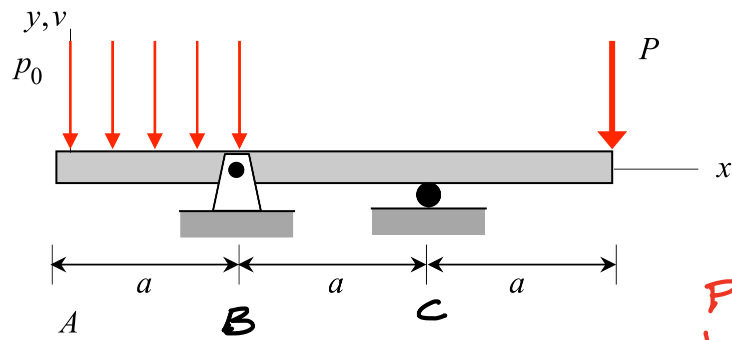
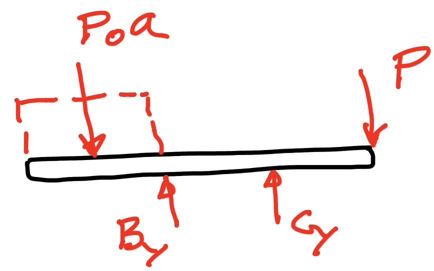
Example 11.24

Determine the deflection curve v(x) for the beam shown below.





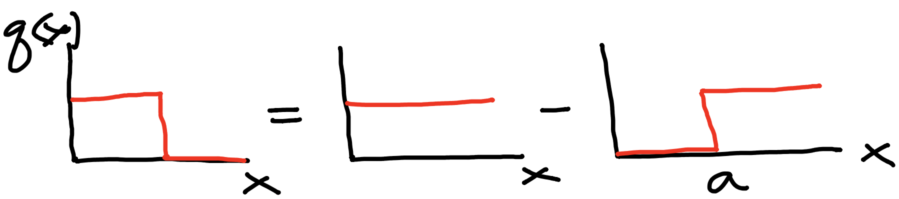
1. Equilibrium

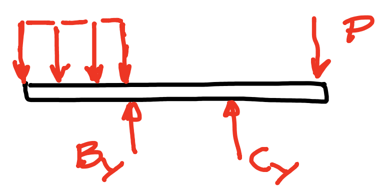
$$\Sigma M_B = (p_0 a)\left(\frac{a}{2}\right) + C_y a - P(2a) = 0$$

$$\hookrightarrow \boxed{C_y = 2P - \tfrac{1}{2} p_0 a}$$

$$\Sigma F_y = B_y + C_y - p_0 a = 0 \Rightarrow \boxed{B_y = p_0 a - C_y = \tfrac{3}{2} p_0 a - 2P}$$

2. Load/Deflection





$$q(x) = (-p_0)\langle x\rangle^0 - (-p_0)\langle x-a\rangle^0 = -p_0\langle x\rangle^0 + p_0\langle x-a\rangle^0$$

$\therefore$

$$p(x) = -p_0\langle x\rangle^0 + p_0\langle x-a\rangle^0 + B_y\langle x-a\rangle^{-1} + C_y\langle x-2a\rangle^{-1} \underbrace{-P\langle x-3a\rangle^{-1}}_{\text{will not contribute (drop)}}$$

$$V(x) = \cancel{V(0)}^{\,0} - p_0\langle x\rangle^1 + p_0\langle x-a\rangle^1 + B_y\langle x-a\rangle^0 + C_y\langle x-2a\rangle^0$$

$$M(x) = \cancel{M(0)}^{\,0} - \frac{p_0}{2}\langle x\rangle^2 + \frac{p_0}{2}\langle x-a\rangle^2 + B_y\langle x-a\rangle^1 + C_y\langle x-2a\rangle^1$$

$$\theta(x) = \underbrace{\theta(0)}_{\theta_A\langle x\rangle^0} + \frac{1}{EI}\left[-\frac{p_0}{6}\langle x\rangle^3 + \frac{p_0}{6}\langle x-a\rangle^3 + \frac{B_y}{2}\langle x-a\rangle^2 + \frac{C_y}{2}\langle x-2a\rangle^2\right]$$

$$v(x) = \underbrace{v(0)}_{v_A\langle x\rangle^0} + \theta_A \langle x\rangle^1 + \frac{1}{EI}\Big[-\frac{p_0}{24}\langle x\rangle^4 + \frac{p_0}{24}\langle x-a\rangle^4 + \frac{B_y}{6}\langle x-a\rangle^3 + \frac{C_y}{6}\langle x-2a\rangle^3\Big]$$

3. <u>Compatibility</u>

$$v(a) = 0 = v_A + \theta_A a + \frac{1}{EI}\Big[-\frac{p_0}{24}a^4\Big] \qquad (1)$$

$$v(2a) = 0 = v_A + \theta_A(2a) + \frac{1}{EI}\Big[-\frac{p_0}{24}(2a)^4 + \frac{p_0}{24}a^4 + \frac{B_y}{6}a^3 + \frac{C_y}{6}\quad{}^3\Big]$$

$$= v_A + 2\theta_A + \frac{1}{EI}\Big[-\frac{15}{24}p_0 a^4 + \frac{B_y}{6} + \frac{C_y}{6}a^3\Big] \qquad (2)$$

Solve (1) and (2) for θ_A & v_A

$\therefore$

$$v(x) = \begin{cases} v_A + \theta_A x + \frac{1}{EI}\Big[-\frac{p_0}{24}x^4\Big] & ;\ 0<x<a \\ v_A + \theta_A x + \frac{1}{EI}\Big[-\frac{p_0}{24}x^4 + \frac{p_0}{24}(x-a)^4 + \frac{B_y}{6}(x-a)^3\Big] & a<x<2a \\ v_A + \theta_A x + \frac{1}{EI}\Big[-\frac{p_0}{24}x^4 + \frac{p_0}{24}(x-a)^4 + \frac{B_y}{6}(x-a)^3 + \frac{C_y}{6}(x-2a)^3\Big] & 2a<x<3a \end{cases}$$

w/
$$\begin{cases} B_y = \frac{3}{2}p_0 a - 2P \\ C_y = 2P - \frac{1}{2}p_0 a \end{cases}$$