The truss shown is part of a system that is used to support a container. Complete the FBDs of the truss and truss section shown below.







Find the *internal* resultants (axial force, shear force, and bending moment) on the beam cross section at B.



The bolt shank is subjected to a tension of 80 lb. Determine the resultant internal loadings acting on the cross section at point C.



Frame member BCD is loaded with a line load  $p_0$  (force/length) over section CD. Ends B and D are connected to ground by roller and pin connections, respectively, as shown in the figure.

- a) Determine the external reactions on member BCD at ends B and D.
- b) Determine the internal shear force, axial force and bending moment acting on the left face of a cut through the member at E, where E is midway between C and D. Write your answers as vectors.



A solid brass rod AB and a solid aluminum rod BC are connected through a coupler at B. Determine the axial stresses in rod AB and rod BC.



A tension specimen having a diameter of  $d_0 = 13mm$  and length  $L_0 = 50mm$  was used to obtain the load-elongation data. Engineering stresses and strains were calculated from this data and plotted in the following figure.

- a) Determine the modulus of elasticity of the material.
- b) Determine the ultimate stress of the material.



A 175-lb woman stands on a vinyl floor wearing stiletto high-heel shoes. If the heel has the dimensions shown, determine the average normal stress she exerts on the floor and compare it with the average normal stress developed when a man having the same weight is wearing flat-heeled shoes. Assume the load is applied slowly, so that dynamic effects can be ignored. Also, assume the entire weight is supported only by the heel of one shoe.



7

Observe the following stress-strain curves for two materials A and B.



Which material is *stronger*?

Which material is *stiffer*?

Which material is more *ductile*?

In the truss shown below, all members have circular cross sections, with BC and BD having cross-sectional areas of A, and CD and DH having cross-sectional areas of 2A. All members are made up of a material having a Young's modulus of E and a Poisson's ratio of v. A vertical force P is applied to joint C of the truss. As a result of this applied load:

- a) Determine the stress in each of the four members. State whether each member is in tension or compression.
- b) Determine the elongation of member DH.
- c) Determine the change in the cross-sectional area of member DH.



# Example 3.4

Determine the average shear stress in the pin.



#### Example 3.7

The frame shown below is made up of links BK, CH and HK. Links BK and CH have lengths of L, and member HK has a length of 0.6L. Member HK is made up of two pieces that are spliced together as shown in the figure. The frame supports a drum of diameter D and weight W, as shown. All pins in the frame have a diameter of d. Link HK has a cross-sectional area of A. Consider the weight of the frame members to be small compared to the weight of the drum, and that all surfaces are smooth.

- a) Determine the axial stress in member HK of the frame.
- b) Determine the normal (n) and tangential (t) components of stress along the splice joint in member HK.
- c) Determine the shear stress in pin J of the frame.



#### Example 4.4

A compressor of weight W is suspended by long rods AB and CD of diameters  $d_1$  and  $d_2$ , respectively, as shown in the figure. Using the data given below, determine the allowable compressor weight  $W_{allow}$ . In doing so, neglect the weight of the platform between A and C, and neglect the weight of the two rods. Also, assume that rod AB and the pins at A and B are large enough that they do not need to be considered. All pin connections in the structure are double-sided.

$$\frac{Rod CD}{Pins C and D}: d_{p} = 100 \text{ mm}, \sigma_{allow} = 85 MPa$$

$$\frac{Pins C and D}{a}: d_{p} = 7 \text{ mm}, \tau_{allow} = 100 MPa$$

$$a = 0.75 \text{ mm}, b = 0.50 \text{ mm}$$



#### Example 4.6

The truss shown below is loaded with a force P at joint C. Member (1) of the truss is made up of two components that are joined with a pin having a diameter of d with a yield strength in shear of  $\tau_{y}$ .

- a) Determine the loads carried by the three members of the truss.
- b) Determine the minimum diameter d of the pin joining the two components of member AC such that the material of the pin does not yield with a factor of safety of *FS*.



Use the following parameter values in your analysis:  $a = 16/15 \ ft$ ,  $b = 3/5 \ ft$ ,  $h = 4/5 \ ft$ ,  $P = 20 \ kips$ , FS = 2 and  $\tau_y = 18 \ ksi$ .

The normal stress  $\sigma_x$  acts over the cross section of a rectangular bar with the distribution below.

