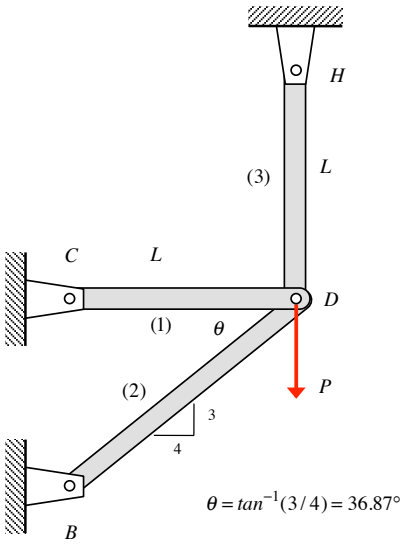
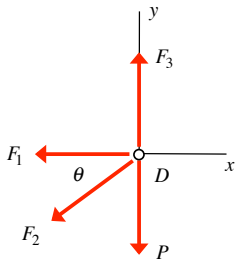
Example 16.9

Determine the vertical component of deflection of node D on the truss. All members have the same cross-sectional area A and are made of the same material having a Young's modulus of E.





Equilibrium analysis of joint D

$$\sum F_y = -P - F_2 sin\theta + F_3 = 0 \quad \Rightarrow \quad F_3 = P + F_2 sin\theta \tag{1}$$

$$\sum F_x = -F_1 - F_2 cos\theta = 0 \quad \Rightarrow \quad F_1 = -F_2 cos\theta \tag{2}$$

From this we see that the problem is INDETERMINATE (two equilibrium equations and three unknowns). We will consider the force F_2 to be the "redundant" force (this was an arbitrary choice).

Strain energy in truss

$$U = U_1 + U_2 + U_2 = \frac{1}{2}\frac{F_1^2 L_1}{EA} + \frac{1}{2}\frac{F_2^2 L_2}{EA} + \frac{1}{2}\frac{F_3^2 L_3}{EA} \quad ; \quad L_2 = \frac{L}{cos\theta} = \frac{5L}{4}$$

$$= \frac{1}{2EA}\left[F_1^2 L + F_2^2 \left(\frac{5L}{4}\right) + F_3^2 L \right]$$

$$= \frac{L}{2EA}\left[\left(-F_2 cos\theta\right)^2 + \frac{5}{4}F_2^2 + \left(P + F_2 sin\theta\right)^2 \right] \quad ; \quad using\ (1)\ and\ (2)$$

Castigliano's theorem as applied to indeterminate structures

Since we chose F_2 as our redundant force:

$$0 = \frac{\partial U}{\partial F_2} = \frac{L}{EA}\left[F_2 cos^2\theta + \frac{5}{4}F_2 + \left(P + F_2 sin\theta\right) sin\theta \right] \Rightarrow$$

$$0 = F_2 cos^2\theta + \frac{5}{4}F_2 + \left(P + F_2 sin\theta\right) sin\theta \Rightarrow$$

$$\left(cos^2\theta + \frac{5}{4} + sin^2\theta \right) F_2 = -P sin\theta \Rightarrow F_2 = -\frac{4}{9} P sin\theta$$

Therefore, the strain energy function becomes:

$$U = \frac{L}{2EA}\left\{ \left[-\left(-\frac{4}{9} P sin\theta \right) cos\theta \right]^2 + \frac{5}{4}\left(-\frac{4}{9} P sin\theta \right)^2 + \left(\frac{5}{9} P sin\theta \right)^2 \right\}$$

$$= \frac{P^2 L sin^2\theta}{162EA}\left(\frac{5}{9} + \frac{16}{81} cos^2\theta \right)$$

Using Castigliano's theorem gives:

$$v_D = \frac{\partial U}{\partial P} = \frac{PL sin^2\theta}{81EA}\left(\frac{5}{9} + \frac{16}{81} cos^2\theta \right)$$ (since "+", in same direction as P - DOWN)