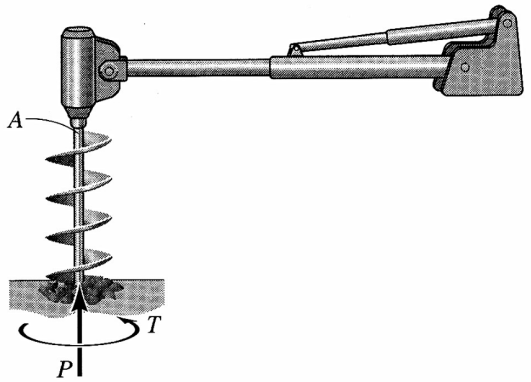
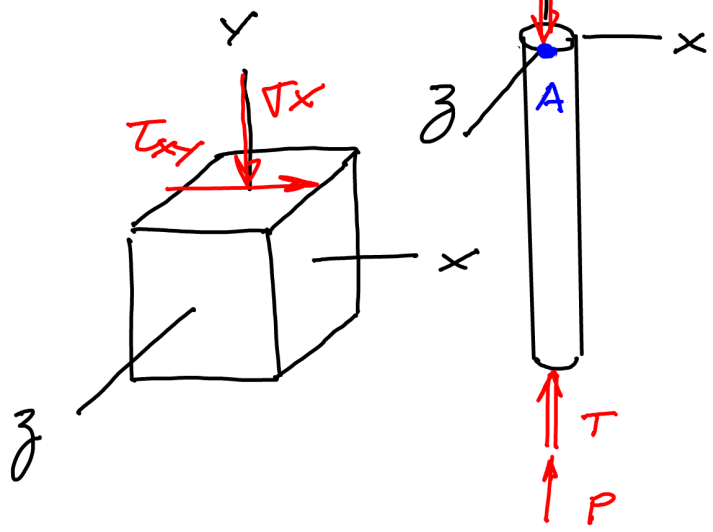
Example 16.3

A post-hole digger is mounted on a tractor. The power unit of the machine applies a torque of $T = 800\ lb \cdot in$ to the auger, and also exerts a downward force of $P = 1500\ lb$ on the auger. If the shaft of the auger is a solid circular rod with a diameter of 2.0 in, determine the principal stresses and the maximum shear stress at a typical point A on the outer surface of the shaft of the auger near the power unit.



Equilibrium: See FBD

Stress element @ A



$$\sigma_x = \frac{P}{A} = \frac{P}{\pi(d^2/4)}$$

$$\tau_{xy} = \frac{T(d/2)}{J} = \frac{T(d/2)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4} = \frac{16T}{\pi d^3}$$

$$\sigma_{ave} = \frac{\sigma_x}{2} = \frac{2P}{\pi d^2}$$

$$R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{2P}{\pi d^2}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \frac{2P}{\pi d^2}\sqrt{1 + 64\left(\frac{T}{Pd}\right)^2}$$

$$\therefore \begin{cases} \sigma_1 = \sigma_{ave} + R = \dfrac{2P}{\pi d^2}\left[1 + \sqrt{1 + 64\left(\dfrac{T}{Pd}\right)^2}\right] > 0 \\ \sigma_2 = \sigma_{ave} - R = \dfrac{2P}{\pi d^2}\left[1 - \sqrt{1 + 64\left(\dfrac{T}{Pd}\right)^2}\right] < 0 \end{cases}$$

Since σ_1 & σ_2 have opposite signs:

$$(\tau_{max})_{abs} = R = \sqrt{1 + 64\left(\frac{T}{Pd}\right)^2}$$