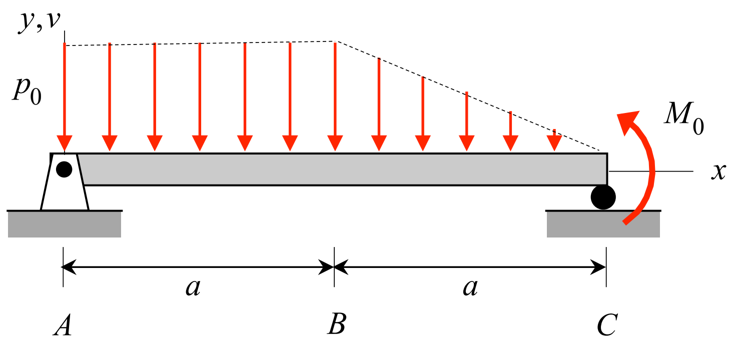
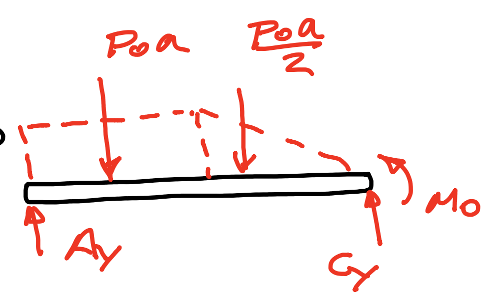
Example 11.23

Determine the deflection curve v(x) for the beam shown below.



1. Equilibrium



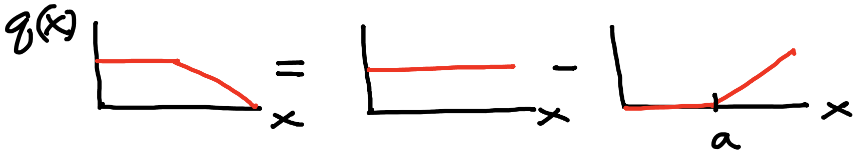
$$\sum M_A = -(p_0 a)\left(\frac{a}{2}\right) - \left(\frac{p_0 a}{2}\right)\left(\frac{4a}{3}\right) + C_y(2a) + M_0 = 0$$

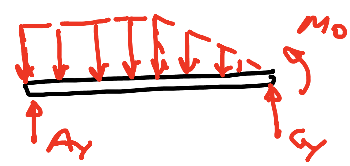
$$\hookrightarrow \; C_y = \frac{7}{12} p_0 a - M_0/a$$

$$\sum F_y = A_y + C_y - p_0 a - p_0 a/2 = 0$$

$$\hookrightarrow \; A_y = -C_y + \frac{3}{2} p_0 a = \frac{11}{12} p_0 a + \frac{M_0}{a}$$

2. Load/Deflection





$$q(x) = (-p_0)\langle x\rangle^0 - \left(-\frac{p_0}{a}\right)\langle x-a\rangle^1 = -p_0\langle x\rangle^0 + \frac{p_0}{a}\langle x-a\rangle^1$$

$$\therefore \; p(x) = \underbrace{-p_0\langle x\rangle^0 + \frac{p_0}{a}\langle x-a\rangle^1}_{q(x)} + A_y\langle x\rangle^{-1} + \underbrace{C_y\langle x-2a\rangle^{-1} - M_0\langle x-2a\rangle^{-2}}_{\text{will not contribute (drop)}}$$

already included

$$V(x) = \boxed{\cancel{V(0)}} - \frac{p_0}{a}\langle x\rangle^1 + \frac{p_0}{2}\langle x-a\rangle^2 + A_y\langle x\rangle^0$$

$$M(x) = \cancel{M(0)}^{\,0} - \frac{p_0}{2}\langle x\rangle^2 + \frac{p_0}{6a}\langle x-a\rangle^3 + A_y\langle x\rangle^1$$

$$\theta(x) = \theta(0) + \frac{1}{EI}\left[-\frac{p_0}{6}\langle x\rangle^3 + \frac{p_0}{24a}\langle x-a\rangle^4 + \frac{A_y}{2}\langle x\rangle^2\right]$$

$$\hookrightarrow \; \theta_A\langle x\rangle^0$$

$$v(x) = \cancel{v(0)}^{\,0} + \theta_A\langle x\rangle^1 + \frac{1}{EI}\left[-\frac{p_0}{24}\langle x\rangle^4 + \frac{p_0}{120a}\langle x-a\rangle^5 + \frac{A_y}{6}\langle x\rangle^3\right]$$

3. Compatibility

$$v(2a) = 0 = \theta_a(2a) + \frac{1}{EI}\left[-\frac{p_0}{24}(2a)^4 + \frac{p_0}{120a}a^5 + \frac{A_y}{6}(2a)^3\right]$$

$$= \theta_a(2a) + \frac{1}{EI}\left[-\frac{79}{120}p_0a^4 + \frac{4}{3}A_ya^3\right]$$

$$\hookrightarrow \theta_A = \frac{1}{EI}\left[-\frac{2}{3}A_ya^3 + \frac{79}{240}p_0a^4\right]$$

$\therefore$

$$v(x) = \begin{cases} \theta_A x + \frac{1}{EI}\left[-\frac{p_0}{24}x^4 + \frac{A_y}{6}x^3\right] & ; 0<x<a \\ \theta_A x + \frac{1}{EI}\left[-\frac{p_0}{24}x^4 + \frac{p_0}{120a}(x-a)^5 + \frac{A_y}{6}x^3\right] & ; a<x<2a \end{cases}$$

w/

$$A_y = \frac{11}{12}p_0a + \frac{M_0}{a}$$