Lectures 37-39: Stress due to combined loading

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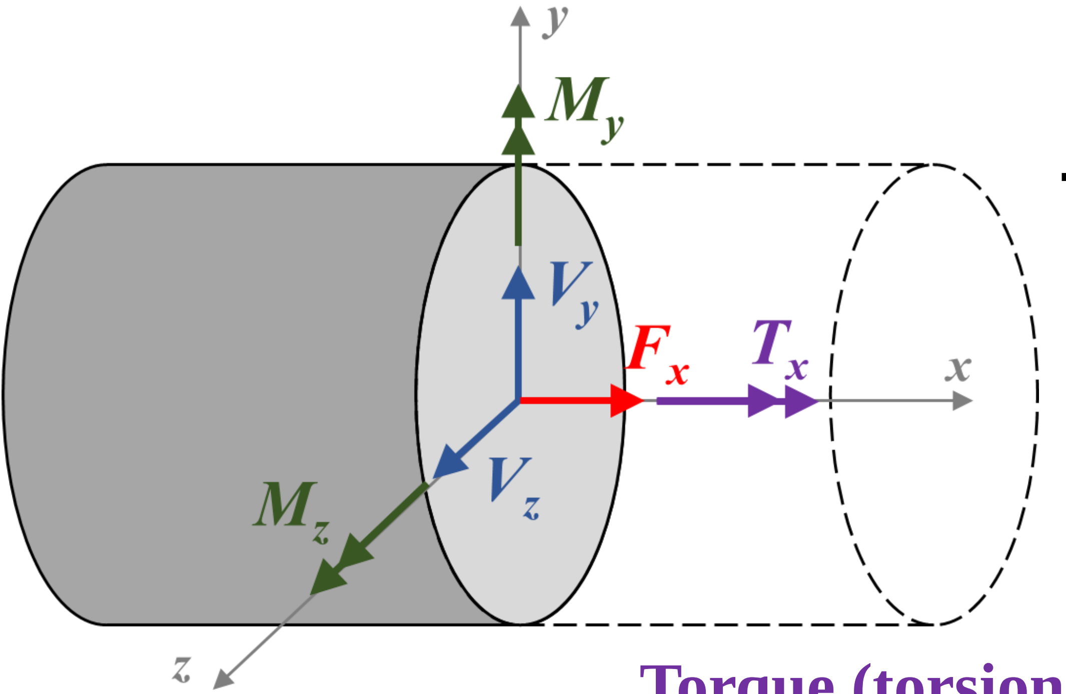
Lecture Book: Ch. 14

PURDUE ENGINEERING

Objectives for combined loading problems

- Determine the normal and shear stresses at points on a cross section due to combined axial, torsion, and bending loading
- Determine the principal stresses and maximum shear stress at these points
  - Use Mohr’s circle – we will always be in a state plane stress, but not necessarily in the x-y plane

Review: Internal resultants in 3 dimensions



Load	Type of stress	Stress distribution	Lecture book ch.
Axial force F_x	**Normal**	$\sigma_x = F_x / A$	**Ch. 6**
Shear force V_y	**Shear**	$\tau_{xy} = \dfrac{V_y Q}{I_{zz} t}$	**Ch. 10**
Shear force V_z	**Shear**	$\tau_{xz} = \dfrac{V_z Q}{I_{yy} t}$	**Ch. 10**
Torque (torsional moment) T_x	**Shear**	$\tau = T \rho / I_p$	**Ch. 8**
Bending moment M_y	**Normal**	$\sigma_x = M_y z / I_{yy}$	**Ch. 10**
Bending moment M_z	**Normal**	$\sigma_x = -M_z y / I_{zz}$	**Ch. 10**

Note: subscripts (xz or xy) for shear stress due to **torque** depend on the point of interest on the cross section

See Superposition of six sets of stress components on the "Animations and Demos" page of the course blog

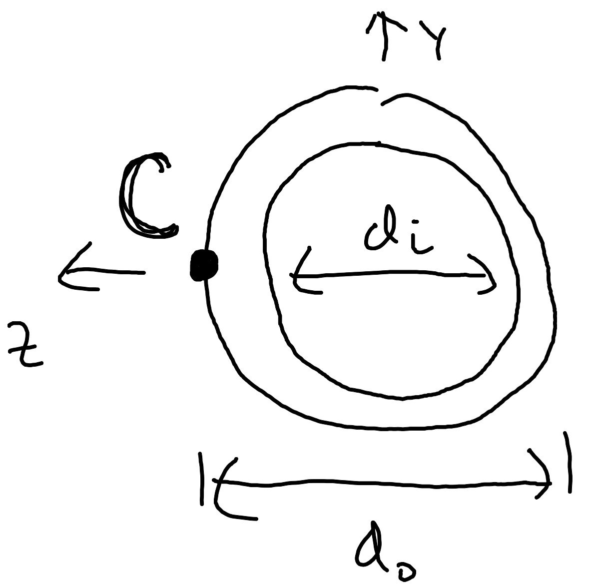
Example 14.6

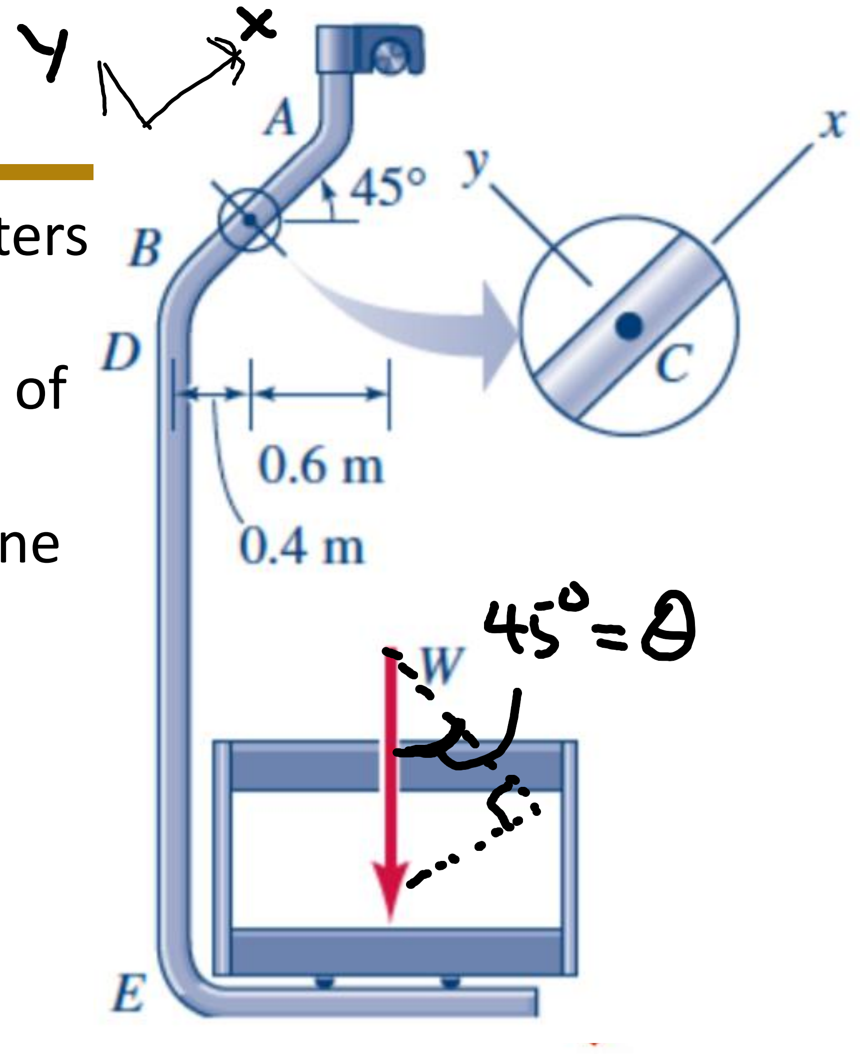
A ski lift is supported by a steel pipe with outer and inner diameters **d_o = 60 mm** and **d_i = 52 mm**, respectively.

a.) Determine the stresses at point C on the front section of the pipe

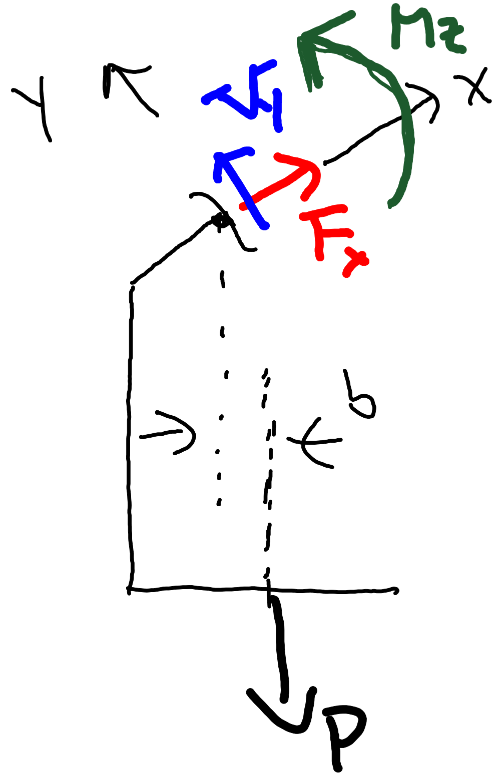
b.) Determine the principal stresses and maximum in-plane shear stress at point C

Use **h = 400 mm**, **b = 600 mm**, **θ = 45º** and **P = W = 2 kN**





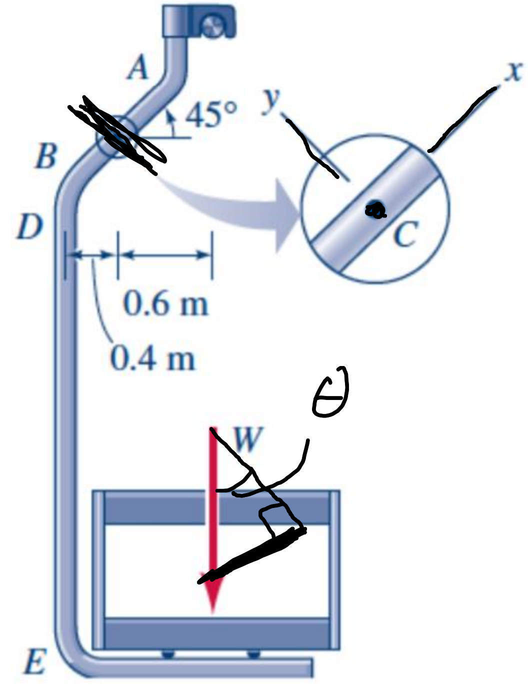
1) FBD + equilibrium $\longrightarrow$ internal resultants

