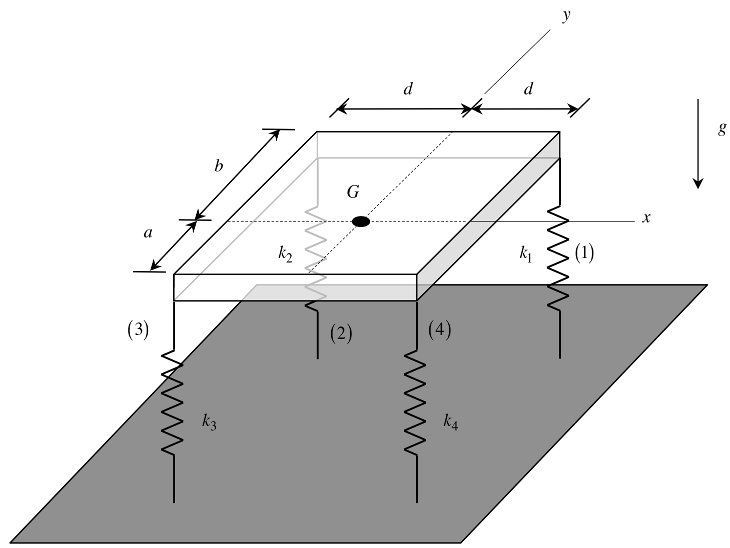
Example 6.9

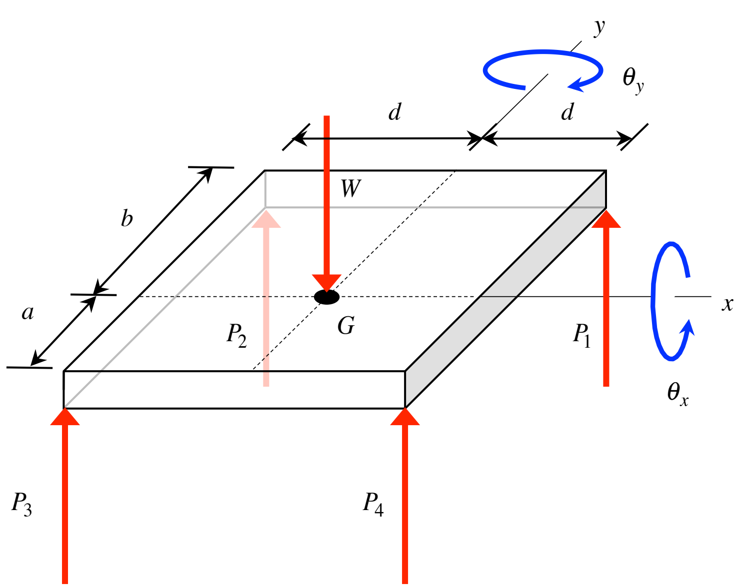
Recall that we discussed earlier that the determination of the load carried by the four tires of an automobile is a statically indeterminate problem. What factors determine the load carried by the tires? Suppose that we model the automobile as a rigid body with center of mass at G supported by four springs representing the stiffness of the four tires. With this model, determine the load carried by each of the four tires.



SOLUTION

<u>*Equilibrium*</u>

The FBD of the automobile body has the support forces of the four tires (P_1, P_2, $_3$ and P_4) along with the body weight force W. Let θ_y represent the rotation of the auto body about is longitudinal axis (y-axis) and θ_x represent the pitch rotation of the auto body.



From FBD above:

$$\sum F_y = P_1 + P_2 + P_3 + P_4 - W = 0 \;\Rightarrow\; P_1 + P_2 + P_3 + P_4 = W \qquad (1)$$

$$\sum M_{Gx} = \left(P_1 + P_2\right)b - \left(P_3 + P_4\right)a = 0 \;\Rightarrow\; P_1 + P_2 - \frac{a}{b}P_3 - \frac{a}{b}P_4 = 0 \qquad (2)$$

$$\sum M_{Gy} = \left(P_2 + P_3\right)d - \left(P_1 + P_4\right)d = 0 \;\Rightarrow\; P_1 - P_2 - P_3 + P_4 = 0 \qquad (3)$$

Problem is INDETERMINATE – three equations and four unknown forces. Note that the weight is shared equally between the right and left pairs of supports, and the weight is shared between the front and right pairs of support depending on the location of the center of mass G.

Elongation equations

$$e_1 = \frac{P_1}{k_1} \qquad (4)$$

$$e_2 = \frac{P_2}{k_2} \qquad (5)$$

$$e_3 = \frac{P_3}{k_3} \qquad (6)$$

$$e_4 = \frac{P_4}{k_4} \qquad (7)$$

Compatibility (kinematics) equations for small rotations of slab

$$e_1 = y_G + b\theta_x - d\theta_y \qquad (8)$$

$$e_2 = y_G + b\theta_x + d\theta_y \qquad (9)$$

$$e_3 = y_G - a\theta_x + d\theta_y \qquad (10)$$

$$e_4 = y_G - a\theta_x - d\theta_y \qquad (11)$$

where y_G is the vertical displacement of the center of mass of the auto body.

Note that the problem can now be solved for eleven unknowns from the eleven equations above.

Solve

Subtract equation (9) from equation (8):

$$e_1 - e_2 = -2d\theta_y \qquad (12)$$

Subtract equation (10) from equation (11):

$$e_4 - e_3 = -2d\theta_y \qquad (13)$$

Equating (12) and (13):

$$e_1 - e_2 = e_4 - e_3 \qquad (14)$$

Substituting (4)-(7) into (14) gives:

$$\frac{P_1}{k_1} - \frac{P_2}{k_2} = \frac{P_4}{k_4} - \frac{P_3}{k_3} \quad \Rightarrow \quad P_1 - \frac{k_1}{k_2}P_2 + \frac{k_1}{k_3}P_3 - \frac{k_1}{k_4}P_4 = 0 \tag{15}$$

Therefore, we now have four equations (1), (2), (3) and (15)) for our four support forces at the tires:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -\frac{a}{b} & -\frac{a}{b} \\ 1 & -1 & -1 & 1 \\ 1 & -\frac{k_1}{k_2} & \frac{k_1}{k_3} & -\frac{k_1}{k_4} \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{Bmatrix} = \begin{Bmatrix} W \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{16}$$

For given values of the location of the center of mass (a/b) and relative stiffness values of k_2/k_1, k_3/k_1 and k_4/k_1, we can solve equation (16) for the reactions at the supports.

Suppose we consider the influence of the stiffness of tire #1 by varying its stiffness k_1 and holding the other tire stiffnesses constant: $k_2 = k_3 = k_4 = k_{nom} = \textit{nominal tire stiffness}$. Shown below are the loads carried by the four times for $a/b = 0.8$.