Exam 2 (don't forget Ch. 9!)

Ch. 10-11 (stress & deflection in beams)

Ch. 16-17 (energy methods & FEM)

HW 6-9

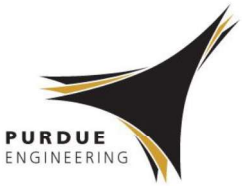
Lec. 15-29

Lecture 30: Thin-walled pressure vessels

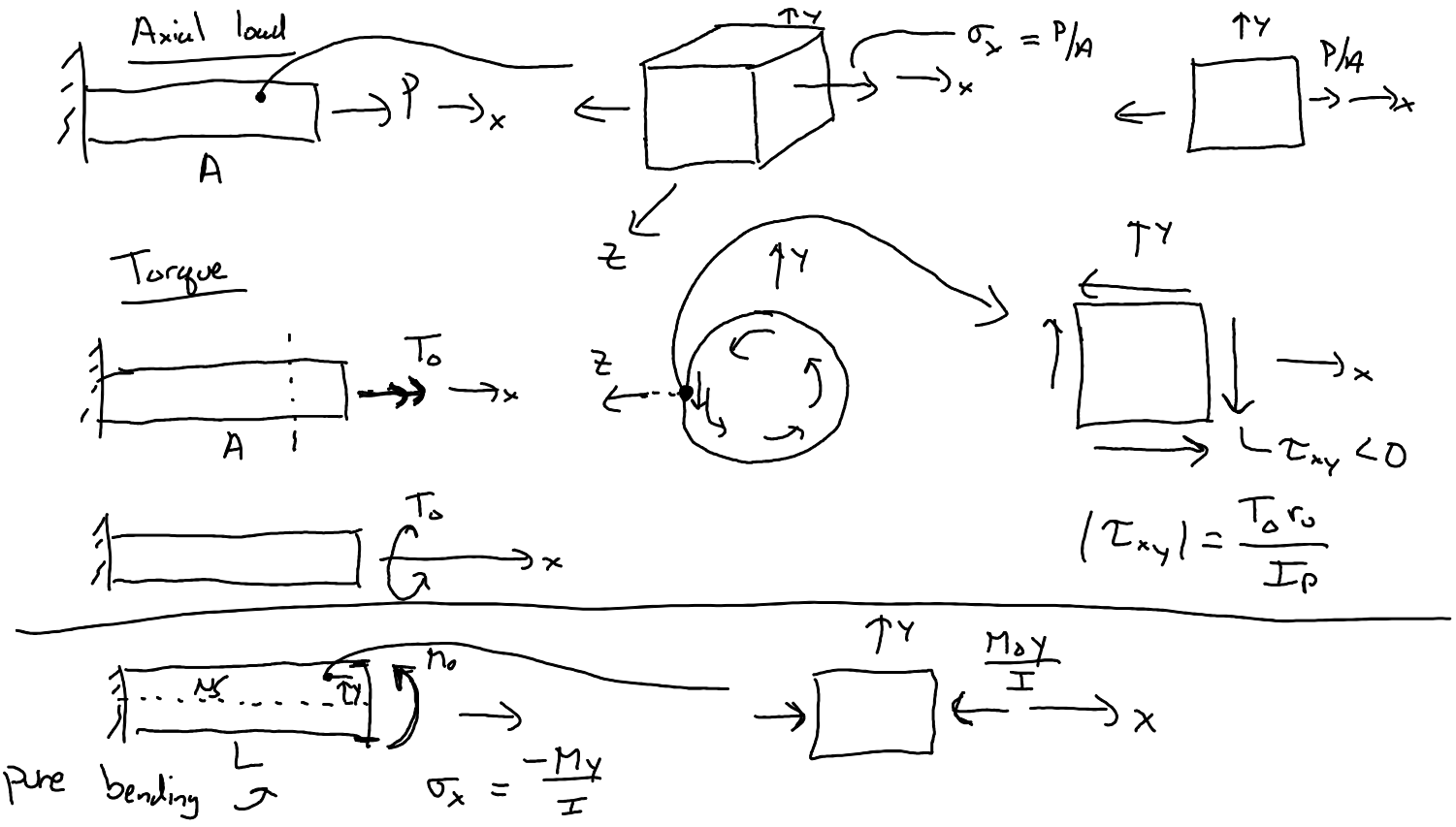
Joshua Pribe

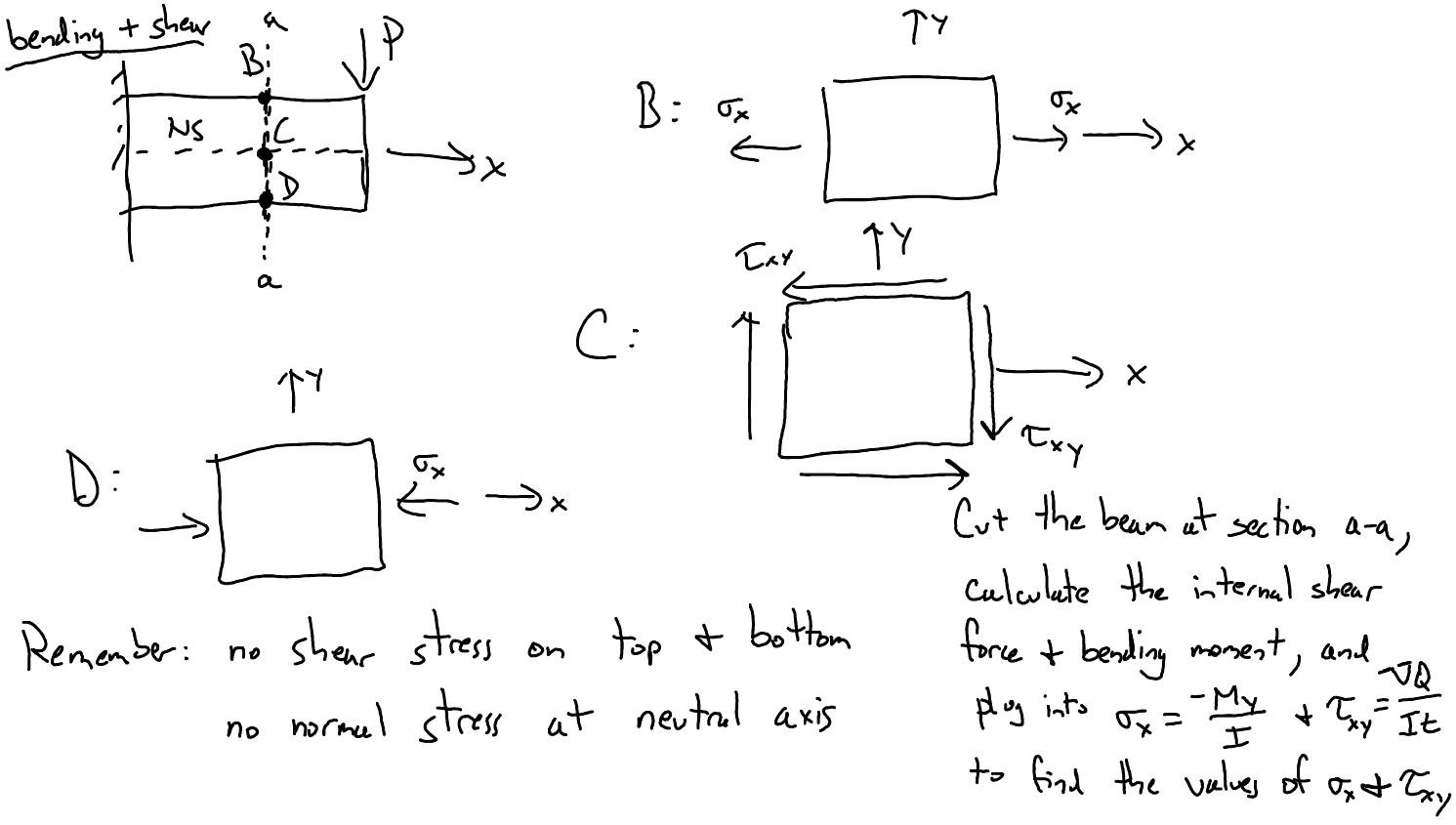
Lecture Book: Chapter 12

Fall 2019



Review of stress states we have looked at so far

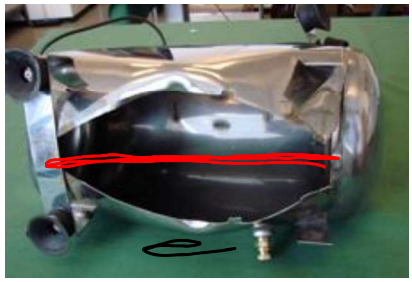


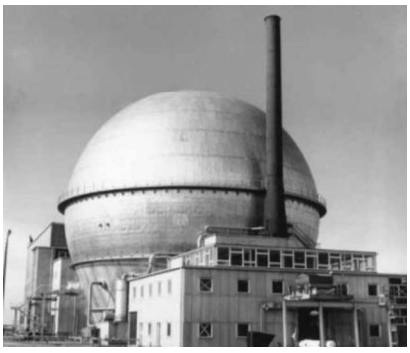


Outline

- **Start of Final Exam material** (this is *not* covered on Exam 2)
