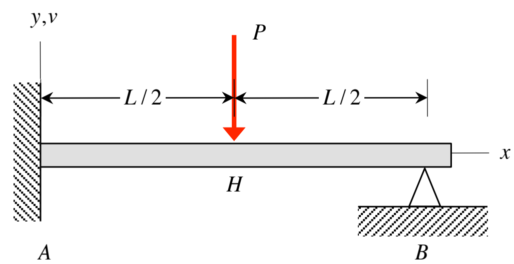
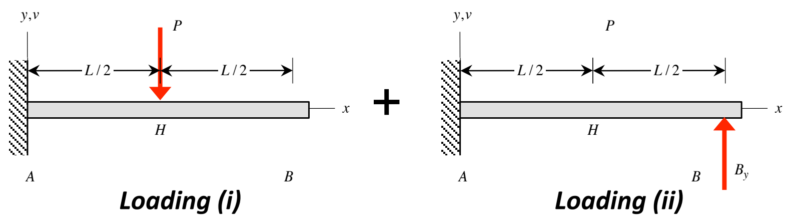
Example 11.22

Consider the indeterminate beam shown below, where the beam is made up of a material having a Young's modulus of E and a cross-sectional second area moment of I. For this beam, determine its deflection at the midpoint H. Use the superposition approach with known beam deflection solutions.



SOLUTION

The loading on this beam can be broken down into the following two parts: Loadings (i) and (ii):



We will use results provided to us in the Appendix of this lecture book.

For Loading (i):

$$v_{(i)}(x) = -\frac{1}{6}\left[x^2\left(3\frac{L}{2} - x\right)\right]\frac{P}{EI} \quad ; \quad 0 < x < L/2$$

$$= -\frac{1}{6}\left[\frac{L^2}{4}\left(3x - \frac{L}{2}\right)\right]\frac{P}{EI} \quad ; \quad L/2 < x < L$$

For Loading (ii):

$$v_{(ii)}(x) = \frac{1}{6}\left[x^2\left(3L - x\right)\right]\frac{B_y}{EI}$$

Enforcing the displacement boundary condition at $x = L$:

$$v(L) = 0 = v_{(i)}(L) + v_{(ii)}(L)$$

$$= -\frac{1}{6}\left[\frac{L^2}{4}\left(3L - \frac{L}{2}\right)\right]\frac{P}{EI} + \frac{1}{6}\left[L^2\left(3L - L\right)\right]\frac{B_y}{EI}$$

$$= -\frac{5}{48}\frac{PL^3}{EI} + \frac{1}{3}\frac{B_y L^3}{EI} \quad \Rightarrow \quad B_y = \frac{5}{16}P$$

Therefore:

$$v(x) = \left\{-\frac{1}{6}\left[x^2\left(3\frac{L}{2} - x\right)\right] - \frac{5}{96}\left[x^2\left(3L - x\right)\right]\right\}\frac{P}{EI} \quad ; \quad 0 < x < L/2$$

$$= \left\{-\frac{1}{6}\left[\frac{L^2}{4}\left(3x - \frac{L}{2}\right)\right] - \frac{5}{96}\left[x^2\left(3L - x\right)\right]\right\}\frac{P}{EI} \quad ; \quad L/2 < x < L$$