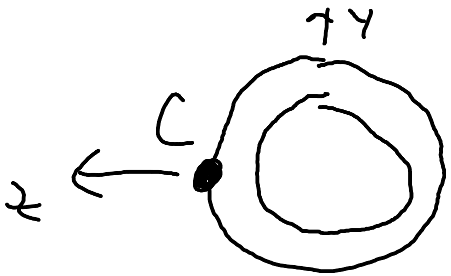


$\xrightarrow{+} \sum F_x = F_x - P\sin\theta = 0$ $\quad \boxed{F_x = P\sin\theta}$

$+\nwarrow \sum F_y = V_y - P\cos\theta = 0$ $\quad \boxed{V_y = P\cos\theta}$

$+\circlearrowleft \sum M_{cut} = M_z - Pb = 0$ $\quad \boxed{M_z = Pb}$





2.) Find the stress at point C due to each internal resultant

Load	stress dist	stress at C

$A = 704\,mm^2$

axial force

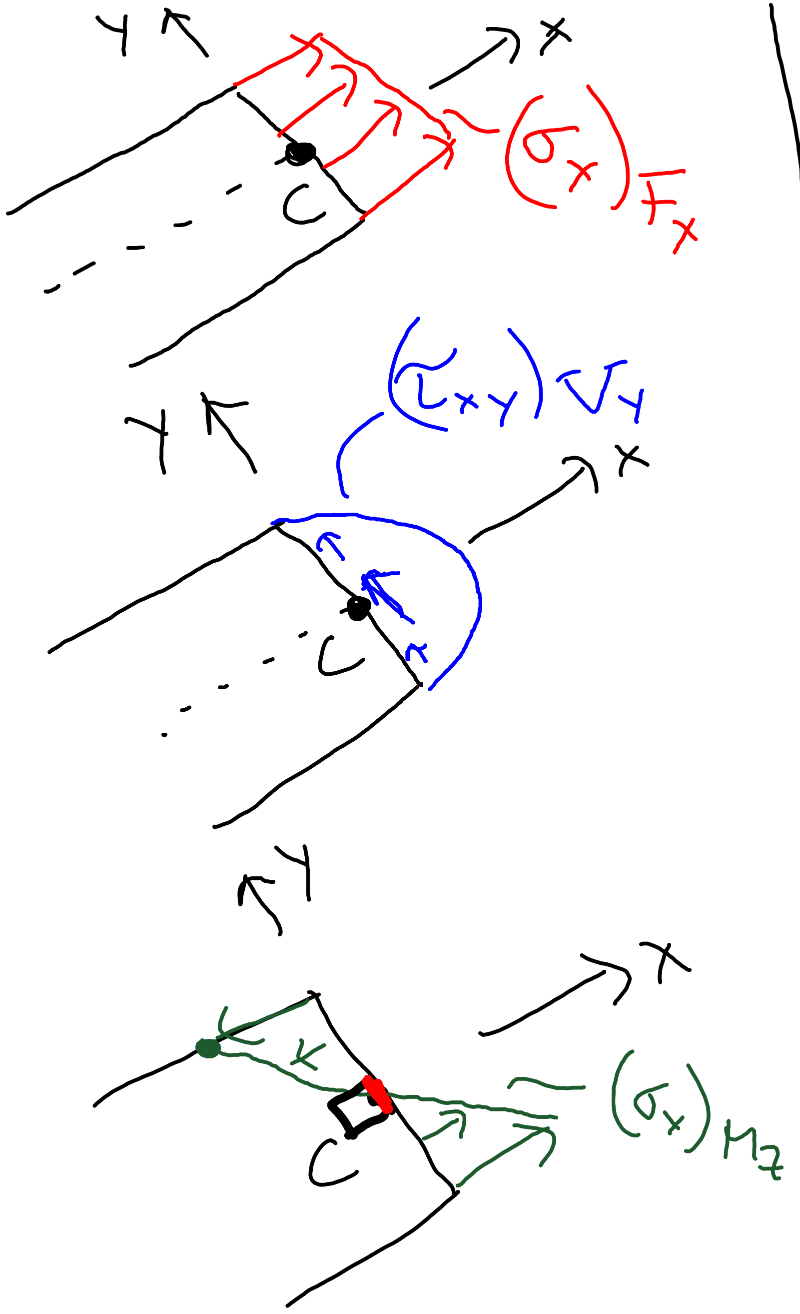
$F_x = P\sin\theta$

shear force

$V_y = P\cos\theta$

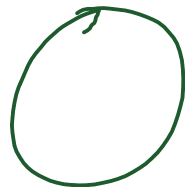
bending moment

$M_z = Pb$



$\sigma_x = F_x/A = \frac{P\sin\theta}{A} = 2\ \text{MPa}$

$\tau_{xy} = \tau_{max} = \frac{\cancel{3}\,2P\cos\theta}{A} = 4.0\ \text{MPa}$

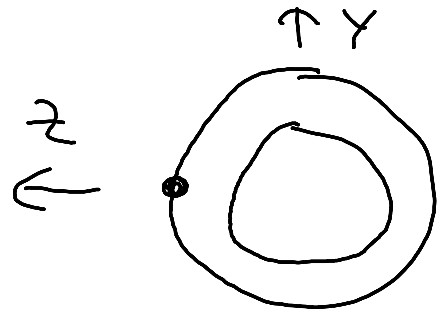


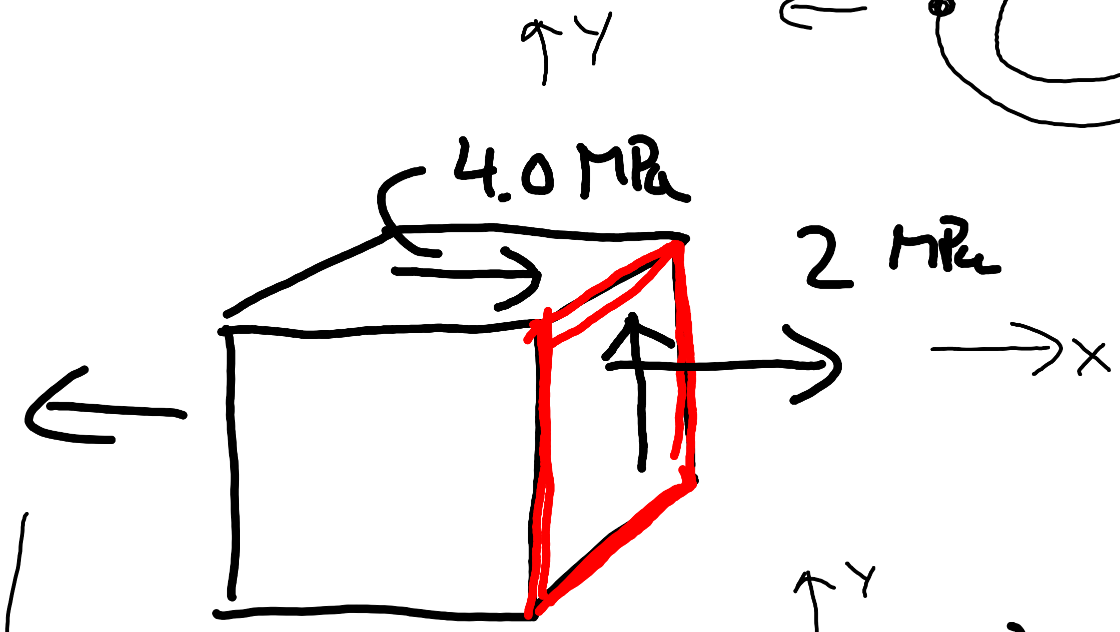
3.) Combine stresses + draw the stress element at C

$$\sigma_x^C = \frac{P\sin\theta}{A} + 0 = 2 \text{ MPa}$$

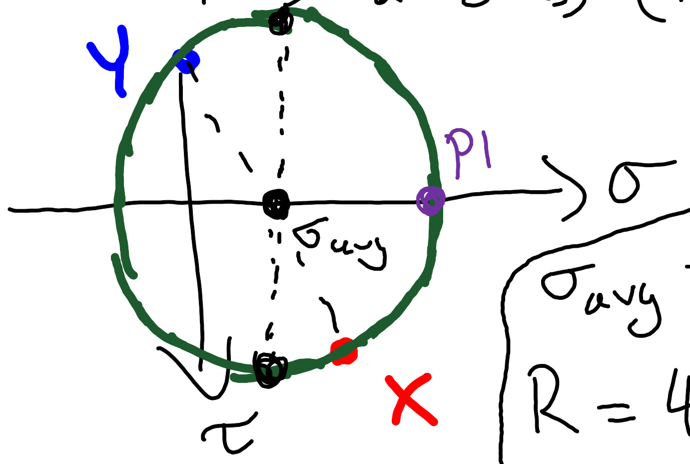
$$\tau_{xy}^C = \frac{2P\cos\theta}{A} = 4.0 \text{ MPa}$$

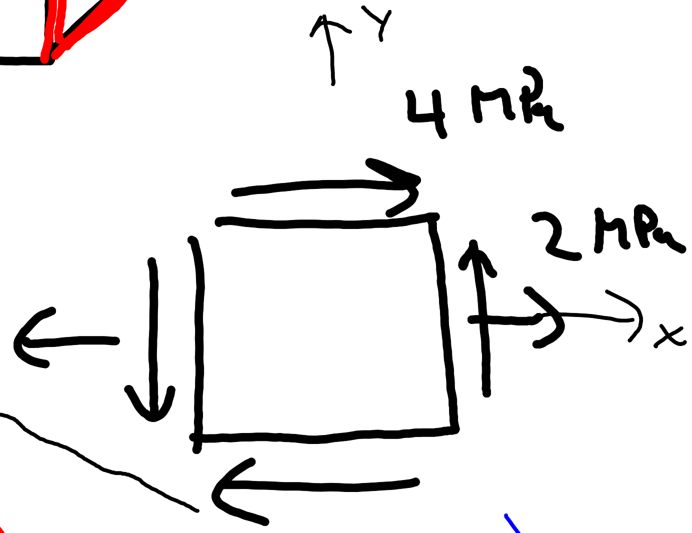
all other stresses at C = 0





4.) Find the principal stresses + max shear stress (Mohr's circle)