- Applications of pressure vessels
- Assumptions for stress analysis in thin-walled pressure vessels
- Stresses in thin-walled pressure vessels
  - Cylindrical pressure vessels
  - Spherical pressure vessels



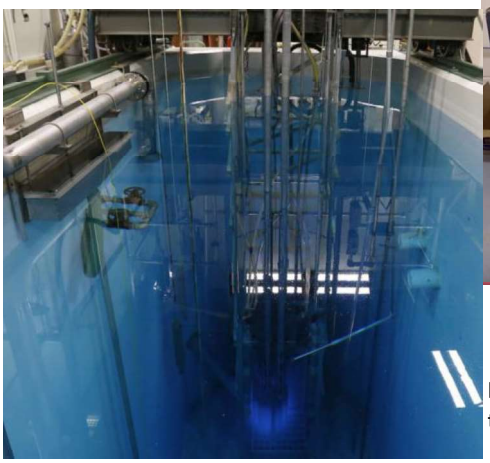




Pressure vessel examples

Design, construction, and maintenance covered by the ASME Boiler and Pressure Vessel Code
Can be subjected to internal *as well as external* pressure
Power generation, fuel containers, pressurized gas storage, …







Coolant processing at PUR-1

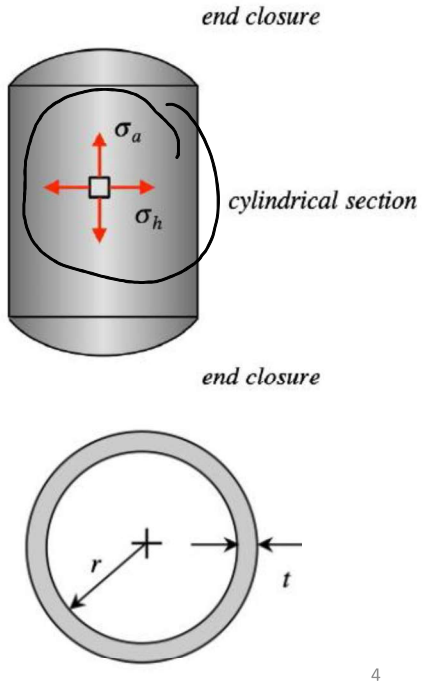
