Lecture 5: Stress and strain—Generalized concepts

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PURDUE ENGINEERING

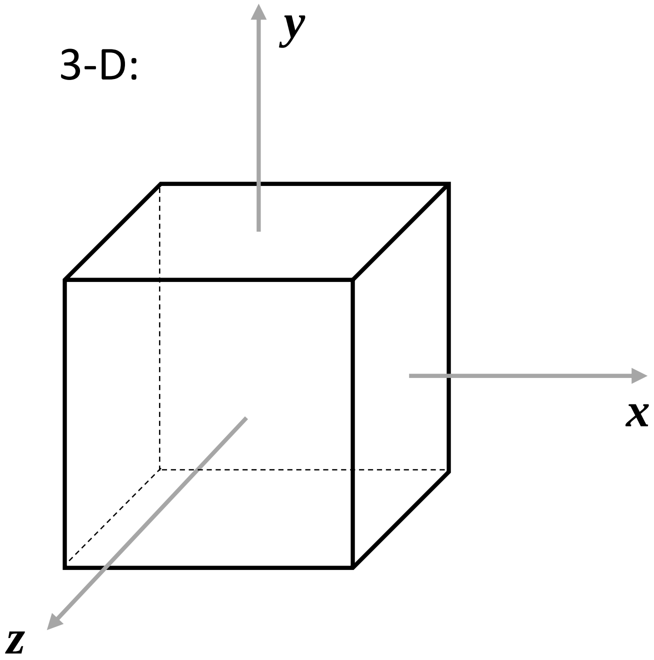
Objectives

- Review states of stress, generalized Hooke's law (Chapter 5 pre-week video)
  - Show how this relates to the normal and shear stresses we saw before
  - Remember: assume homogeneous, isotropic materials
  - Combine thermal and mechanical strains
- Illustrate these ideas with examples

Normal and shear stress

Normal stress

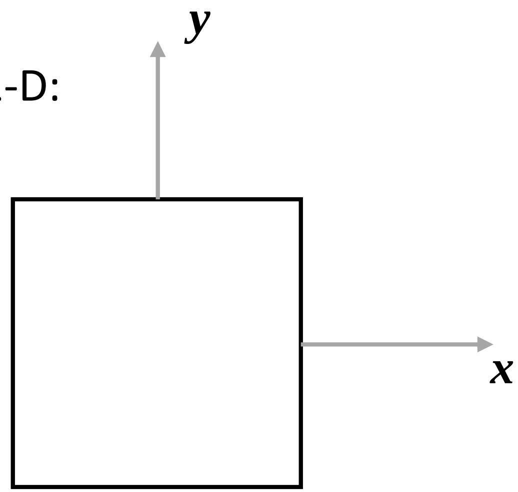
3-D:



Sign convention?

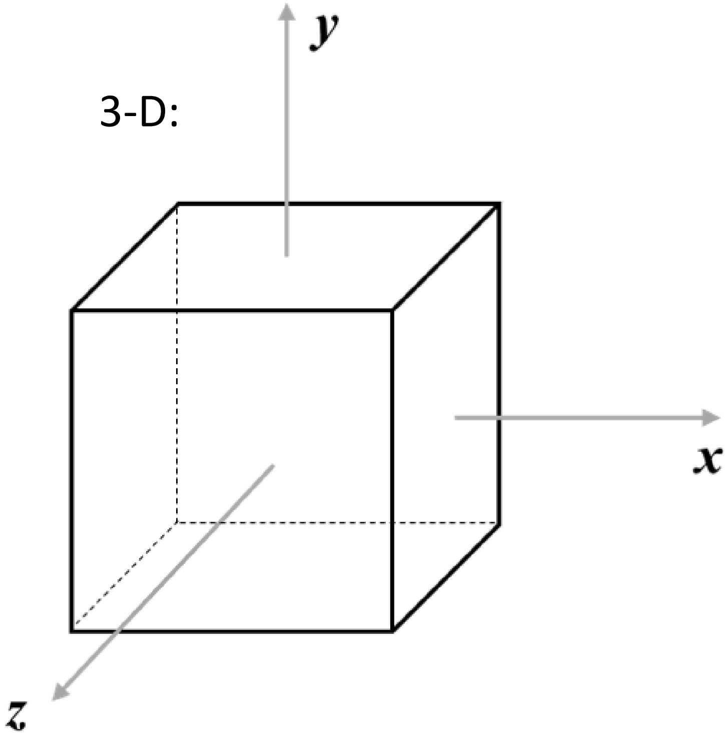
Shear stress

2-D:



Sign convention?