- a) Determine the equivalent force/couple system at the middle of the cross section of the bar due to this stress.
- b) Determine the single-force equivalent resultant of this stress and its location on the cross-section.



On a particular cross section of a rectangular beam (with the beam having a depth of t into the page), there is shear stress whose distribution has the form of

$$\tau = \tau_{max} \left[ 1 - \left(\frac{y}{b}\right)^2 \right]$$

where y is measured from the centroid of the cross section. If the shear stress  $\tau$  may not exceed  $\tau_{allow}$ , what is the maximum shear force V that may be applied to the beam at this cross section?



A short cylinder ( $d_0 = 0.6$  in.,  $L_0 = 1.0$  in.) is compressed between two smooth rigid plates by an axial force P = 5 kips. (a) If the measured shortening of the cylinder due to this force is 0.00105 in., what is the modulus of elasticity, E, of the material? (b) If the increase in diameter is 0.00021 in., what is the value of Poisson's ratio, v, for this material?



The flat-bar plastic test specimen shown in the figure has a reduced-area rectangular cross-section test section that measures 0.5 in. × 1.0 in. Within the test section a strain gage oriented in the axial direction measures  $\varepsilon_x = 0.002 \frac{\text{in}}{\text{in}}$ , while a strain gage mounted in the transverse direction measures  $\varepsilon_y = -0.0008 \frac{\text{in}}{\text{in}}$ , when the load on the specimen is P = 300 lb. Determine the values of the modulus of elasticity, *E*, and Poisson's ratio, *v*.



Consider the 3D, parallelepiped body shown below that has experienced  $\sigma_x$ ,  $\sigma_y$  and  $\sigma_z$  components of stress. The body originally had dimensions of  $L_x$ ,  $L_y$  and  $L_z$ , and after the application of the loads, the body has dimensions of  $L_x^*$ ,  $L_y^*$  and  $L_z^*$ . The "volumetric strain" as a result of such loading is defined as the change in volume divided by the original volume:

$$\varepsilon_V = \frac{\Delta V}{V}$$

Suppose that the material making up the body is linearly elastic with a Young's modulus of E and a Poisson's ratio of v.

- a) Show that for small strains<sup>1</sup>, the volumetric strain can be written as:  $\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$
- b) Consider a state of stress with the following known information:  $\sigma_x = 20 \text{ ksi}$ ,  $\sigma_z = 25 \text{ ksi}$  and  $\varepsilon_y = -0.001 \text{ in / in}$ , and with the remaining components of stress and strain being unknown. The material has a Young's modulus of  $E = 30 \times 10^6 \text{ psi}$  and v = 0.3. Determine the volumetric strain for this state of stress.



**BEFORE** loading

AFTER loading

<sup>&</sup>lt;sup>1</sup> Consider strains small enough that products of strains are negligible as compared to the strains themselves; e.g.,  $\varepsilon_x \varepsilon_v \ll \varepsilon_z$ .

Example shown below is made up of two members extending the full length of the composite element. The two members below experience identical strains  $\varepsilon_1 = \varepsilon_2 = \varepsilon$  due to end connections to rigid plates. Determine the stresses in each member and determine the distance d locating the point of application of the load P needed for equal strains in the two members.



A rod having a cross sectional area A is acted upon by a linearly-varying force/length p(x), as shown below, where p(0) = 0 and  $p(L) = p_0$ . The material of the rod has a Young's modulus of E that is constant throughout the rod. Determine the total axial elongation of the road as a result of the axial loading p(x).



Recall that we discussed earlier that the determination of the load carried by the four tires of an automobile is a statically indeterminate problem. What factors determine the load carried by the tires? Suppose that we model the automobile as a rigid body with center of mass at G supported by four springs representing the stiffness of the four tires. With this model, determine the load carried by each of the four tires.



A rod is made up of elements (1), (2) and (3), with the elements being made up of materials with Young's moduli of  $E_1 = E$ ,  $E_2 = 2E$  and  $E_3 = 2E$ , respectively. Elements (1) and (3) are solid, each with an outer diameter of *d*, whereas (2) is a tube with inner and outer diameters of *d* and 2*d*, respectively. An axial load F acts at connector D. The goal of this problem is to determine the stresses in each element, leaving your answers in terms of, at most, *d* and *F*. To this end, please show the following four-steps:

- 1. *Equilibrium equations*: FBD and equilibrium equation of each connector.
- 2. *Load/deformation equations*: Write down the force/elongation equation for each element.
- 3. *Compatibility equations*: Write down the compatibility equations relating the elongations and displacements.
- 4. *Solve*: First checking to see if you have enough equations, solve for element loads, from which you find the stresses.