$\sigma_{avg} = 1$ MPa

$R = 4.1$ MPa

$\sigma_{p1} = 5.1$ MPa

$\sigma_{p2} = -3.1$ MPa

$\tau_{max} = 4.1$ MPa

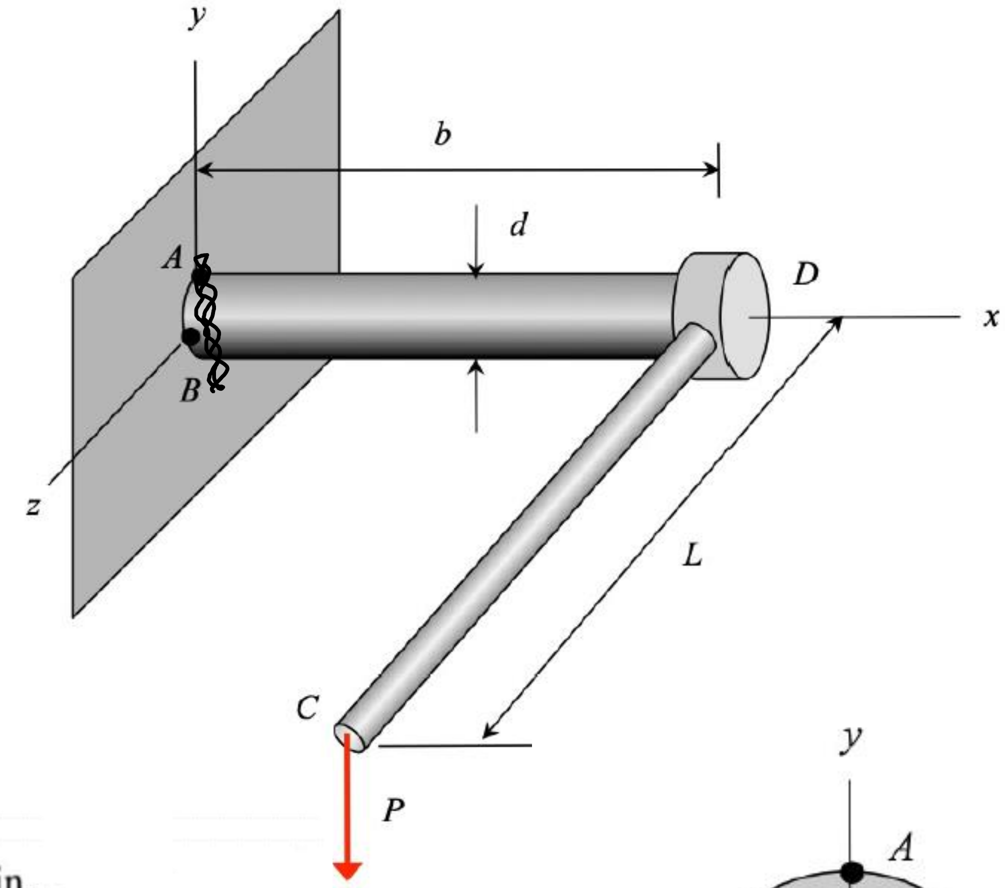
$\sigma_{p3} = 0$

X(2, 4)

Y(0, −4)

Example 14.4

A vertical force of ***P* = 40 lb** is applied to the end of a pipe wrench, whose handle is parallel to the z axis. The pipe has an outer diameter ***d* = 1 in**. and wall thickness ***t* = 0.1 in**.
Determine the principal stresses at points A and B on the cross section of the pipe
Use ***b* = 12 in**. and ***L* = 9 in**.



