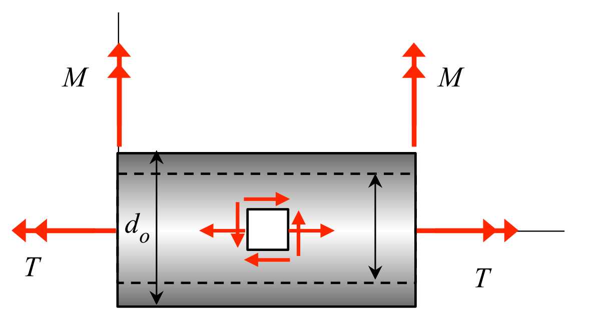
Example 15.3

A section of pipe is loaded as shown below with bending couple $M = 35\,kip \cdot in$ and axial torque $T = 175\,kip \cdot in$. The yield strength of the pipe's ductile material is known to be $\sigma_Y = 100\,ksi$, with the inner and outer diameters of the pipe given as $d_i = 3\,in$ and $d_o = 3.5\,in$, respectively.

a) What is the factor of safety FS_S predicted by the *maximum shear stress* theory of failure for this loading on the pipe section?

b) What is the factor of safety FS_D predicted by the *maximum distortional energy* theory of failure for this loading on the pipe section?



SOLUTION

Stress element

For this loading:

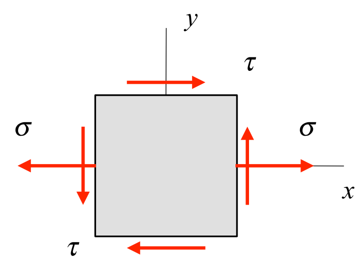
$$\sigma = \frac{M(d_o/2)}{I}$$

$$\tau = \frac{T(d_o/2)}{J}$$

where:

$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right) = \frac{\pi}{64}\left(3.5^4 - 3^4\right) = 3.39\,in^4$$

$$J = \frac{\pi}{32}\left(d_o^4 - d_i^4\right) = \frac{\pi}{32}\left(3.5^4 - 3^4\right) = 6.78\,in^4 = 2I$$



Therefore:

$$\sigma = \frac{(35\,kip \cdot in)(3.5\,in/2)}{3.39\,in^4} = 18.1\,ksi$$

$$\tau = \frac{(175\,kip \cdot in)(3.5\,in/2)}{6.78\,in^4} = 45.2\,ksi$$

Stress transformation/Mohr's circle

From the above state of stress:

$$\sigma_{ave} = \frac{\sigma}{2} = \frac{18.1}{2} = 9.03\,ksi$$

$$R = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{18.1}{2}\right)^2 + (45.2)^2} = 46.1\,ksi$$

Therefore:

$$\sigma_{P1} = \sigma_{ave} + R = 9.03 + 46.1 = 55.1\,ksi = \sigma_1$$

$$\sigma_{P2} = \sigma_{ave} - R = 9.03 - 46.1 = -37.1\,ksi = \sigma_3$$

From this, we have:

$$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} = \frac{55.1 - (-37.1)}{2} = 46.1\ ksi$$

$$\sigma_M = \sqrt{\sigma_{P1}^2 - \sigma_{P1}\sigma_{P2} + \sigma_{P2}^2} = \sqrt{55.1^2 + (55.1)(37.1) + (37.1)^2} = 80.4\ ksi$$

Therefore:

$$FS_S = \frac{\sigma_Y}{2\tau_{max,abs}} = \frac{100}{(2)(46.1)} = 1.08$$

$$FS_D = \frac{\sigma_Y}{\sigma_M} = \frac{100}{80.4} = 1.24$$