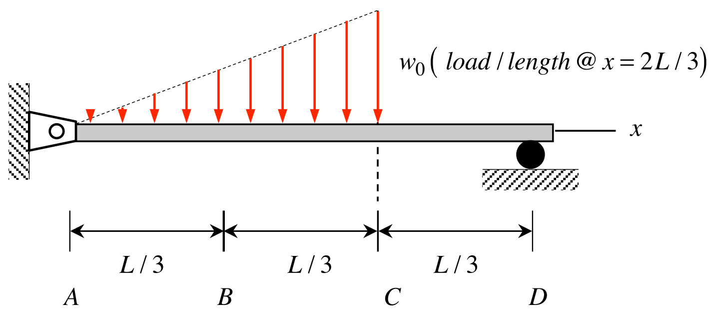
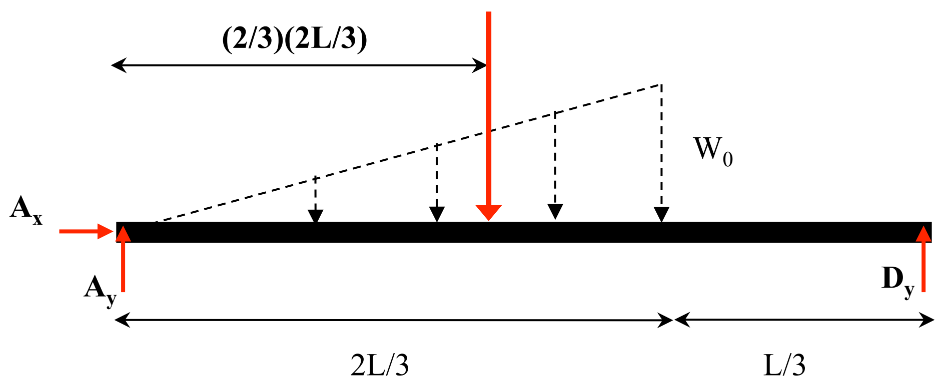
Example 1.4

Find the *internal* resultants (axial force, shear force, and bending moment) on the beam cross section at B.



SOLUTION

<u>*Step 1: Determine the values of the support reactions*</u>



$$\sum F_x = 0 \quad \Rightarrow \quad A_x = 0$$

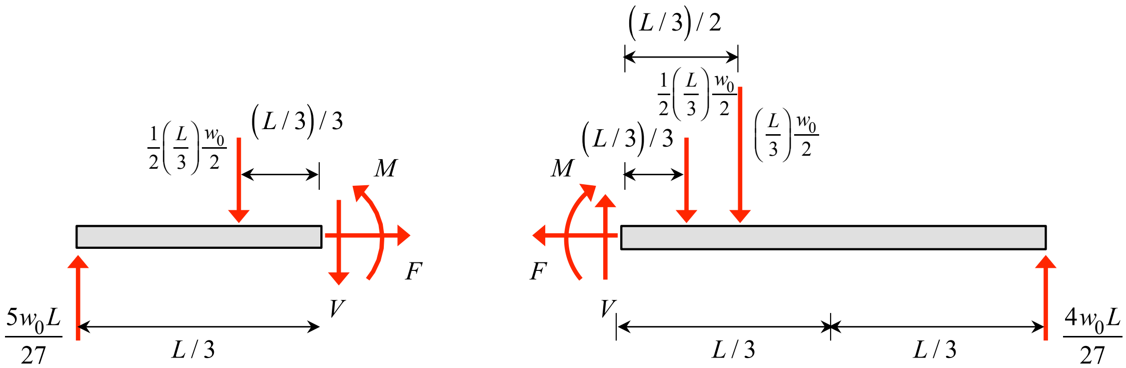
$$\sum F_y = 0 \quad \Rightarrow A_y + D_y - \frac{1}{2}\left(\frac{2L}{3}\right)w_0 = 0$$

$$\left(\sum M_z\right)_A = 0 \quad \Rightarrow LD_y - \frac{2}{3}\left(\frac{2L}{3}\right)\frac{1}{2}\left(\frac{2L}{3}\right)w_0 = 0$$

Solving the above two equations gives:

$$D_y = \frac{4w_0L}{27} \qquad A_y = \frac{5w_0L}{27}$$

Step 2: Make a cut in the beam at B



Step 3: Equilibrium analysis

Using FBD of body to the *left* of the cut:

$$\sum F_x = 0 \quad \Rightarrow \quad F = 0$$

$$\sum F_y = 0 \Rightarrow \frac{5w_0L}{27} - V - \frac{1}{2}\left(\frac{L}{3}\right)\frac{w_0}{2} = 0 \quad \Rightarrow \quad V = \frac{33}{324}w_0L$$

$$\left(\sum M_z\right)_B = 0 \Rightarrow M - \frac{L}{3}\left(\frac{5w_0L}{27}\right) + \frac{1}{3}\left(\frac{L}{3}\right)\frac{1}{2}\left(\frac{L}{3}\right)\frac{w_0}{2} = 0 \quad \Rightarrow \quad M = \frac{17}{324}w_0L^2$$

Using FBD of body to the *right* of the cut:

$$\sum F_x = 0 \quad \Rightarrow \quad F = 0$$

$$\sum F_y = 0 \Rightarrow \frac{4w_0L}{27} + V - \frac{w_0}{2}\left(\frac{L}{3}\right) - \frac{1}{2}\left(\frac{L}{3}\right)\frac{w_0}{2} = 0 \quad \Rightarrow \quad V = \frac{33}{324}w_0L$$

$$\left(\sum M_z\right)_B = 0 \Rightarrow -M + \frac{2L}{3}\left(\frac{4w_0L}{27}\right) - \frac{2}{3}\left(\frac{L}{3}\right)\frac{1}{2}\left(\frac{L}{3}\right)\frac{w_0}{2} - \frac{1}{2}\left(\frac{L}{3}\right)\left(\frac{L}{3}\right)\frac{w_0}{2} = 0 \quad \Rightarrow$$

$$M = \frac{17}{324}w_0L^2$$

Note that we get the same results for the internal resultants at the cut regardless of which section of the cut beam that we use for equilibrium. This is an expected result.