$\Sigma M_x = 0$

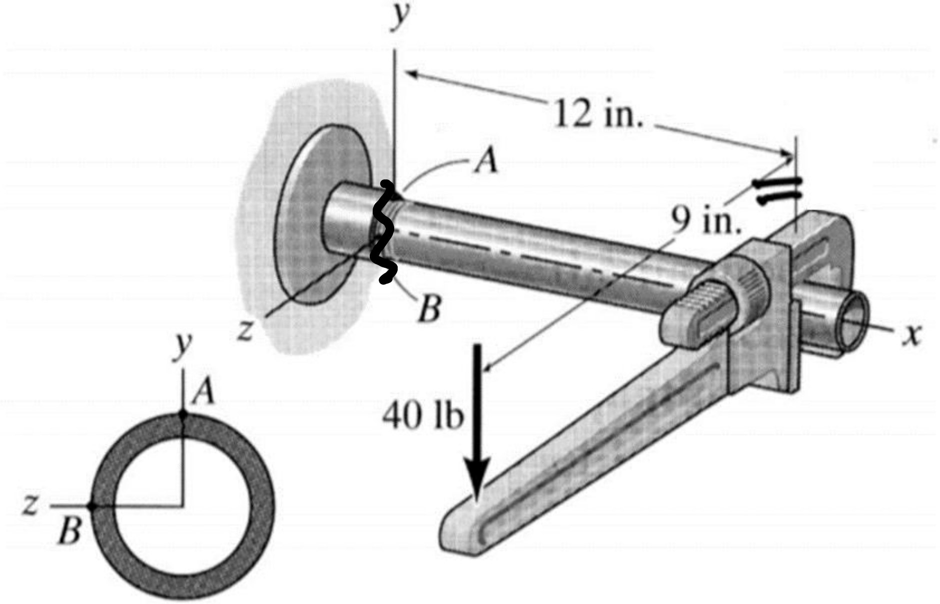
$\Sigma M_y = 0$

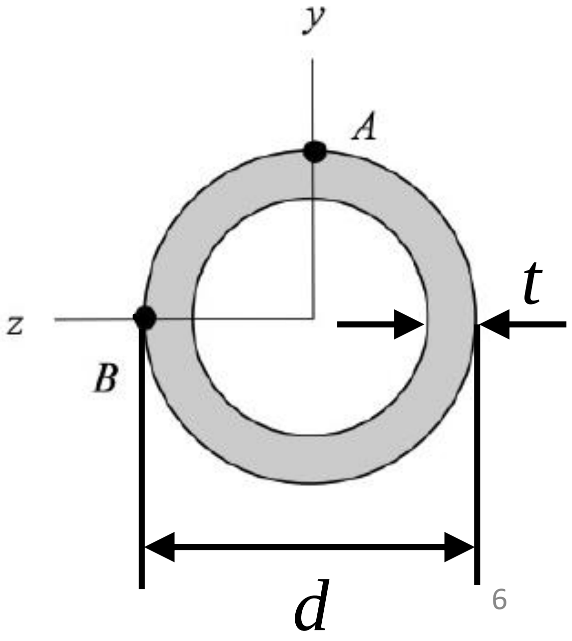
$\Sigma M_z = 0$

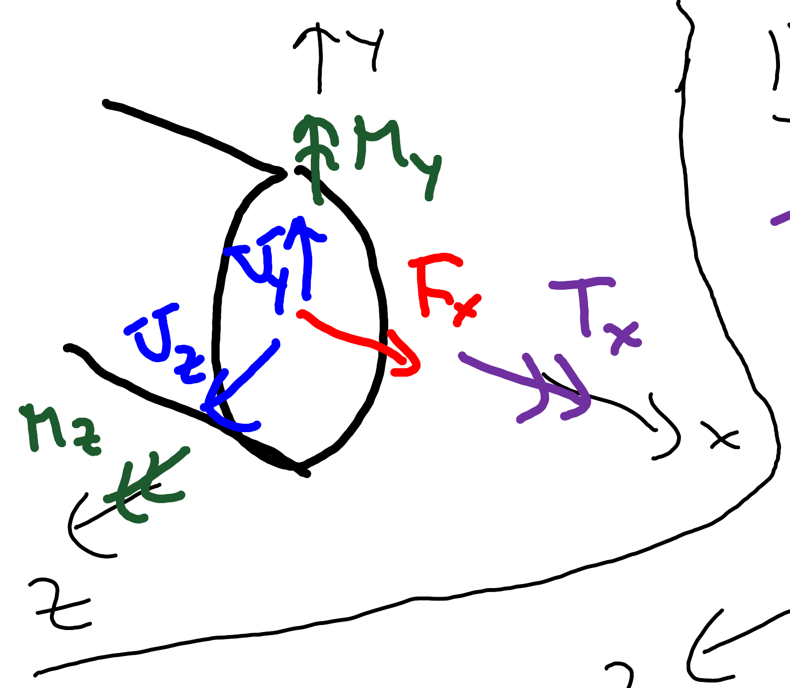
$\Sigma \vec{M} = \vec{0}$

$\vec{r} \times \vec{F}$

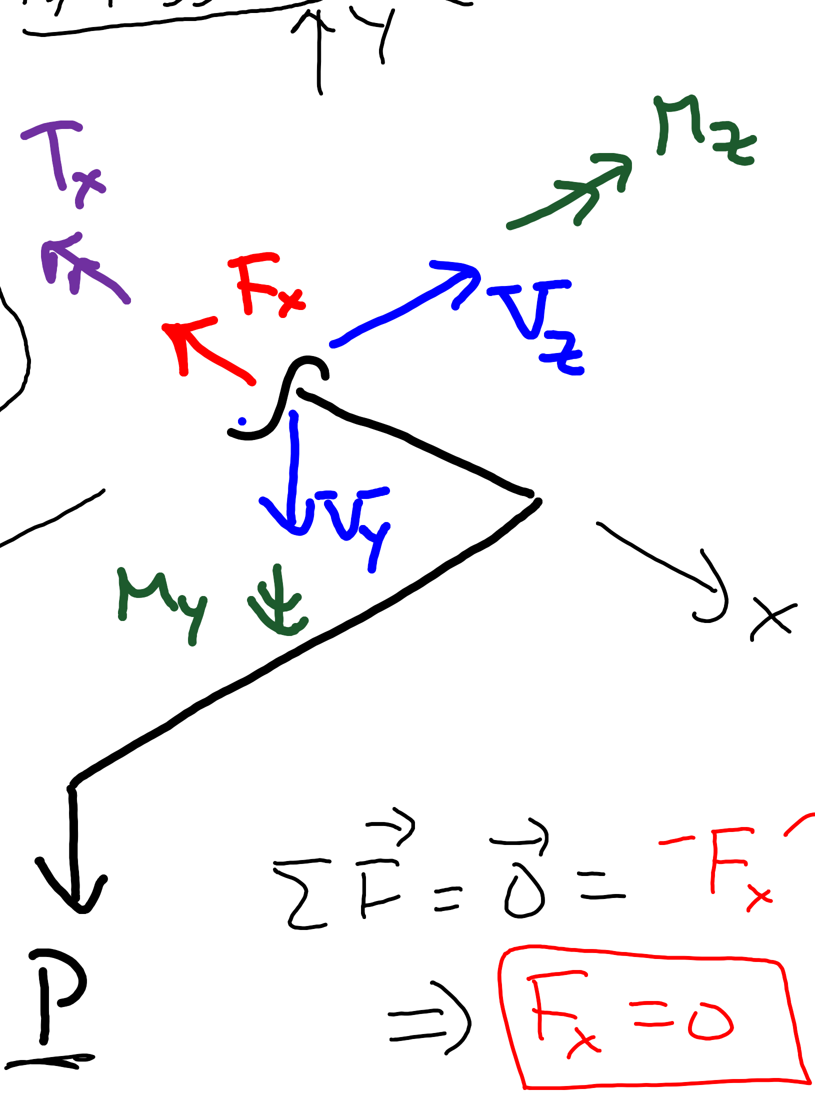
Goal

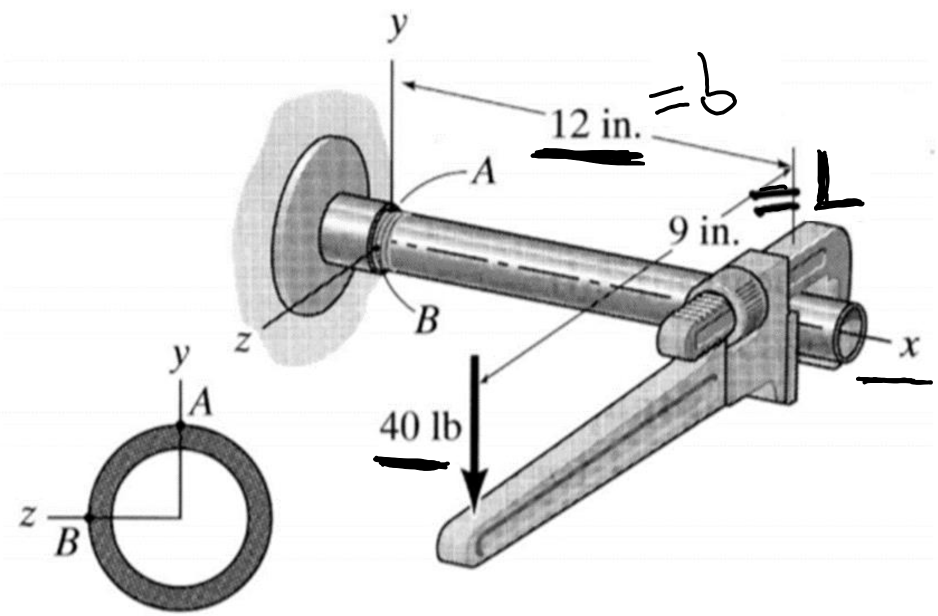






1.) FBD + equil





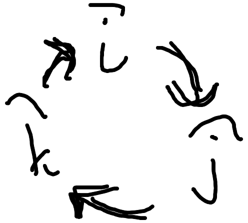
$$\vec{r} = b\hat{i} + 0\hat{j} + L\hat{k}$$

$$\vec{F} = 0\hat{i} - P\hat{j} + 0\hat{k}$$

$$\sum \vec{F} = \vec{0} = -F_x\hat{i} - V_y\hat{j} - V_z\hat{k} - P\hat{j}$$

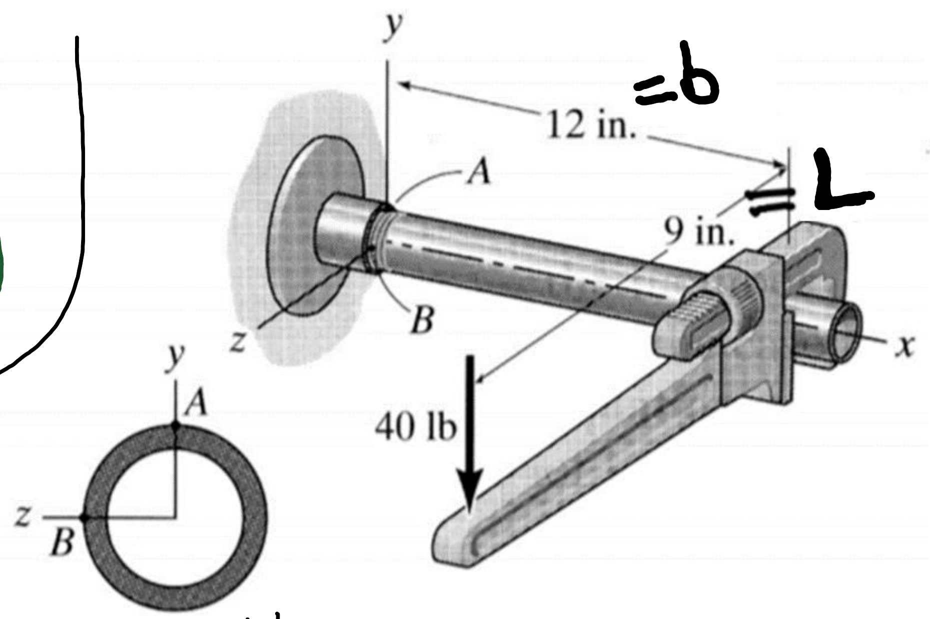
$$\Rightarrow \boxed{F_x = 0} \quad \boxed{V_y = -P} \quad \boxed{V_z = 0}$$

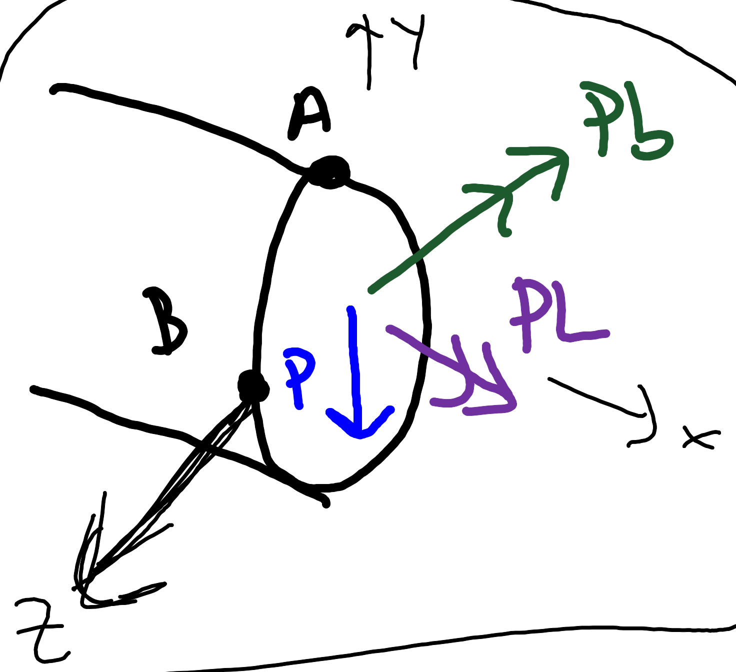
$$\sum \vec{M} = \vec{0} = -T_x\hat{i} - M_y\hat{j} - M_z\hat{k} + \vec{r} \times \vec{F}$$



$\vec{r} \times \vec{F} = -Pb\,\hat{k} + PL\,\hat{i}$

$\boxed{T_x = PL}$ $\boxed{M_y = 0}$ $\boxed{M_z = -Pb}$





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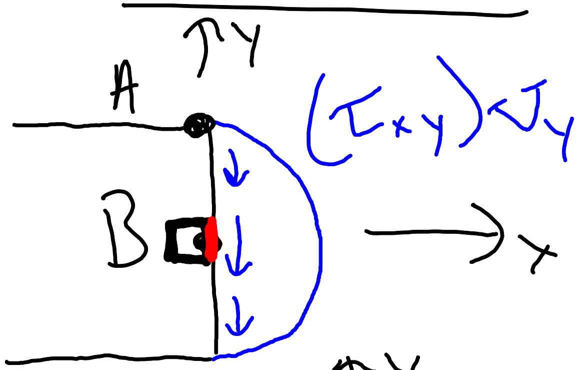
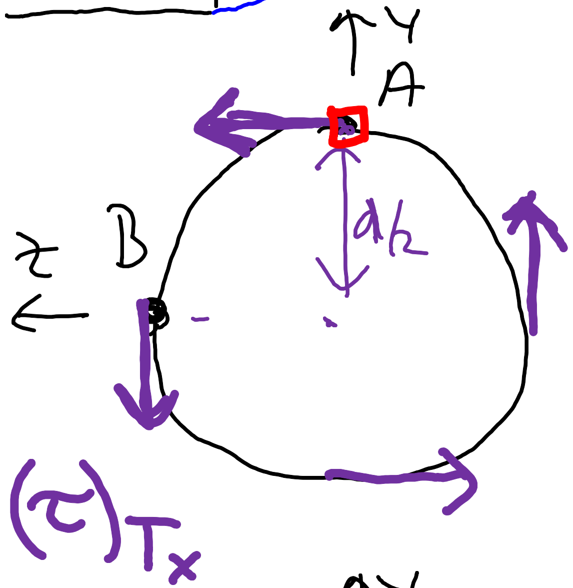
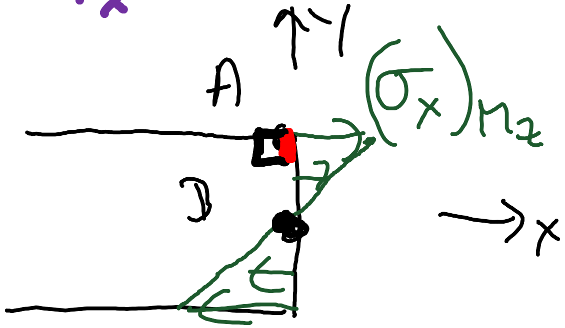
Aside: Using 3 separate eqs. for moment equilibrium

$+\sum M_x = -T_x + PL = 0 \Rightarrow \underline{T_x = PL}$ ✓

$+\sum M_y = \underline{-M_y = 0}$ ✓ (P is parallel to the y axis, so it does not cause an M_y)

$+\sum M_z = -M_z + Pb = 0 \Rightarrow \underline{M_z = Pb}$ ✓

2.) Find stresses at A+B

Load	Stress dist.	Stress at A	Stress at B
shear force $V_y = -P$		0 ✓	$\tau_{xy} = \tau_{max} = -\frac{2P}{A}$
torque $T_x = PL$		✓ $\tau_{xz} = \frac{(PL)(d/2)}{I_p}$ ✓	$\tau_{xy} = \frac{-(PL)(d/2)}{I_p}$
bending moment $M_z = -Pb$		$\sigma_x = \frac{+(Pb)(d/2)}{I_{zz}}$	0