Nuclear reactor at Missouri S&T (any piping in the "pool" is subjected to hydrostatic pressure)

Assumptions

Lecture Book: Ch. 12, Pg. 3

- Geometry
  - Radius is at least 10 times the wall thickness
- Deformation
  - The strain varies insignificantly across the wall thickness
  - *The walls are in a state of "plane stress"*
- Material behavior
  - Linear elastic, small deformations (like usual)

$r \geq 10t$



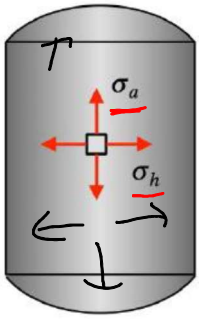
Stresses in cylindrical pressure vessels

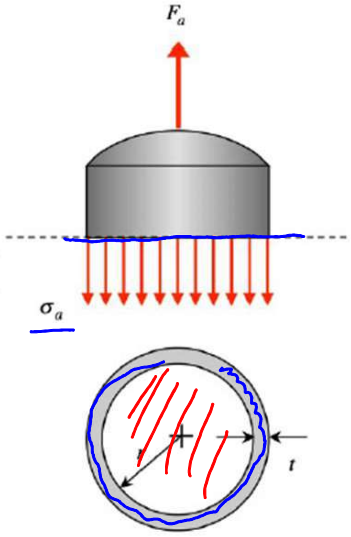
Cylindrical pressure vessel with internal pressure p

