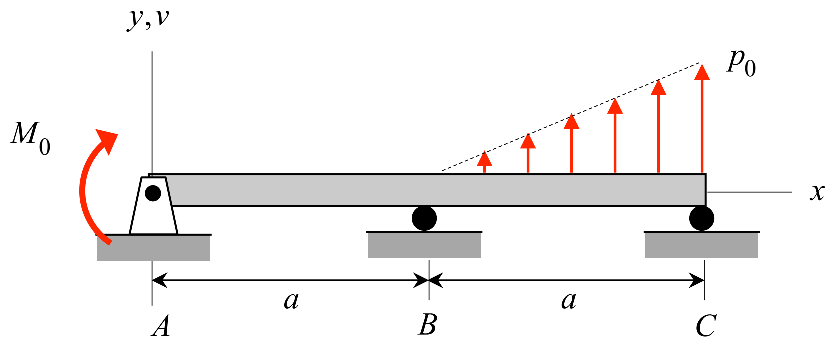
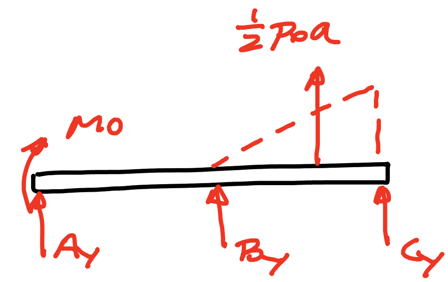
Example 11.25

Determine the deflection curve v(x) for the beam shown below.





1. Equilibrium

$$\sum M_A = B_y(a) + C_y(2a) + \left(\frac{p_o a}{2}\right)\left(\frac{5}{3}a\right) - M_o = 0$$

$$\hookrightarrow \quad B_y + 2C_y = \frac{M_o}{a} - \frac{5}{6}p_o a \qquad (1)$$

$$\sum F_y = A_y + B_y + \frac{1}{2}p_o a = 0$$

$$\hookrightarrow \quad A_y = -B_y - \frac{1}{2}p_o a = -\frac{M_o}{a} + \frac{1}{3}p_o a \qquad (2)$$

INDETERMINATE

2. Load/Deflection

$$p(x) = -(-M_o)\langle x\rangle^{-2} + A_y\langle x\rangle^{-1} + B_y\langle x-2a\rangle^{-1} + \frac{p_o}{a}\langle x-a\rangle^{1} + \underbrace{C_y\langle x-2a\rangle^{-1}}_{\text{will not contribute (drop)}}$$

$$V(x) = \boxed{\cancel{V(0)}} + M_o\langle x\rangle^{-1} + A_y\langle x\rangle^{0} + B_y\langle x-a\rangle^{0} + \frac{p_o}{2a}\langle x-a\rangle^{2}$$

$$M(x) = \cancel{M(0)}^{\,0} + M_o\langle x\rangle^{0} + A_y\langle x\rangle^{1} + B_y\langle x-a\rangle^{1} + \frac{p_o}{6a}\langle x-a\rangle^{3}$$

$$\theta(x) = \underbrace{\theta(0)}_{\theta_A\langle x\rangle^0} + \frac{1}{EI}\left[M_o\langle x\rangle^{1} + \frac{A_y}{2}\langle x\rangle^{2} + \frac{B_y}{2}\langle x-a\rangle^{2} + \frac{p_o}{24a}\langle x-a\rangle^{4}\right]$$

$$v(x) = \cancel{v(0)}^{\,0} + \theta_A\langle x\rangle^{1} + \frac{1}{EI}\left[\frac{M_o}{2}\langle x\rangle^{2} + \frac{A_y}{6}\langle x\rangle^{3} + \frac{B_y}{6}\langle x-a\rangle^{3} + \frac{p_o}{120a}\langle x-a\rangle^{5}\right]$$

3. Compatibility

- $v(a) = 0 = \theta_A a + \frac{1}{EI}\left[\frac{M_o}{2}a^2 + \frac{A_y}{6}a^3\right]$

$\hookrightarrow \quad \theta_A = -\frac{1}{EI}\left[\frac{M_0}{2}a + \frac{A_y}{6}a^2\right]$

- $v(2a) = 0 = \theta_A(2a) + \frac{1}{EI}\left[\frac{M_0}{2}(2a)^2 + \frac{A_y}{6}(2a)^3 + \frac{B_y}{6}a^3 + \frac{p_0}{120a}a^5\right]$

$\therefore$

$$8A_y + B_y = -\frac{12}{a^2}EI\theta_A - 12\frac{M_0}{a} - \frac{p_0 a}{20}$$

$$= +\frac{12\cancel{EI}}{\cancel{a^2}}\left[\frac{1}{\cancel{EI}}\left(\frac{M_0}{2}\cancel{a} + \frac{A_y}{6}\cancel{a^2}\right)\right]$$

$$= \frac{6M_0}{a} + 2A_y$$

$\hookrightarrow$

$$\boxed{6A_y + B_y = 6\frac{M_0}{a}} \qquad (3)$$

Solve (1)–(3) for reactions: A_y, B_y, C_y

$\therefore$

$$v(x) = \begin{cases} \theta_A x + \frac{1}{EI}\left[\frac{M_0}{2}x^2 + \frac{A_y}{6}x^3\right] & ; 0 < x < a \\ \theta_A x + \frac{1}{EI}\left[\frac{M_0}{2}x^2 + \frac{A_y}{6}x^3 + \frac{B_y}{6}(x-a)^3 + \frac{p_0}{120a}(x-a)^5\right] & ; a < x < 2a \end{cases}$$