3.) Combine stresses + draw stress element at each point

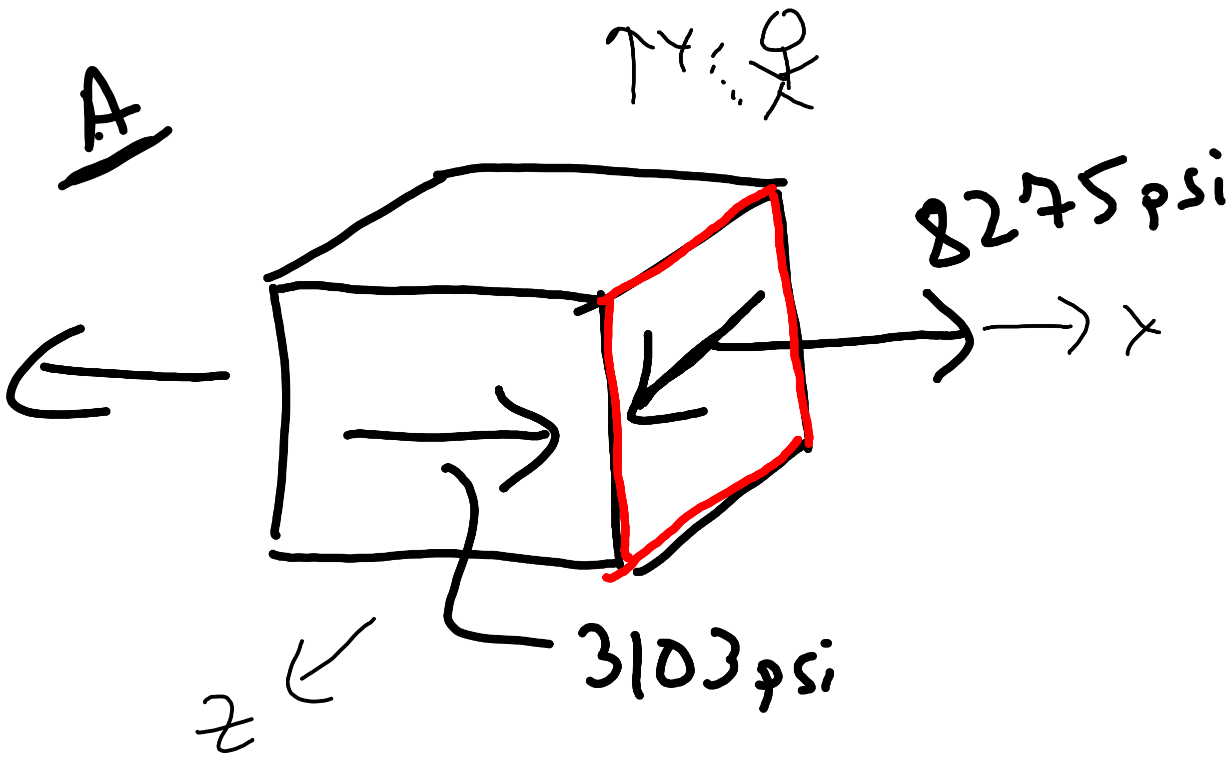
$$A: \ \sigma_x^a = \frac{+Pbd}{2I_{zz}} = +8275 \text{ psi}$$

$$I_{zz} = 0.03 \text{ in.}^4$$

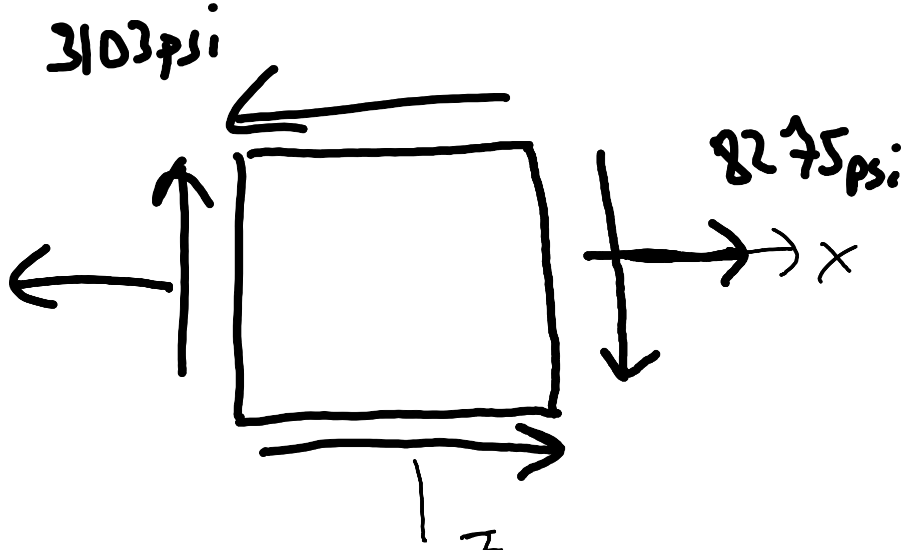
$$\tau_{xz}^a = \frac{+PLd}{2I_p} = +3103 \text{ psi}$$

$$I_p = 0.06 \text{ in.}^4$$

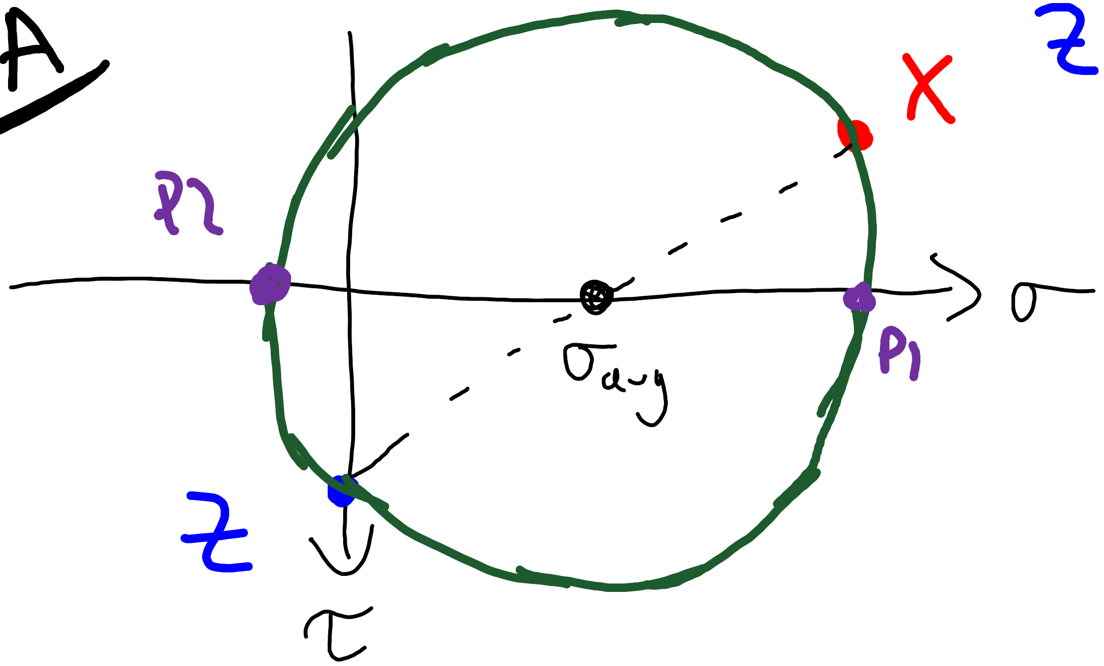
all other stresses at $A = 0$



4.) Find principal stresses + max shear stress at A



A



$\sigma_{avg} = 4138$ psi

$R = 5172$ psi

$\sigma_{p1} = 9310$ psi

$\sigma_{p2} = -1034$ psi

$\sigma_{p3} = 0$

$\tau_{max} = 5172$ psi

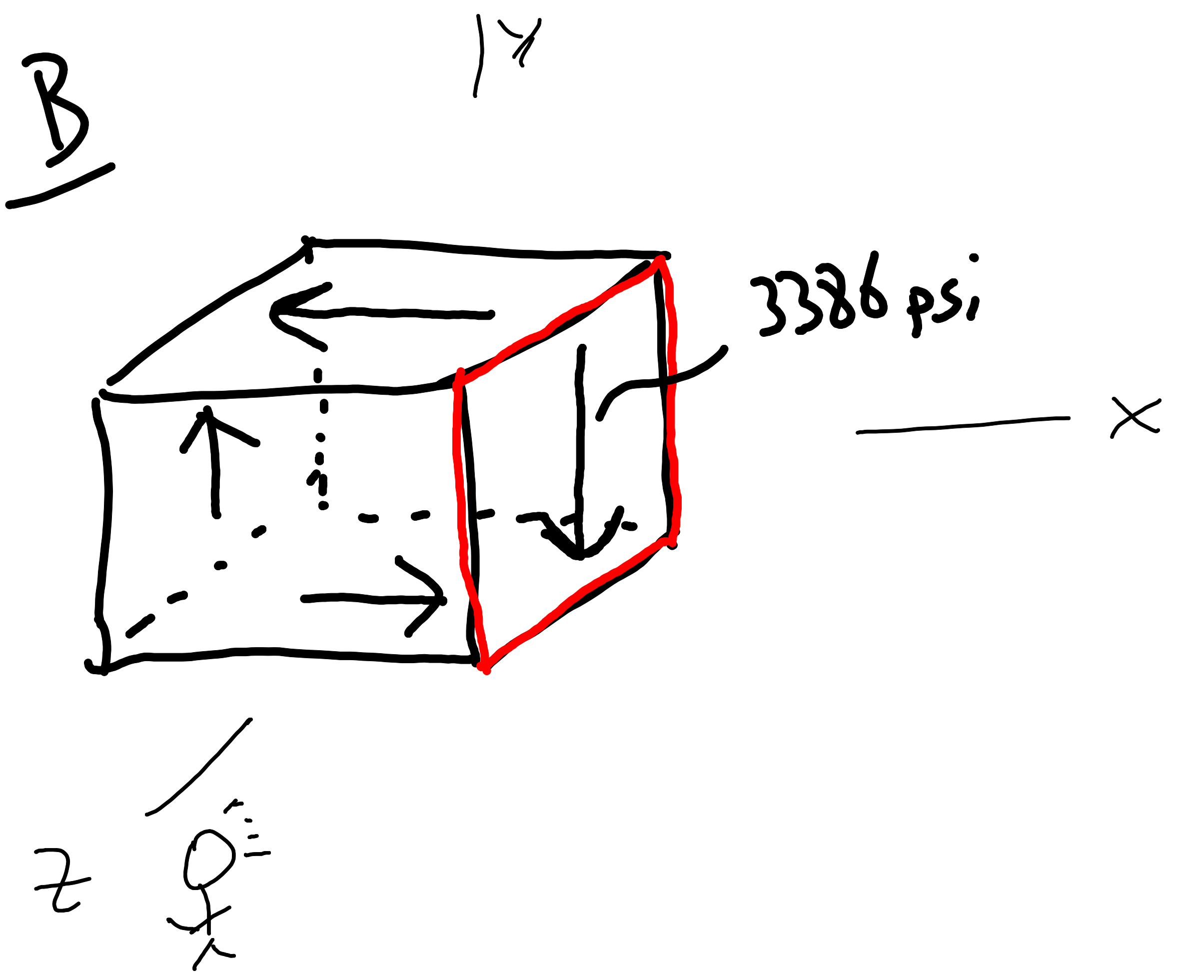
3.) Combine stress + draw stress element at B