3-D:

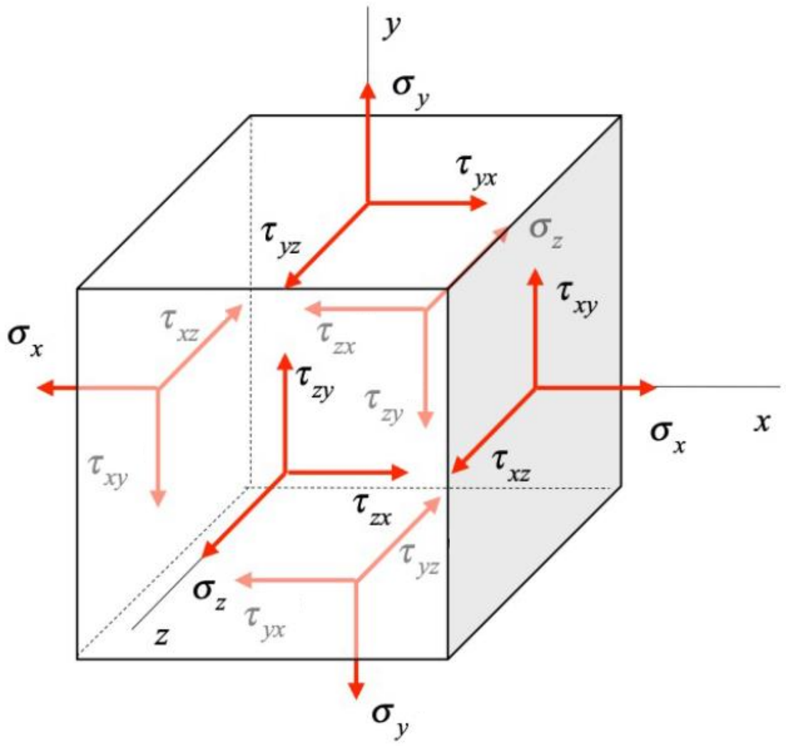


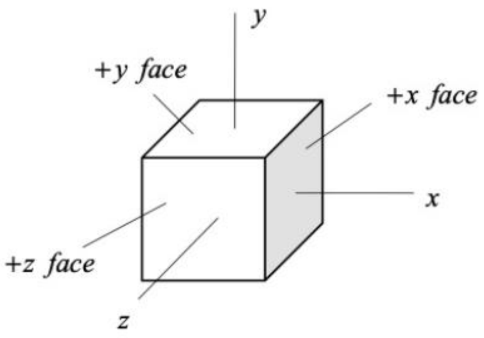
States of stress

<u>Three dimensions</u>

6 independent components of stress

- 3 normal: $\sigma_x, \sigma_y, \sigma_z$
- 3 shear: $\tau_{xy}, \tau_{xz}, \tau_{yz}$