Neglect strains through the wall thickness → two stress components: **axial** and **hoop stress**

First, determine the **axial stress**, σ_a

$$\Sigma F = 0 \rightarrow \sigma_a = \frac{F_a}{A_{cut}}$$





$$F_a = p(\pi r^2)$$

$$A_{cut} = \pi(r+t)^2 - \pi r^2$$

$$t/r << 1 \Rightarrow A_{cut} \approx 2\pi r t$$

$$\sigma_a = \frac{pr}{2t}$$

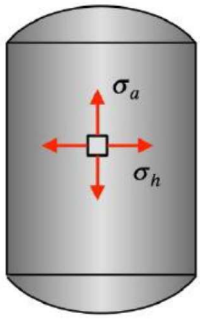
Lecture Book:
Ch. 12, Pg. 3

Stresses in cylindrical pressure vessels

Cylindrical pressure vessel with internal pressure p

Next, determine the **hoop stress**, σ_h

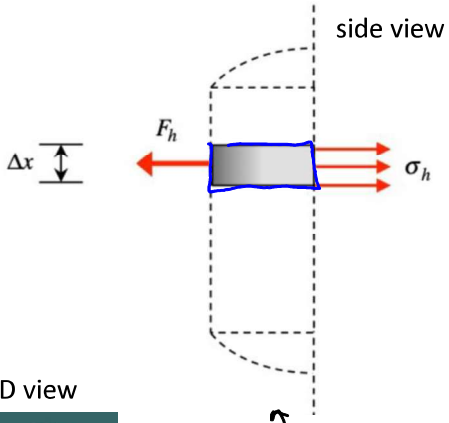
$$\Sigma F = 0 \rightarrow \sigma_h = \frac{F_h}{A_{cut}}$$



$$F_h = p(2r\Delta x)$$

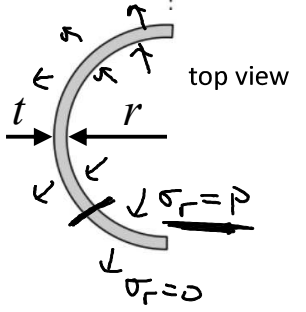
$$A_{cut} = 2t\Delta x$$

$$\sigma_h = \frac{pr}{t}$$



3-D view





Lecture Book:
Ch. 12, Pg. 4

Stresses in cylindrical pressure vessels

Axial stress $\sigma_a = \frac{pr}{2t}$

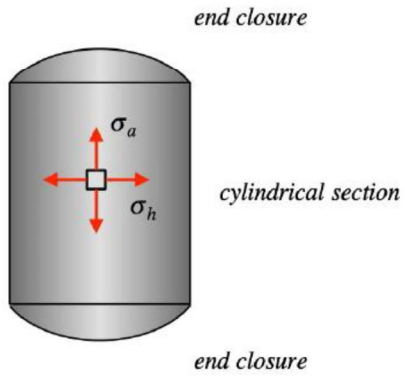
Hoop stress $\sigma_h = \frac{pr}{t}$

what about the radial stress σ_r? $\sigma_r = 0$ on the outer surface; $\sigma_r = p$ on the inner surface

