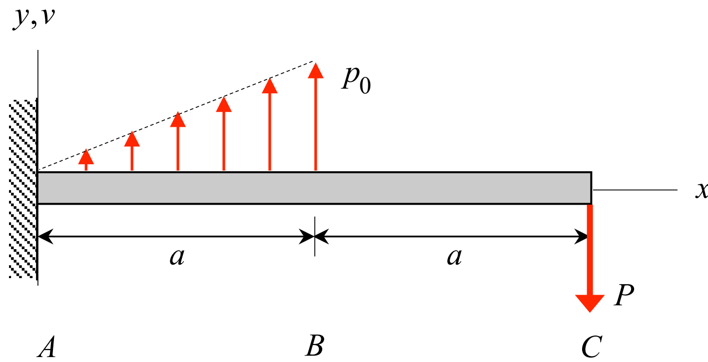
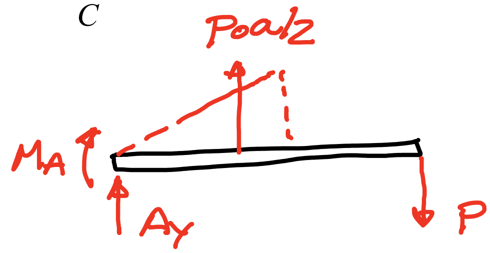
Example 11.22

Determine the deflection curve v(x) for the beam shown below.





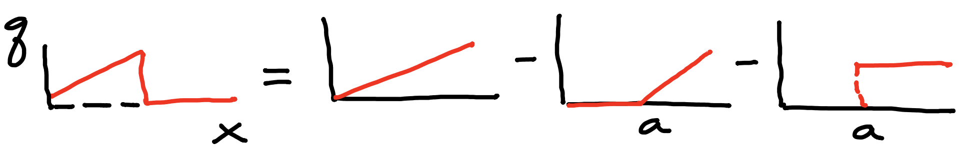
1. Equilibrium

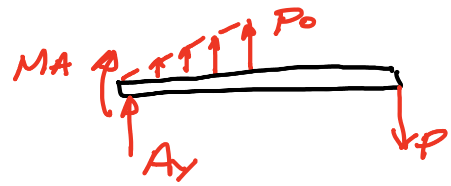
$$\Sigma M_A = +\frac{p_o a}{2}\left(\frac{2a}{3}\right) - P(2a) - M_A = 0$$

$$\hookrightarrow \boxed{M_A = -2Pa + \frac{p_o a^2}{3}}$$

$$\Sigma F_y = \frac{p_o a}{2} + A_y - P = 0 \Rightarrow \boxed{A_y = P - \frac{p_o a}{2}}$$

2. Load/deflection





$$\hookrightarrow q(x) = \frac{p_o}{a} - \frac{p_o}{a}\langle x-a\rangle^1 - p_o\langle x-a\rangle^0$$

$$\therefore \quad p(x) = \frac{p_o}{a}\langle x\rangle^1 - \frac{p_o}{a}\langle x-a\rangle^1 - p_o\langle x-a\rangle^0 + A_y\langle x\rangle^{-1} - (-M_A)\langle x\rangle^{-2} \underbrace{- P\langle x-2a\rangle^{-1}}_{\text{will not contribute (drop)}}$$

already included

$$V(x) = \cancel{V(0)} + \frac{p_o}{2a}\langle x\rangle^2 - \frac{p_o}{2a}\langle x-a\rangle^2 - p_o\langle x-a\rangle^1 + A_y\langle x\rangle^0 + M_A\langle x\rangle^{-1}$$

$$M(x) = \cancel{M(0)} + \frac{p_o}{6a}\langle x\rangle^3 - \frac{p_o}{6a}\langle x-a\rangle^3 - \frac{p_o}{2}\langle x-a\rangle^2 + A_y\langle x\rangle^1 + M_A\langle x\rangle^0$$

(BC)

$$\theta(x) = \cancel{\theta(0)}^{\,0} + \frac{1}{EI}\left[\frac{p_o}{24a}\langle x\rangle^4 - \frac{p_o}{24a}\langle x-a\rangle^4 - \frac{p_o}{6}\langle x-a\rangle^3 + \frac{A_y}{2}\langle x\rangle^2 + M_A\langle x\rangle^1\right]$$

$$v(x) = \cancel{v(0)}^{0\ (BC)} + \frac{1}{EI}\left[\frac{p_0}{120a}\langle x\rangle^5 - \frac{p_0}{120a}\langle x-a\rangle^5 - \frac{p_0}{24}\langle x-a\rangle^4 + \frac{A_y}{6}\langle x\rangle^3 + \frac{M_A}{2}\langle x\rangle^2\right]$$

$\therefore$

$$v(x) = \begin{cases} \frac{1}{EI}\left[\frac{p_0}{120a}x^5 + \frac{A_y}{6}x^3 + \frac{M_A}{2}x^2\right] & ;\ 0<x<a \\ \frac{1}{EI}\left[\frac{p_0}{120a}x^5 - \frac{p_0}{120a}(x-a)^5 - \frac{p_0}{24}(x-a)^4 + \frac{A_y}{6}x^3 + \frac{M_A}{2}x^2\right] & ;\ a<x<2a \end{cases}$$

w/ $\begin{cases} A_y = P - \frac{p_0 a}{2} \\ M_A = -2Pa + \frac{p_0 a^2}{3} \end{cases}$