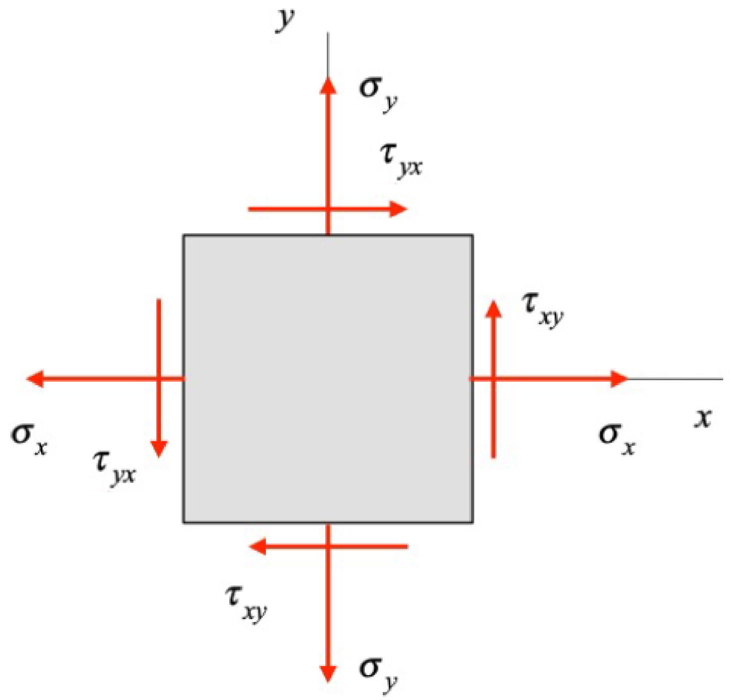




<u>Plane stress (two dimensions)</u>

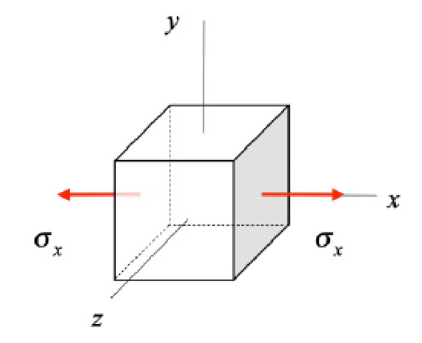
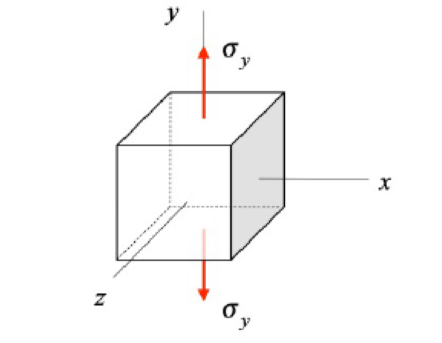
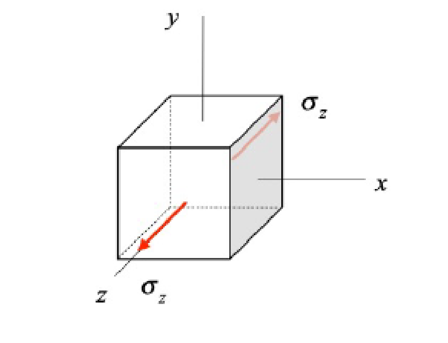
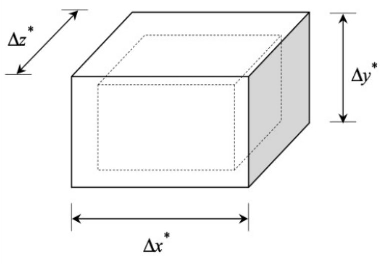
3 independent components of stress

- 2 normal: σ_x, σ_y
- 1 shear: τ_{xy}



Generalized Hooke's law

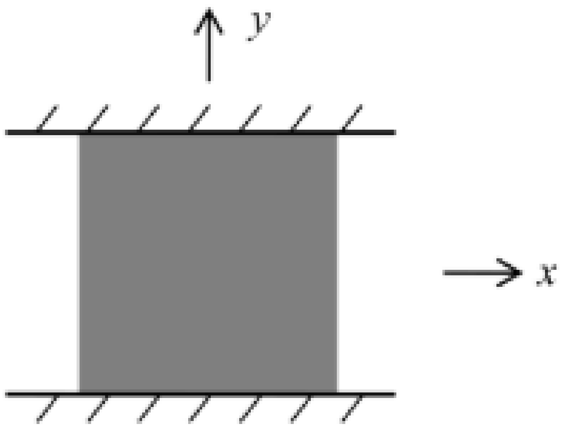
Add together <u>uniaxial load cases</u> and <u>temperature change</u> → extensional (normal) strain for arbitrary loading