# Example 6.14 <u>PART A</u>

A three-segment rod is constructed as shown below. Segments (1) and (2) have a length of L, whereas segment (3) has a length of 2L. Segments (1) and (2) have solid, circular cross sections with diameters of 2d and d, respectively, whereas segment (3) is a tube with outer and inner diameters of 3d and 2d, respectively. Segments (1) and (2) are joined by a rigid connecter at C, and segments (2) and (3) are joined by a rigid connector at D. Ends B and H of the rod are fixed to rigid walls. All three segments are made of the same material, with E being the Young's modulus of the material. A force P acts on connector D.

- a) Determine the stresses in each of the three segments of the rod.
- b) Determine the displacements of connectors C and D.



# PART B

Consider the two structures below, (i) and (ii). In each case, let  $F_1$  and  $F_2$  represent the axial loads carried by members (1) and (2), with the sign conventions that  $F_i > 0$  and  $e_i > 0$  for the *i*<sup>th</sup> member being in tension. For each structure, write down the *compatibility equation* relating the elongations  $e_1$  and  $e_2$ .



Additional lecturebook examples

A truss is constructed using three identical members (each of length L, cross-sectional area A and made up of a material having a Young's modulus of E). A vertical load P acts on joint D.

- a) Determine the stresses in each of the three segments of the truss. Leave your answers in terms of P and A.
- b) Determine the x- and y-components of displacement of joint D. Leave your answers in terms of, at most, P, L, E and A.



# Example 7.4

Elements (1) and (2), each having a solid circular cross section, are made up of the same material with the material having a Young's modulus of E. Initially when the elements are unstressed, a gap of  $\delta$  exists between end C of element (1) and the rigid connector D attached to element (2). The temperature of (1) is increased by an amount of  $\Delta T$  while the temperature of (2) is held constant. Assuming that the temperature increase of (1) is sufficient to close the gap between C and D, what are the load in (1) and (2) that result from the temperature increase of (1)?



#### Example 7.7

A structure is made up of a rigid member CK and three rod elements (1), (2) and (3). The cross-sectional area for each element is A. The material makeup of the three elements is such that the Young's moduli are related by  $E_1 = E_3 = E$  and  $E_2 = 2E$ . The coefficient of thermal expansion for each member is  $\alpha$ . A load P is applied to member CK as shown, with the temperature of elements (1) and (3) increased by  $\Delta T$ . The temperature of element (2) remains unchanged. The load P is given by  $P = 2\alpha\Delta TEA$ .

- a) Determine the stresses in each of the three elements of the structure. Leave your answers in terms of the variables given here in the problem statement.
- b) Indicate whether the stress in each member is compressive or tensile.
- c) Indicate whether the strain in each member is compressive or tensile.



# Example 8.2

The solid circular shaft of diameter d shown in (a) below has a maximum shear stress of  $\tau_{max} = \tau_a$  under the action of an applied torque  $T_a$ . If the solid shaft is replaced by a tubular shaft shown in (b) below with  $d_o / d_i = 1.2$  but weighing the same as the solid shaft of (a), by what percentage would the torque have to be increased in order to produce the same maximum shear stress?



# Example 8.8

A shaft is made up of elements (1), (2) and (3). Elements (1) and (2) have solid cross sections each with a diameter of d, and have lengths L and 2L, respectively. Element (3) has a tubular cross section with inner and outer diameters of d and 2d, respectively. Elements (1) and (2) are joined with a rigid connector at C, whereas elements (2) and (3) are joined with a rigid connector at H. Elements (1) and (3) are rigidly connected to ground at ends B and D, respectively. The material makeup of each element is the same, having a shear modulus of G. Torques are applied at connectors C and H, as shown in the figure.

- a) Determine the torques carried by each element. Express your answers in terms of T.
- b) What is the maximum shear stress in the shaft? Express your answer in terms of T and d. At what points on the shaft does this maximum shear stress exist?