$\boxed{\sigma_x^B = 0}$

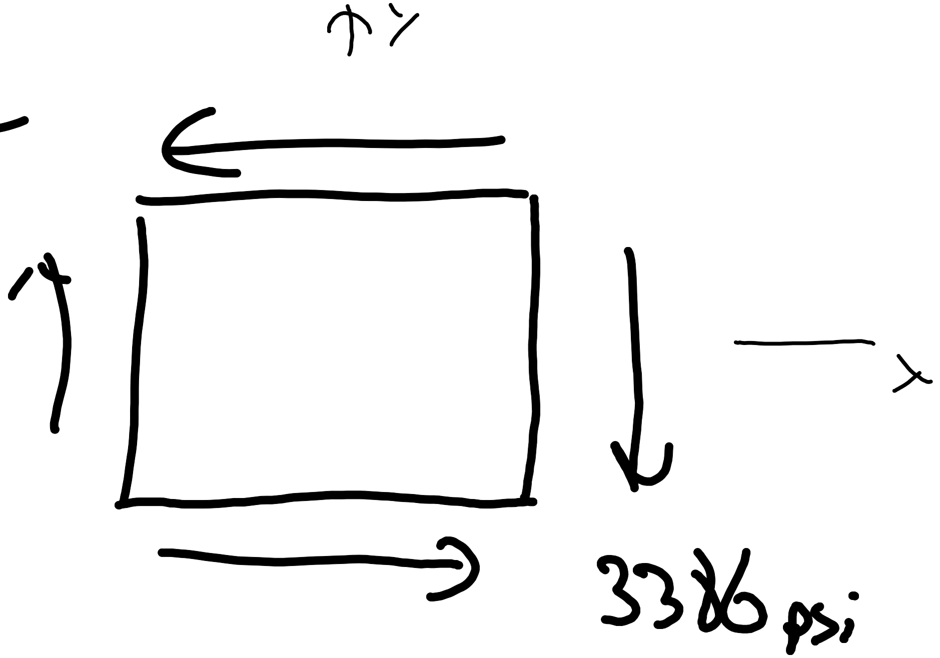
$$\tau_{xy}^B = -\frac{2P}{A} - \frac{PLd}{2I_p} = -283\,psi - 3103\,psi$$

$$\boxed{\tau_{xy}^B = -3386\,psi}$$

B

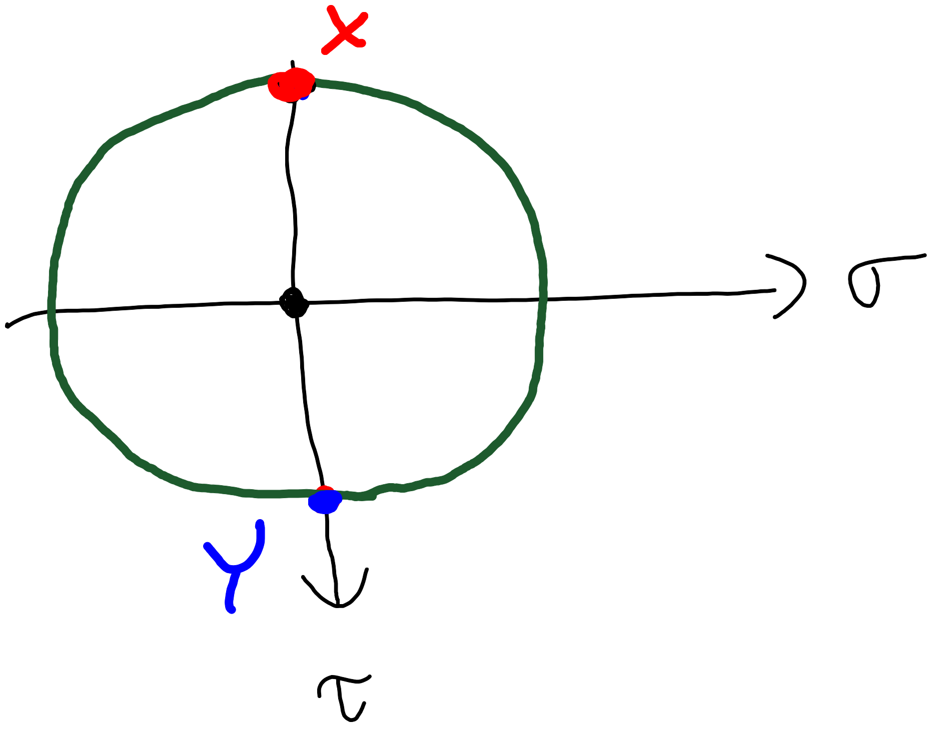


B



4.) Draw Mohr's circle + find principal stresses + max shear stress

B



X $(0, -3386)$ psi

Y $(0, 3386)$ psi

$\sigma_{avg} = 0$

$R = 3386 \text{ psi} = \tau_{max}$

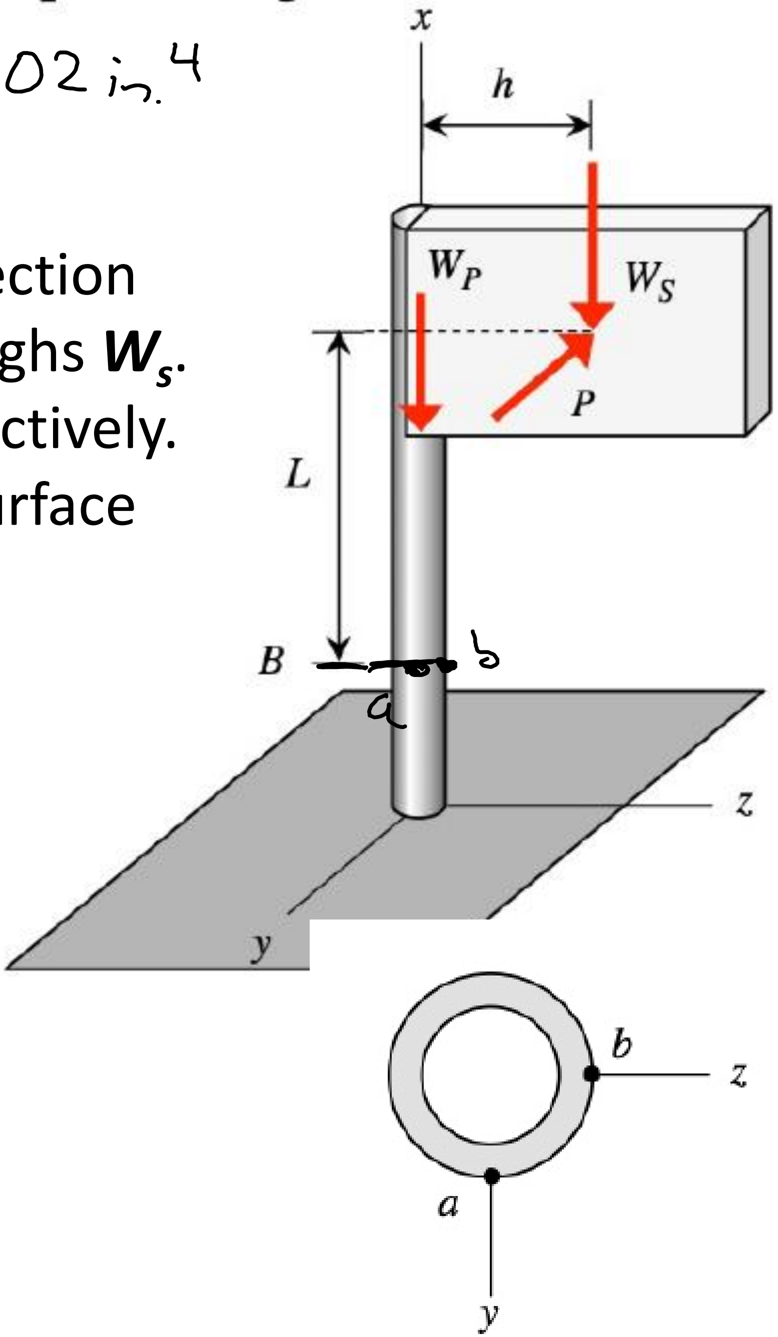
$$\boxed{\begin{aligned} \sigma_{p1} &= \sigma_{avg} + R = 3386 \text{ psi} \\ \sigma_{p2} &= \sigma_{avg} - R = -3386 \text{ psi} \\ \sigma_{p3} &= 0 \end{aligned}}$$

Example 14.12

$A = 2.23 \text{ in}^2$ $I_{zz} = I_{yy} = 3.02 \text{ in.}^4$

$I_p = 6.03 \text{ in.}^4$

Wind blowing on a sign produces a resultant force ***P*** in the –y direction at the point shown. The support pole weighs $\boldsymbol{W_P}$ and the sign weighs $\boldsymbol{W_s}$. The pole is a pipe with outer and inner diameters $\boldsymbol{d_o}$ and $\boldsymbol{d_i}$, respectively. Determine the principal stresses at points *a* and *b* on the outer surface of the pole at location B along the pole's length



pipe cross section at B

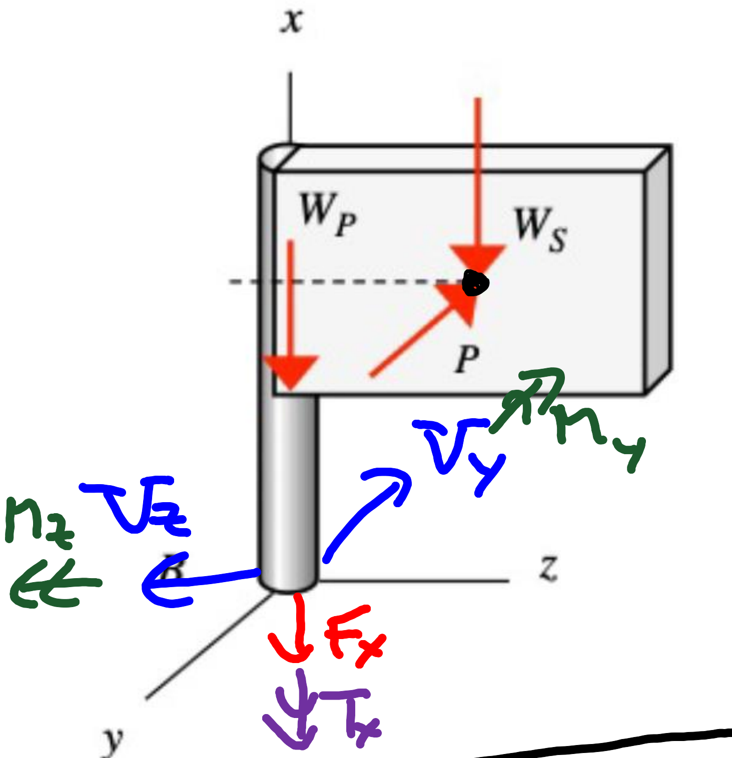
$h = 40$ in. $d_o = 3.5$ in. $W_P = 160$ lb

