena to take into account during design.

□