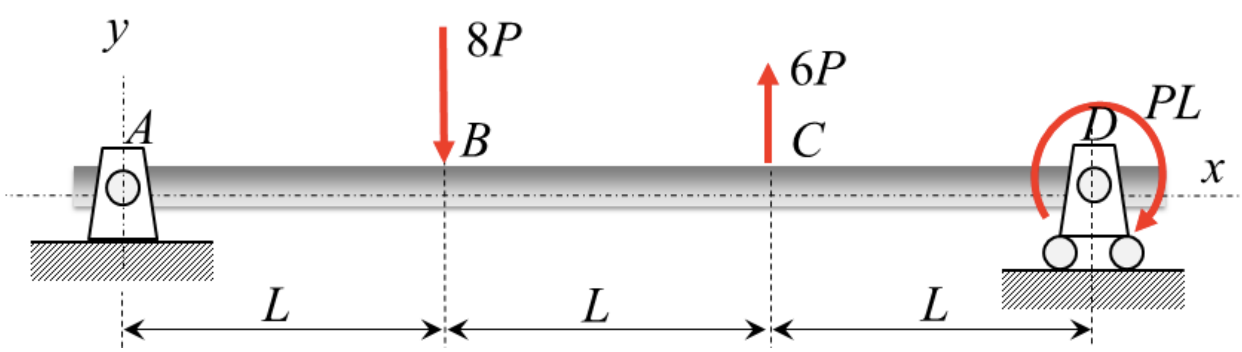
ME 323: Mechanics of Materials ***Homework Set 7***

Fall 2018 ***Due: Friday, Oct. 19***

Problem 7.1 (10 points)

Recall the simply supported beam analyzed in Homework Problem 6.2.

The beam is loaded as shown below. The beam is made up of a material with an elastic modulus E and a cross-sectional second area moment I, both of which are constant along the length of the beam.



Determine the following using the integration method:

a) The deflection curve for the beam, $v(x)$.
b) Sketch the beam's deflection.
c) The slope at the left end of the beam, θ_A.
d) The slope at the right end of the beam, θ_D.

Note: This problem can be solved using either the 2nd order method or the 4th order method. Recall that you have already solved for M(x) in HW6 and can use those expressions as a starting point for this assignment. Correct expressions for M(x) will be provided on the course blog after the HW6 submission deadline.

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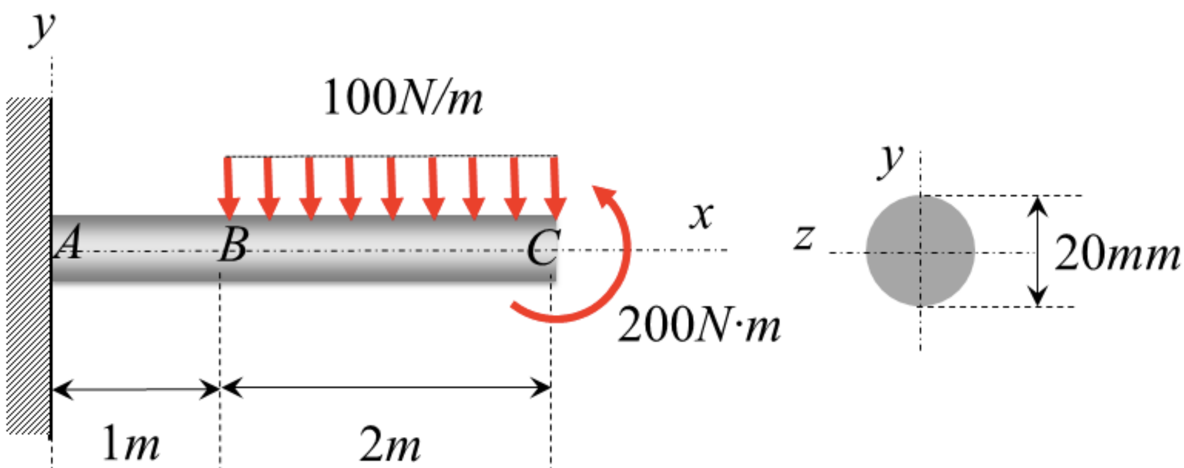
Problem 7.2 (10 points)

Recall the cantilever beam analyzed in Homework Problem 6.3.

The beam is aluminum with a modulus of elasticity of 70 GPa.



Determine the following using the integration method:

a) The deflection curve for the beam, $v(x)$.
b) Sketch the beam's deflection.
c) The slope at the right end of the beam, θ_C.
d) The deflection at the right end of the beam, v_C.

Note: This problem can be solved using either the 2nd order method or the 4th order method. Recall that you have already solved for M(x) in HW6 and can use those expressions as a starting point for this assignment. Correct expressions for M(x) will be provided on the course blog after the HW6 submission deadline.

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Problem 7.3 (10 points)

Determine the following using the superposition method:

a) For the simply supported beam in Problem 7.1, find the deflection at point B. (Hint: consider checking this answer using the expression for $v(x)$ obtained in Problem 7.1)

b) For the cantilever beam in Problem 7.2, find the deflection curve for the beam, $v(x)$, and the deflection at point B.