$L = 220$ in. $d_i = 3.068$ in. $W_s = 125$ lb

$P = 75$ lb

(~40 mph wind gust)

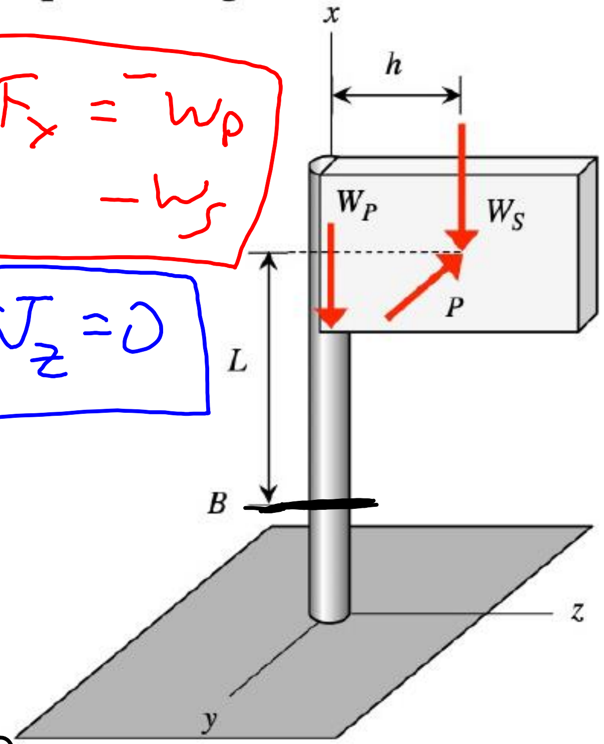
1.) FBD & int. resultants



$$\sum F_x = 0 = -F_x - W_p - W_s = 0 \Rightarrow \boxed{F_x = -W_p - W_s}$$

$$\sum F_y = 0 \Rightarrow \boxed{V_y = -P}$$

$$\sum F_z = 0 \Rightarrow \boxed{V_z = 0}$$

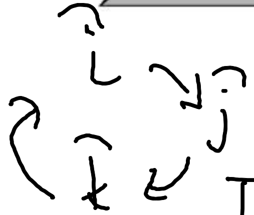


$$+\sum \vec{M} = 0 = -T_x\hat{i} - M_y\hat{j} - M_z\hat{k}$$

$$\cancel{+\vec{r}_1 \times \vec{F}_1} \to 0 + \vec{r}_2 \times \vec{F}_2$$

$$\vec{F}_1 = -W_p\hat{i} \qquad \cancel{\vec{r}_1 \times \vec{F}_1} = 0$$

$$\vec{r}_1 = L\hat{i}$$

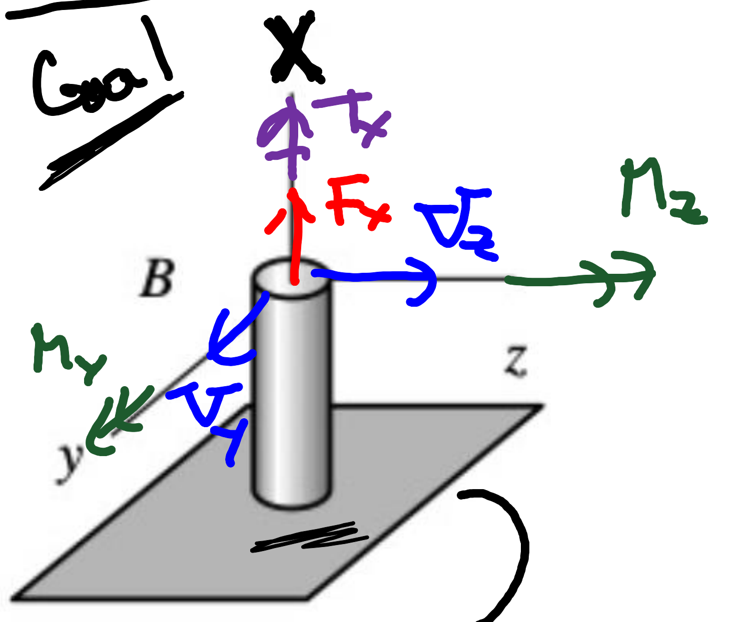


$$\vec{r}_2 = L\hat{i} + h\hat{k}$$

$$\vec{F}_2 = -W_s\hat{i} - P\hat{j} + 0\hat{k}$$

$$\boxed{\vec{r}_2 \times \vec{F}_2 = \boxed{Ph}\,\hat{i}\ \boxed{-W_s h}\,\hat{j}\ \boxed{-PL}\,\hat{k}}$$

$$\boxed{\begin{aligned} T_x &= Ph \\ M_y &= -W_s h \\ M_z &= -PL \end{aligned}}$$

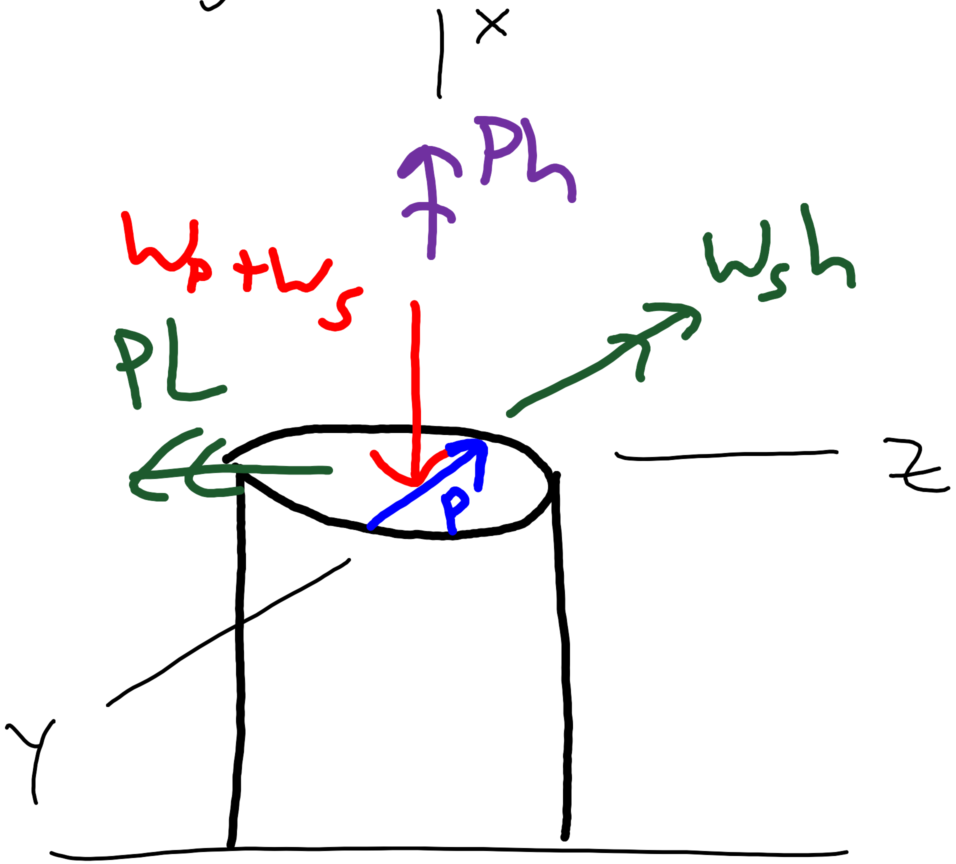


$$F_x\hat{i} + V_y\hat{j} + V_z\hat{k} = \vec{F}_1 + \vec{F}_2 = \underline{(-W_p - W_s)\hat{i} - P\hat{j} + 0\hat{k}}$$

$$T_x\hat{i} + M_y\hat{j} + M_z\hat{k} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = \underline{Ph\hat{i} - W_s h\hat{j} - PL\hat{k}}$$

These equations give us the internal resultant loads on the $+x$ face

Redrawing our FBD on the positive x face based on the forces + moments we calculated