	Mechanical strains			Thermal strains
	Strains due to mechanical loading in the x-direction	*Strains due to mechanical loading in the y-direction*	*Strains due to mechanical loading in the z-direction*	***Strains due to temperature change, ΔT***
				
ε_x	$\varepsilon_x = \sigma_x / E$	$\varepsilon_x = -\nu\varepsilon_y = -\nu\sigma_y / E$	$\varepsilon_x = -\nu\varepsilon_z = -\nu\sigma_z / E$	$\varepsilon_{x,T} = \alpha\Delta T$
ε_y	$\varepsilon_y = -\nu\varepsilon_x = -\nu\sigma_x / E$	$\varepsilon_y = \sigma_y / E$	$\varepsilon_y = -\nu\varepsilon_z = -\nu\sigma_z / E$	$\varepsilon_{y,T} = \alpha\Delta T$
ε_z	$\varepsilon_z = -\nu\varepsilon_x = -\nu\sigma_x / E$	$\varepsilon_z = -\nu\varepsilon_y = -\nu\sigma_y / E$	$\varepsilon_z = \sigma_z / E$	$\varepsilon_{z,T} = \alpha\Delta T$

Shear strains *always* related to shear stresses by $\gamma_{xy} = \tau_{xy}/G,\ \gamma_{xz} = \tau_{xz}/G,\ \gamma_{yz} = \tau_{yz}/G$

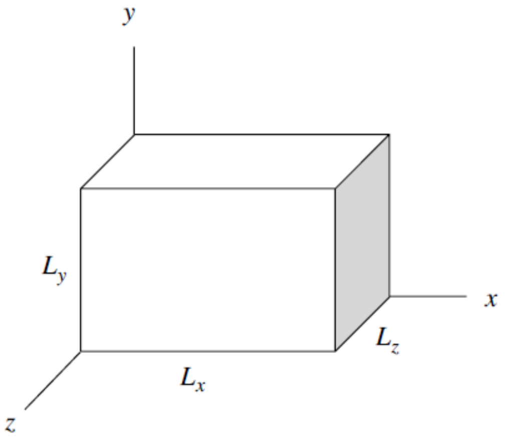
Example: Pre-week quiz, Problem 3

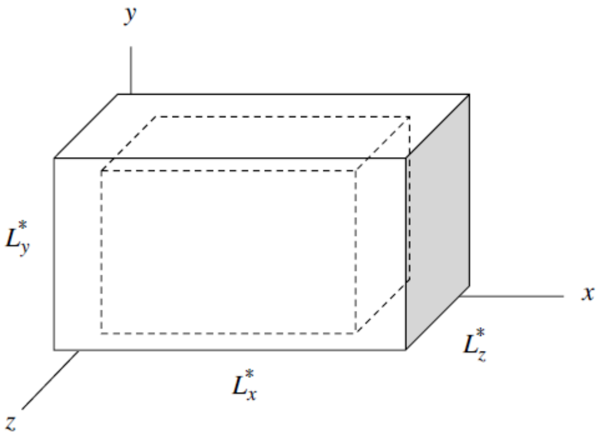
The 2-D body (i.e. plane stress) is *fully constrained in the y direction* and is *free to expand in the x direction*. If we increase the temperature by ΔT, what are the normal stresses and strains in the body?



Example: Volumetric strain

The "volumetric strain" is defined as the change in volume divided by the original volume (that is, $\varepsilon_V = \Delta V / V_0$). Show that, for small strains, the volumetric strain is also given by $\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$.





Summary

- Use a *stress element* to show the state of stress (normal and shear) at a point in a body
- Use the **generalized Hooke's law** to relate stress and strain for any arbitrary loading and temperature change
  - Assumptions: homogeneous, isotropic material; linear elastic deformation; small strains

<u>Normal strains</u>

$$\varepsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\sigma_y - \frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_z + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\sigma_z - \frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \alpha\Delta T$$

<u>Shear strains</u>

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

<u>Shear modulus</u>

$$G = \frac{E}{2(1+\nu)}$$