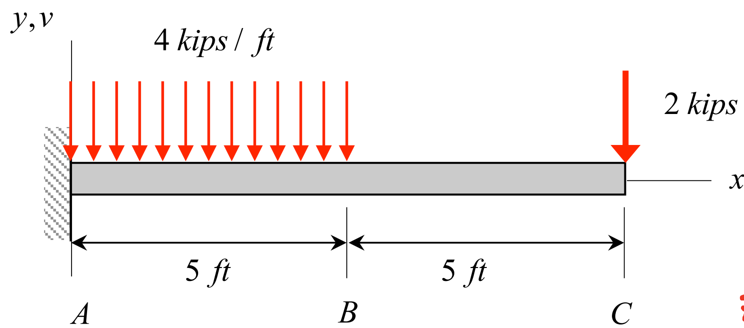
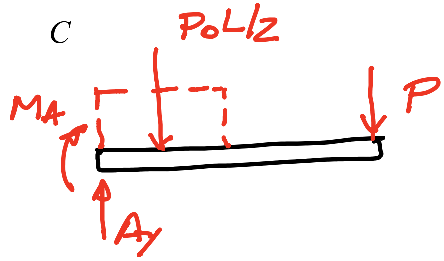
Shown below is a cantilevered $W14\times120$ wide flange beam made up of steel, with $E = 29\times10^3\,ksi$. As shown in the Appendix of the textbook, this beam has a cross-sectional area of $A = 35.3\,in^2$ and a second area moment of $I = 1380\,in^4$. Determine the slope and deflection at end C of the beam due to the loading shown.





1. Equilibrium

$$\sum M_A = -\left(\frac{P_oL}{2}\right)\left(\frac{L}{4}\right) - PL - M_A = 0$$

$$\hookrightarrow \quad M_A = -PL - \frac{P_oL^2}{8}$$

$$\sum F_y = A_y - \frac{P_oL}{2} - P = 0 \Rightarrow A_y = P + \frac{P_oL}{2}$$

2. Load Deflection

$$p(x) = A_y\langle x\rangle^{-1} - (-M_A)\langle x\rangle^{-2} \underbrace{- P_o\left[\langle x\rangle^{0} - \langle x-\tfrac{L}{2}\rangle^{0}\right]}_{\text{line load}} \underbrace{- P\langle x-L\rangle^{-1}}_{\text{will not contribute}}$$

already in p(x)

$$V(x) = \cancel{V(0)} + \int p(x)\,dx$$

$$= A_y\langle x\rangle^{0} + M_A\langle x\rangle^{-1} - p_o\left[\langle x\rangle^{1} - \langle x-\tfrac{L}{2}\rangle^{1}\right]$$

$$M(x) = \cancel{M(0)} + \int V(x)\,dx$$

already in p(x)

$$= A_y\langle x\rangle^{1} + M_A\langle x\rangle^{0} - p_o\left[\tfrac{1}{2}\langle x\rangle^{2} - \tfrac{1}{2}\langle x-\tfrac{L}{2}\rangle^{2}\right]$$

$$\theta(x) = \cancel{\theta(0)}^{\,0} + \frac{1}{EI}\int M(x)\,dx$$

$$= \frac{1}{EI}\left[\frac{A_y}{2}\langle x\rangle^{2} + M_A\langle x\rangle^{1} - p_o\left[\tfrac{1}{6}\langle x\rangle^{3} - \tfrac{1}{6}\langle x-\tfrac{L}{2}\rangle^{3}\right]\right]$$

$$v(x) = \cancel{v(0)}^{\,0} + \int \theta(x)\,dx$$

$$= \frac{1}{EI}\left[\frac{A_y}{6}\langle x\rangle^3 + \frac{M_A}{2}\langle x\rangle^2 - p_0\left[\frac{1}{24}\langle x\rangle^4 - \frac{1}{24}\left\langle x - \frac{L}{2}\right\rangle^4\right]\right]$$

$\therefore$

$$\boxed{\begin{aligned}
\theta(L) &= \frac{1}{EI}\left[\frac{A_y}{2}L^2 + M_A L - p_0\left(\frac{L^3}{6} - \frac{1}{6}\left(\frac{L}{2}\right)^3\right)\right] \\
&= \frac{1}{EI}\left[\frac{A_y}{2}L^2 + M_A L - \frac{7}{48}p_0 L^3\right] \\
v(L) &= \frac{1}{EI}\left[\frac{A_y}{6}L^3 + \frac{M_A}{2}L^2 - p_0\left(\frac{L^4}{24} - \frac{1}{24}\left(\frac{L}{2}\right)^4\right)\right] \\
&= \frac{1}{EI}\left[\frac{A_y}{6}L^3 + \frac{M_A}{2}L^2 - \frac{15}{384}p_0 L^4\right]
\end{aligned}}$$

w/ $$\begin{cases} A_y = P + \dfrac{p_0 L}{2} \\ M_A = -PL - \dfrac{p_0 L^2}{8} \end{cases}$$