since $F_x = -W_p - W_s$

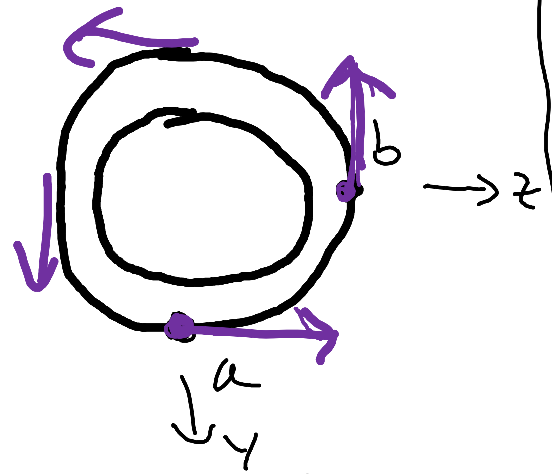
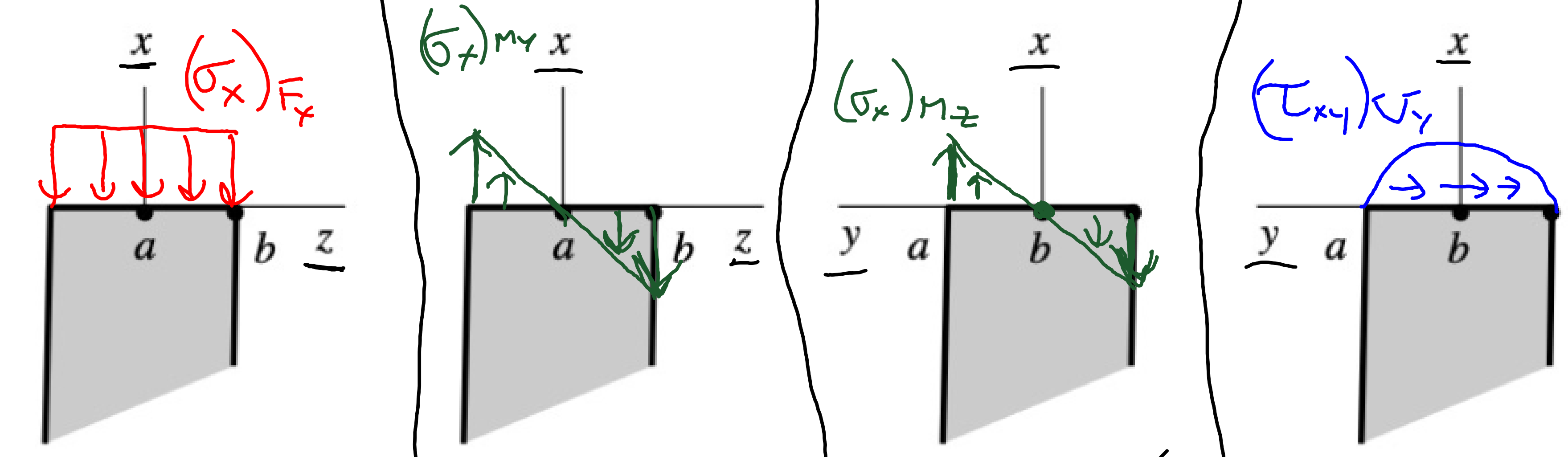
$V_y = -P$

$V_z = 0$

$T_x = +Ph$

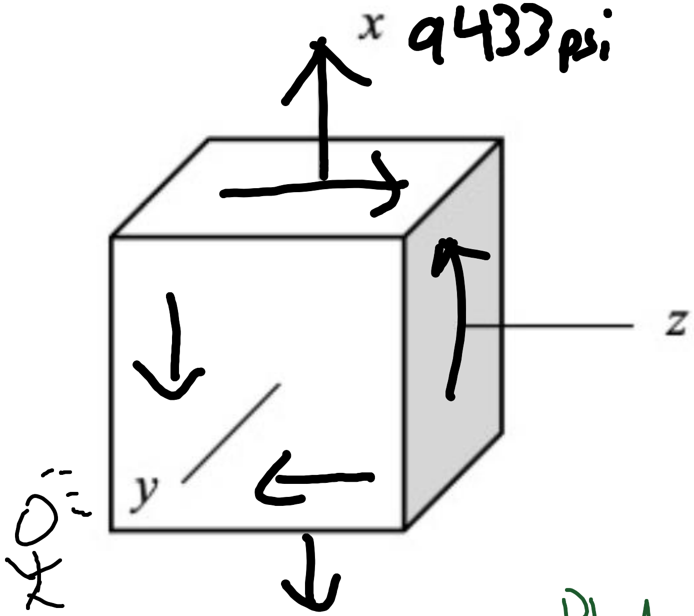
$M_y = -W_s h$

$M_z = -PL$

Load	T_x (torque)	F_x (axial)	M_y (bending)	M_z (bending)	V_y (shear)
Stress distribution					
Stress at point *a*	$\tau_{xz} = +(Ph)(d_o/2)/I_p = 871\,psi$	$\sigma_x = \frac{-W_p - W_s}{A} = -128\,psi$	0	$\sigma_x = \frac{+(PL)(d_o/2)}{I_{zz}} = 9561\,psi$	0
Stress at point *b*	$\tau_{xy} = -(Ph)(d_o/2)/I_p$	$\sigma_x = \frac{-W_p - W_s}{A}$	$\sigma_x = \frac{-(W_s h)(d_o/2)}{I_{yy}} = -2847\,psi$	0	$\tau_{xy} = \frac{-2V}{A} = -67\,psi$

pipe cross section at B

stress element at “a”



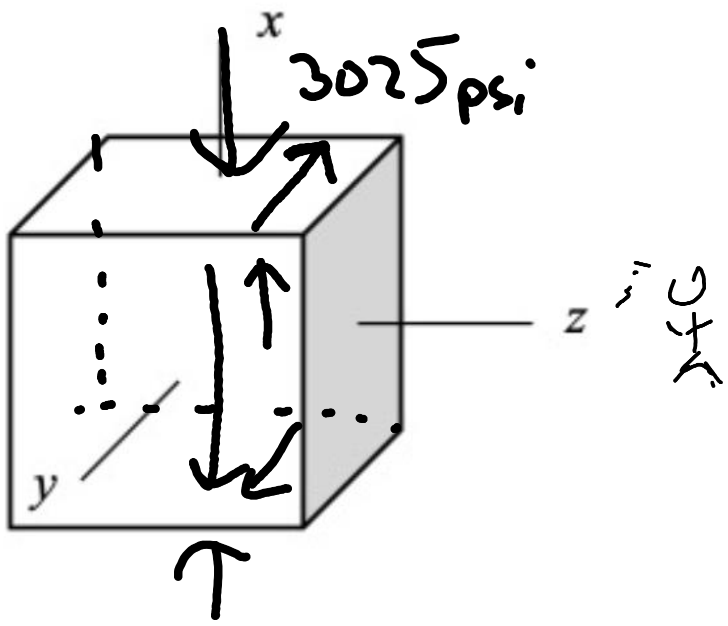
$$\sigma_x^a = \frac{-W_p - W_s}{A} + 0 + \frac{P L d_o}{2 I_{zz}}$$

$$= -128\text{psi} + 9561\text{psi}$$

$$\boxed{\sigma_x^a = {}^{+}9433\text{psi}}$$

$$\boxed{\tau_{xz} = \frac{+P h d_o}{2 I_p} = 871\text{psi}}$$

stress element at “b”



$$\sigma_x^b = \frac{-W_p - W_s}{A} - \frac{W_s h d_o}{2 I_{yy}} + 0 = {}^{-}128\text{psi} - 2897\text{psi}$$

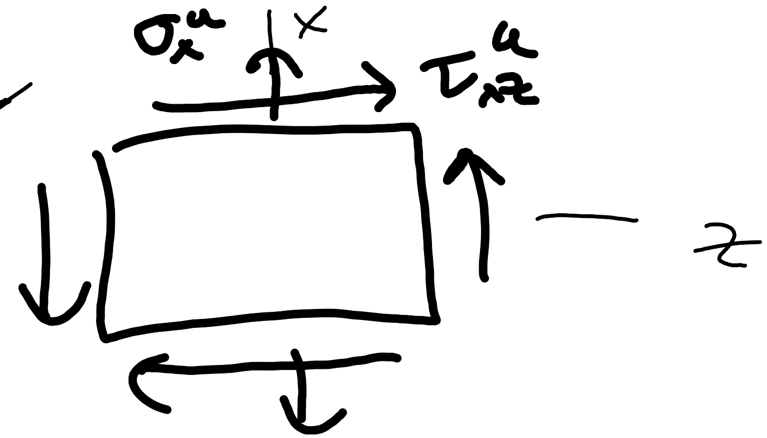
$$= \boxed{{}^{-}3025\text{psi} = \sigma_x^b}$$

$$\tau_{xy}^b = \frac{-P h d_o}{2 I_p} - \frac{2P}{A} = {}^{-}871\text{psi} - 67\text{psi}$$

$$\boxed{\tau_{xy}^b = {}^{-}938\text{psi}}$$

4.) Mohr's circle...

A



$$\sigma_{avg} = 4716.5\,psi = \frac{\sigma_x^a}{2}$$

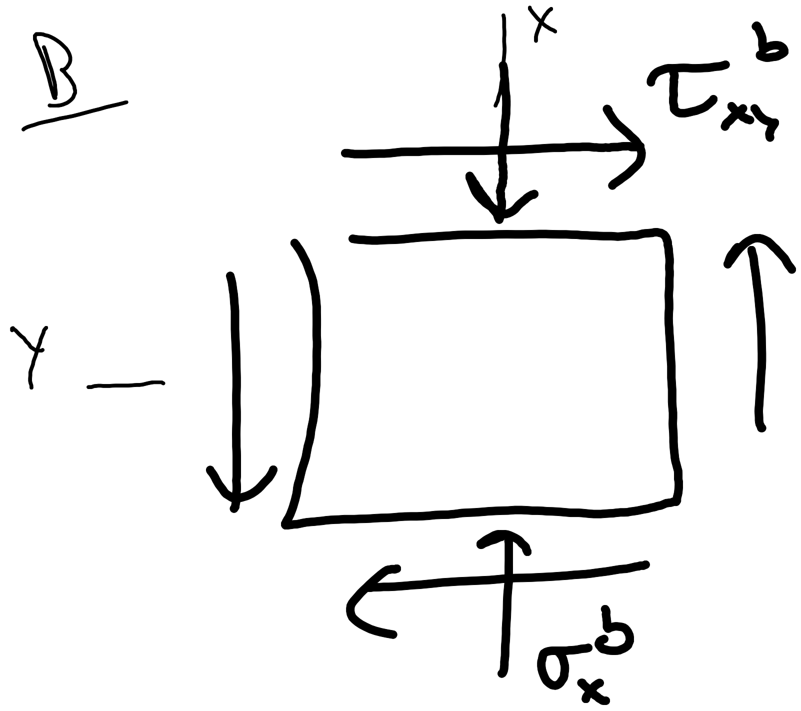
$$R = 4796\,psi = \tau_{max} = \tau_{max}^{abs}$$

$$\sigma_{p1} = \sigma_{avg} + R = 9513\,psi$$

$$\sigma_{p2} = \sigma_{avg} - R = -80\,psi$$

$$\sigma_{p3} = 0$$

B



$$\sigma_{avg} = -1512.5\,psi$$

$$R = 1780\,psi = \tau_{max}^{abs}$$

$$\sigma_{p1} = \sigma_{avg} + R = 267\,psi$$

$$\sigma_{p2} = \sigma_{avg} - R = -3292\,psi$$

$$\sigma_{p3} = 0$$

Procedure for combined loading problems

- Find the internal resultants at a cross section

- Calculate the stress at the point of interest due to each internal resultant

  mag. first; then sign

- Combine the individual stresses, and draw the stress element
  - For example, $\sigma_x = (\sigma_x)_{F_x} + (\sigma_x)_{M_y} + (\sigma_x)_{M_z}$

- Use Mohr's circle to determine the principal stresses, max shear stress, etc.
  - Make sure you identify the plane corresponding to the state of plane stress