Note: Please refer to the following table for the necessary beam deflection equations.

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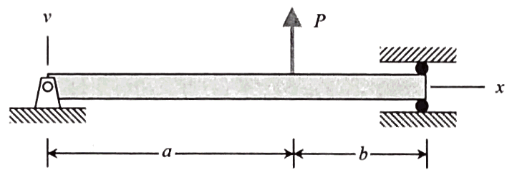
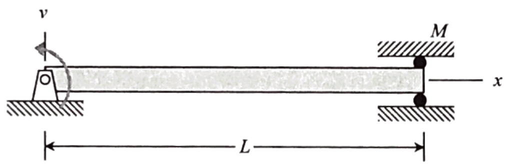
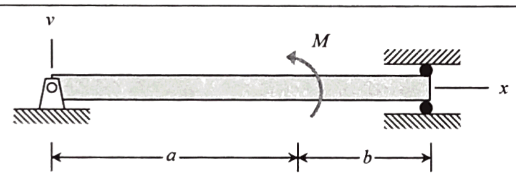
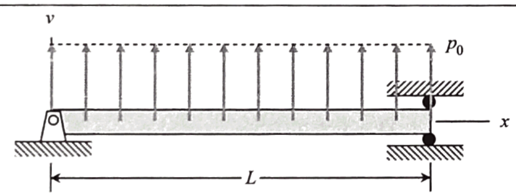
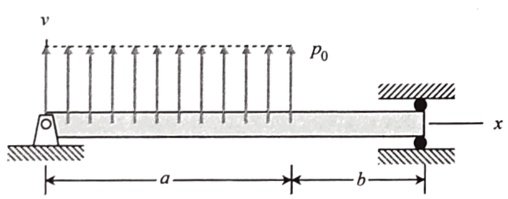
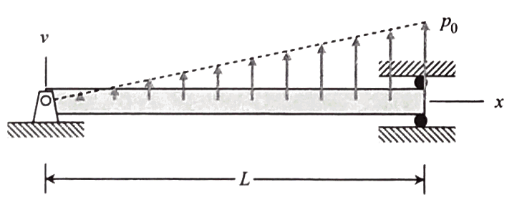
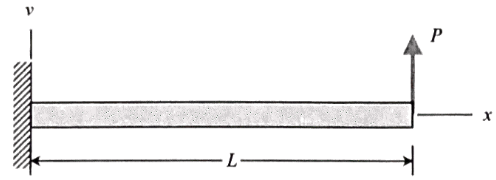
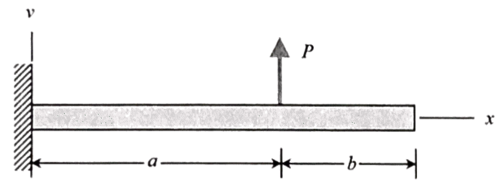
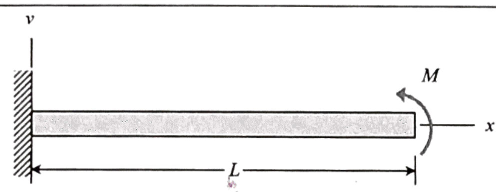
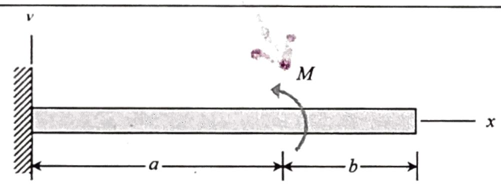
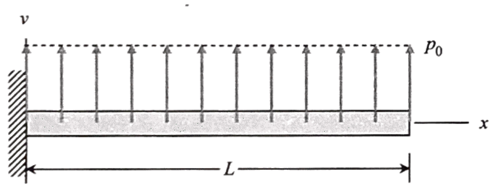
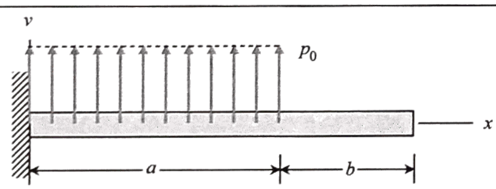
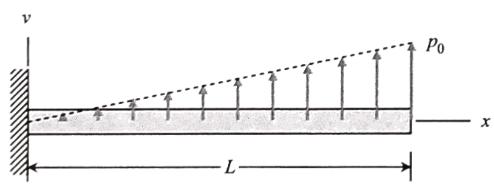
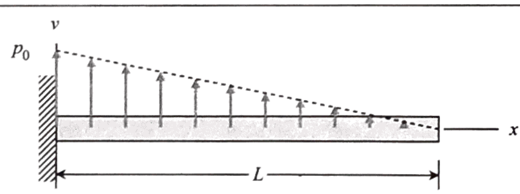
A.2 Beam deflection equations

Formulas are provided below for selected beams and beam loadings, where EI is the flexural rigidity for the beam material/cross section and L is the beam length.