Since $\sigma_h \propto \frac{r}{t}$ and $\frac{r}{t} \geq 10$,

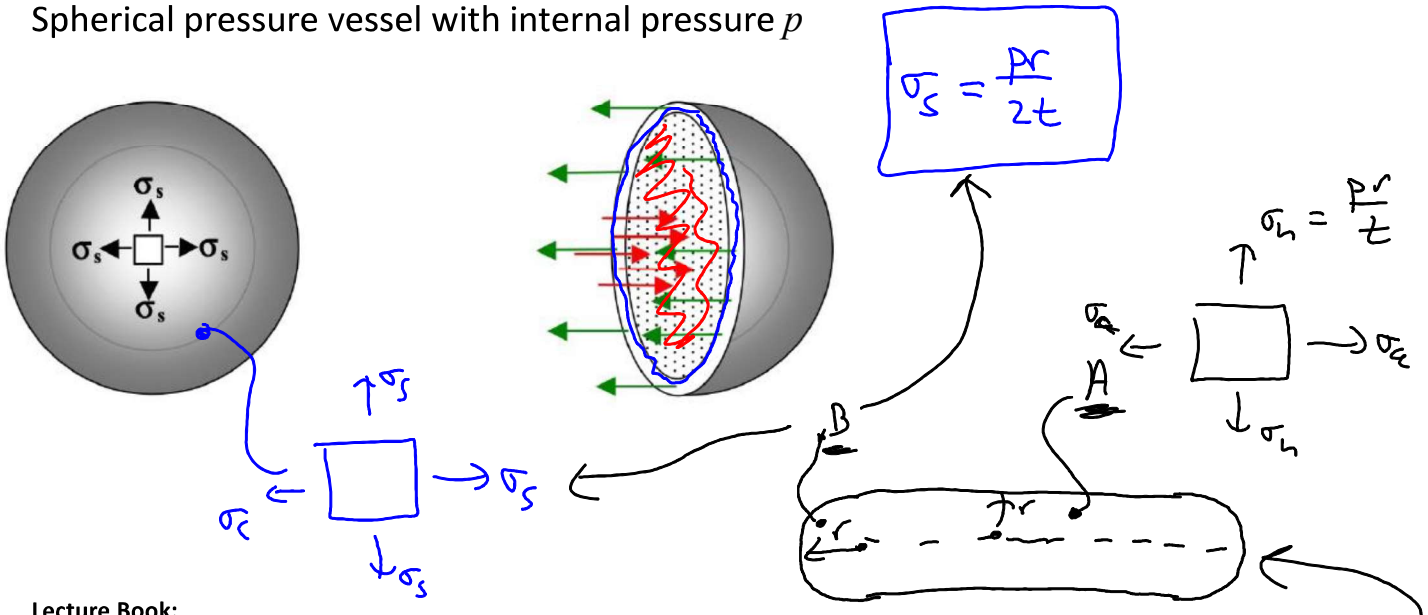
$\frac{\sigma_h}{\sigma_r} \geq 10 \Rightarrow \sigma_r$ is negligible

The hoop stress is exactly 2x the axial stress!



Stresses in spherical pressure vessels

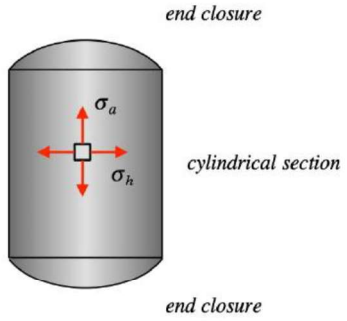
Spherical pressure vessel with internal pressure p



Lecture Book:
Ch. 12, Pg. 5

Pressure vessel summary

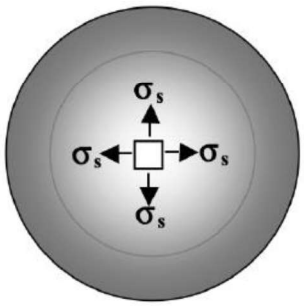
- Cylindrical pressure vessels
  - Axial stress: $\sigma_a = \frac{pr}{2t}$
  - Hoop stress: $\sigma_h = \frac{pr}{t}$



$r \geq 10t$

$\Rightarrow$ plane stress

- Spherical pressure vessels
  - Normal stress in any direction: $\sigma_s = \frac{pr}{2t}$

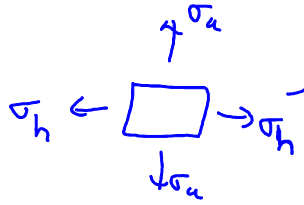


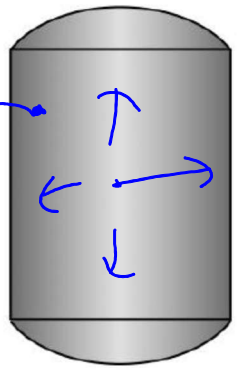
Example 12.1

A steel propane tank for a barbecue grill has a 12-in inside diameter and a wall thickness of 1/8 in. The tank is pressurized to 200 psi. Determine the axial and hoop components of stress in the wall of the tank.

$d = 12$ in. $\quad t = 1/8$ in.

