Summary - some conceptual questions related to beam bending

Question #1
A cantilevered beam has a second area moment of I and is made up of a material with an elastic modulus of E. Let M(x) and V(x) represent the internal bending moments and shear forces for the beam.

a) Do M(x) and V(x) depend on EI? Explain.

b) Do the normal and shear stresses in the beam depend on E? Explain.

Question #2
A propped cantilevered beam has a second area moment of I and is made up of a material with an elastic modulus of E. Let M(x) and V(x) represent the internal bending moments and shear forces for the beam.

a) Do M(x) and V(x) depend on EI? Explain.

b) Do the normal and shear stresses in the beam depend on E? Explain.

Question #3
Consider the stress analysis of statically-determinate and statically-indeterminate beam problems.

a) Compare the methods of analysis for the two classes of problems: how do they differ?

b) Compare the results of analysis for the two classes of problems in terms of their dependence on the elastic modulus of the material: how do they differ?
Question #4
Consider two different cross sections (A and B) for the cantilevered beam shown below, where cross section B is simply a 90° rotation of cross section A about the x-axis. Let $|M_{\text{max}}|_A$, $|V_{\text{max}}|_A$, $|\sigma_{\text{max}}|_A$, $|\tau_{\text{max}}|_A$ and $|\delta_{\text{max}}|_A$ represent the maximum (magnitude) internal bending moment, internal shear force, normal stress, shear stress and deflection for cross section A. Let $|M_{\text{max}}|_B$, $|V_{\text{max}}|_B$, $|\sigma_{\text{max}}|_B$, $|\tau_{\text{max}}|_B$ and $|\delta_{\text{max}}|_B$ represent the corresponding values for cross section B.

a) Determine the x-locations for $|M_{\text{max}}|_A$ and $|V_{\text{max}}|_A$.

b) Determine the x-locations for $|M_{\text{max}}|_B$ and $|V_{\text{max}}|_B$.

c) Determine the x- and y-locations for $|\sigma_{\text{max}}|_A$ and $|\tau_{\text{max}}|_A$.

d) Determine the x- and y-locations for $|\sigma_{\text{max}}|_B$ and $|\tau_{\text{max}}|_B$.

e) Which is larger, $|\sigma_{\text{max}}|_A$ or $|\sigma_{\text{max}}|_B$? Explain.

f) Which is larger, $|\tau_{\text{max}}|_A$ or $|\tau_{\text{max}}|_B$? Explain.

g) Which is larger, $|\delta_{\text{max}}|_A$ or $|\delta_{\text{max}}|_B$? Explain.


**Question #5**

A simply-supported beam (supports at A and D) of length \( L = 9 \text{m} \) is acted upon by a single couple \( M_C = 90 \text{kN} \cdot \text{m} \) at C (shown) and a number of concentrated forces (not shown). The beam has a square cross section of dimensions \( 0.25 \text{m} \times 0.25 \text{m} \).

a) Construct the bending moment diagram in the axes provided below.

b) Show the concentrated forces acting on the beam, providing numerical values for these forces.

c) Determine the value of \( |\sigma_{\text{max}}| \) and the x- and y-locations of where this occurs. Make a sketch of the normal stress distribution at one of these locations.

d) Determine the value of \( |\tau_{\text{max}}| \) and the x- and y-locations of where this occurs. Make a sketch of the shear stress distribution at one of these locations.
Question #6

The cross section of a beam is shown below.

a) Determine the y-z coordinates for the neutral axis for the beam.

b) Determine the neutral axis second area moment I for the beam.

\[ \frac{h}{2} \quad \frac{h}{2} \quad \frac{h}{2} \quad \frac{h}{2} \]

\[ \begin{array}{c}
\frac{h}{2} \\
\hline
h / 2 \\
\hline
\end{array} \]

\[ z \quad \begin{array}{c}
y \\
\hline
d \\
\hline
h \\
\hline
\end{array} \]

Question #7

A beam of length L has a second area moment I and is made of a material having an elastic modulus of E. For a given loading, the deflection of the neutral axis of the beam is known to be:

\[ v(x) = x \left( a + b x^2 + c x^4 \right) \quad ; \quad 0 < x < L \]

Determine \( V(x) \), \( M(x) \) and \( \theta(x) \) for the beam.

Question #8

Consider point A at the connection of the vertical web to the horizontal top flange. Let \( A^- \) and \( A^+ \) be points immediately below A (on the web) and immediately above A (on the flange), respectively. What is the ratio of the shear stress at \( A^- \) to the shear stress at \( A^+ \)?
Question #9
Consider the identical two beams shown below loaded with identical concentrated loads W.

a) What is the numerical value of $\left(\delta_{\text{max}}\right)_B / \left(\delta_{\text{max}}\right)_A$?

b) If the length of Beam A is doubled, what is the new value of $\left(\delta_{\text{max}}\right)_A$?

c) If the length of Beam B is doubled, what is the new value of $\left(\delta_{\text{max}}\right)_B$?

Question #10
The cantilevered beam shown is loaded with a constant force/length of $p_0$. The maximum deflection of the beam under this loading is $\delta_{\text{max}}$. Suppose the length of the beam is doubled, with $p_0$ being extended over the new length of the beam. What is the new value of the maximum deflection of the beam?
**Question #11**

A beam with the cross section shown below is loaded with vertical forces such that the cross section experiences an internal bending moment $M$ and an internal shear force $V$. Let $\sigma$ and $\tau$ be the normal stress and shear stress, respectively, acting at a location on the cross section.

a) TRUE or FALSE: $\sigma_B = 0$

b) TRUE or FALSE: $|\sigma_A| = |\sigma_C|

c) TRUE or FALSE: $|\tau_A| = |\tau_C|$

**Question #12**

The beam shown below has a square cross section of dimensions $h \times h$.

a) Determine the maximum (magnitude) normal stress in the section of the beam between locations B and C.

b) Determine the maximum (magnitude) shear stress in the section of the beam between locations B and C.