SIMPLY-SUPPORTED BEAMS

Loading on beam	Deflection equation
	$v(x)=\frac{1}{6}\left[bx\left(L^2-b^2-x^2\right)\right]\frac{P}{LEI}\quad;\quad 0<x<a$
	$v(x)=\frac{1}{6}\left[x\left(2L^2-3Lx+x^2\right)\right]\frac{M}{LEI}$
	should be $+x^2$ $v(x)=\frac{1}{6}\left[x\left(-6aL+3a^2+2L^2-x^2\right)\right]\frac{M}{LEI}\quad;\quad 0<x<a$
	$v(x)=\frac{1}{24}\left[x\left(L^3-2Lx^2+x^3\right)\right]\frac{p_0}{EI}$
	$\begin{aligned} v(x)&=\frac{1}{24}\Big[x\Big(a^4-4a^3L+4a^2L^2 \\ &\quad +2a^2x^2-4aLx^2+Lx^3\Big)\Big]\frac{p_0}{LEI}\quad;\quad 0<x<a \\ &=\frac{1}{24}\Big[a^2\Big(-a^2L+4L^2x+a^2x \\ &\quad -6Lx^2+2x^3\Big)\Big]\frac{p_0}{LEI}\quad;\quad a<x<L \end{aligned}$
	$v(x)=\frac{1}{360}\left[x\left(7L^4-10L^2x^2+3x^4\right)\right]\frac{p_0}{LEI}$

CANTILEVERED BEAMS

<table>
<tr><th>Loading on beam</th><th>Deflection equation</th></tr>
<tr><td>
</td><td>$$v(x)=\frac{1}{6}\left[x^2(3L-x)\right]\frac{P}{EI}$$</td></tr>
<tr><td>
</td><td>$$v(x)=\frac{1}{6}\left[x^2(3a-x)\right]\frac{P}{EI} \quad ; \quad 0<x<a$$
$$=\frac{1}{6}\left[a^2(3x-a)\right]\frac{P}{EI} \quad ; \quad a<x<L$$</td></tr>
<tr><td>
</td><td>$$v(x)=\frac{1}{2}\left[x^2\right]\frac{M}{EI}$$</td></tr>
<tr><td>
</td><td>$$v(x)=\frac{1}{2}\left[x^2\right]\frac{M}{EI} \quad ; \quad 0<x<a$$
$$=\frac{1}{2}\left[a(2x-a)\right]\frac{M}{EI} \quad ; \quad a<x<L$$</td></tr>
<tr><td>
</td><td>$$v(x)=\frac{1}{24}\left[x^2\left(6L^2-4Lx+x^2\right)\right]\frac{p_0}{EI}$$</td></tr>
<tr><td>
</td><td>$$v(x)=\frac{x^2}{24}\left[6a^2-4ax+x^2\right]\frac{p_0}{EI} \quad ; \quad 0<x<a$$
$$=\frac{a^3}{24}\left[4x-a\right]\frac{p_0}{EI} \quad ; \quad a<x<L$$</td></tr>
<tr><td>
</td><td>$$v(x)=\frac{1}{120}\left[x^3\left(20L^3-10L^2x+x^3\right)\right]\frac{p_0}{LEI}$$</td></tr>
<tr><td>
</td><td>$$v(x)=\frac{1}{120}\left[x^2\left(10L^3-10L^2x+5Lx^2-x^3\right)\right]\frac{p_0}{LEI}$$</td></tr>
</table>

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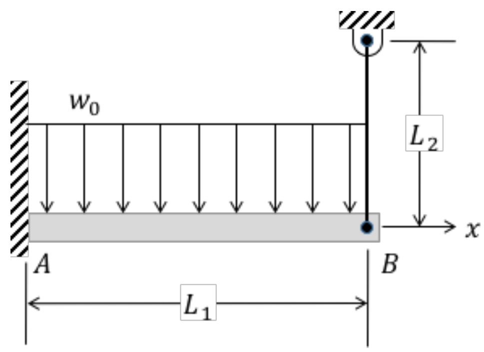
Problem 7.4 (10 points)

The cantilever beam, made of a material with an elastic modulus E_1 and a cross-sectional second area moment I_1, supports a uniformly distributed load with intensity w_0. The beam is pinned to a uniform rod at end B. The uniform rod has cross-sectional area A_2, modulus of elasticity E_2, and length L_2.

Use the following parameters for your analysis. $L_1 = 1$ m, $L_2 = 0.75$ m, $E_1 = 2E_2$, $A_2 = 20$ cm^2, $I_1 = 80$ cm^4, and $w_0 = 2$ kN/m.



Determine the following using the <u>integration method</u>:

a) The reactions at end A.
b) The tension in the rod, F_2.

<u>Note</u>: To solve this indeterminate problem, you will need to write a compatibility equation relating the deflection at B, $v_B = v(L_1)$, to the axial elongation of the uniform rod.

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