$p = 200$ psi





$$\sigma_a = \frac{pr}{2t} = \frac{p(d/2)}{2t}$$

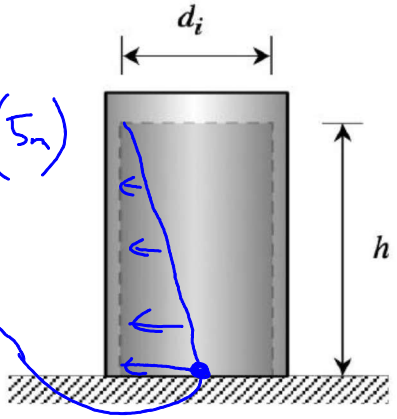
$$= \frac{(200\ \text{psi})(6\ \text{in.})}{2(1/8\ \text{in.})} = 4800\ \text{psi} = 4.8\ \text{ksi} = \sigma_a$$

$$\sigma_h = \frac{pr}{t} = \frac{p\,d/2}{t} = 9600\ \text{psi} = \sigma_h = 2\sigma_a$$

Example 12.2

A vertical standpipe has an inside diameter of $d_i = 3m$ and is filled with water to depth of h = 5m. If the allowable hoop stress is 80MPa, what is the minimum wall thickness of the tank?

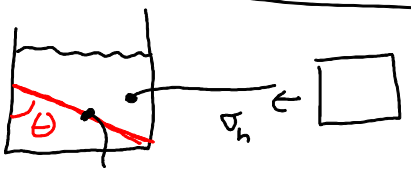
$d = 3m$ $\quad h = 5m$ $\quad \sigma_{allow} = 80\ MPa = \sigma_h$



$\sigma_h = \frac{pr}{t} = \sigma_{allow}$ $\quad p = \rho g h = \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{N}{kg}\right)(5m)$

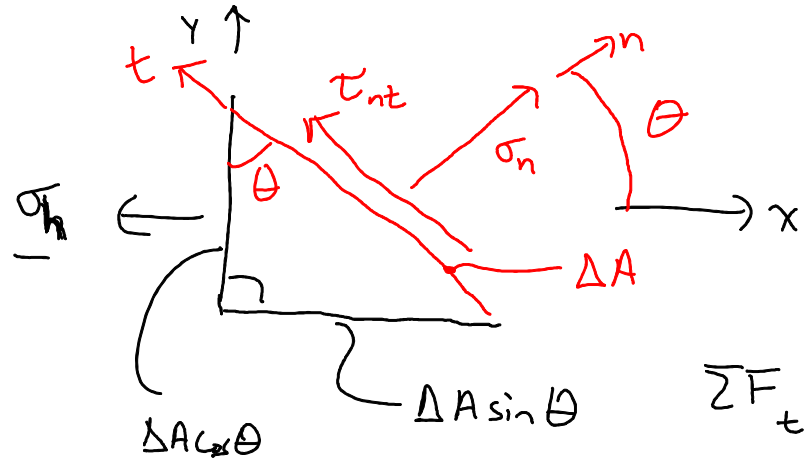
$t_{min} = \frac{pr}{\sigma_{allow}} \Rightarrow \boxed{t_{min} \approx 0.9\ mm}$

What if the top is open?



$\rightarrow \sigma_h = \frac{pr}{t}$ (no axial stress)

What if there is a weld at an angle θ?



$\Sigma F_n = 0$

$= \sigma_n \Delta A - (\sigma_h \cos\theta)(\Delta A \cos\theta)$

$\boxed{\sigma_n = \sigma_h \cos^2\theta}$

$\Sigma F_t = 0$

$\tau_{nt} \Delta A + (\sigma_h \sin\theta)(\Delta A \cos\theta) = 0$

$\boxed{\tau_{nt} = -\sigma_h \cos\theta \sin\theta}$

σ_n & τ_{nt} are the normal & shear stress on the weld surface

Example 12.4

A compressed air tank having an inner radius of 2 ft. and a wall thickness of 0.25 in. is manufactured by welding two steel hemispheres as shown in the figure.

(a) If the allowable tensile stress is 14000 psi and the allowable shear stress is 6000 psi, what is the maximum permissible air pressure in the tank? ↳ need Ch. 13 to solve this

(b) The welded seam would fail if the tensile load on the weld exceeds 8 kips per inch of the weld. If the required factor of safety against failure of the weld is 2.5, what is the maximum permissible pressure?

