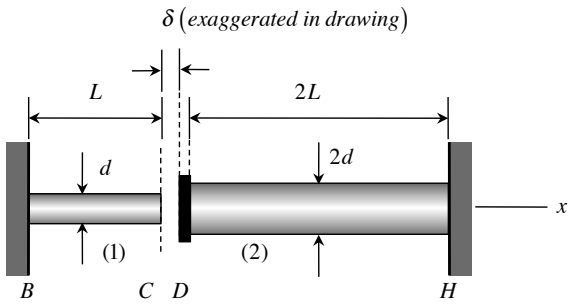
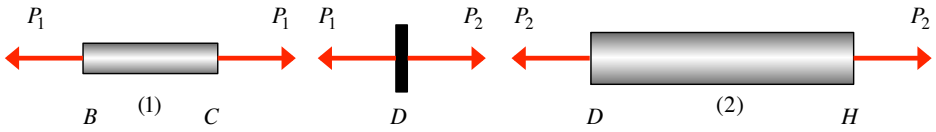
Example 7.4

Elements (1) and (2), each having a solid circular cross section, are made up of the same material with the material having a Young's modulus of E. Initially when the elements are unstressed, a gap of δ exists between end C of element (1) and the rigid connector D attached to element (2). The temperature of (1) is increased by an amount of ΔT while the temperature of (2) is held constant. Assuming that the temperature increase of (1) is sufficient to close the gap between C and D, what are the loads in (1) and (2) that result from the temperature increase of (1)?



Equilibrium: (by our convention, show all members in *tension*)



$$D: \quad \sum F_x = -P_1 + P_2 = 0 \quad \Rightarrow \quad P_1 = P_2 \qquad (INDETERMINATE!) \tag{1}$$

Elongation

$$e_1 = \frac{P_1 L}{E\pi\left(d/2\right)^2} + \alpha \Delta T L = \frac{4P_1 L}{\pi E d^2} + \alpha \Delta T L \tag{2}$$

$$e_2 = \frac{P_2 (2L)}{E\pi\left(2d/2\right)^2} = \frac{2P_2 L}{\pi E d^2} \tag{3}$$

Compatibility (kinematics)

$$u_C = u_B + e_1 = e_1$$

$$u_D = u_C - \delta \quad \left(assuming\ gap\ closed\right)$$

$$= e_1 - \delta$$

$$u_H = 0 = u_D + e_2 = e_1 - \delta + e_2 \quad \Rightarrow \quad e_1 + e_2 = \delta \tag{4}$$

Solve

Solving equations (1)-(4): $P_1 = P_2 = \frac{\pi E d^2}{6L}\left(\delta - \alpha \Delta T L\right)$