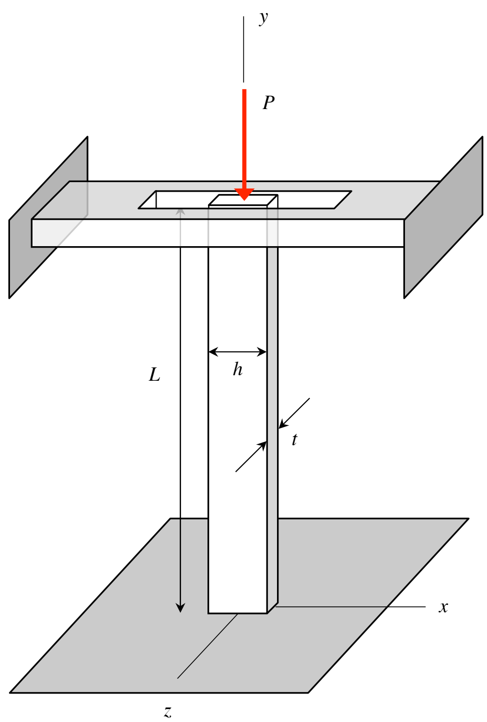
Example 18.5

The column shown below is clamped onto to ground at its bottom, with the top of the beam able to slide within a slot. The column carries an axial load of P. What is the largest load P that the column can withstand without buckling? Use $h = 3t$ and $L = 10h$.



SOLUTION

Boundary conditions and radius of gyration

- For buckling about the x-axis, we have fixed (bottom) and pinned (top) BCs. Therefore: $L_{eff} = 0.7L$. The second aread moment, radius of gyration and slenderness ratio for this buckling are:

$$I = \frac{1}{12}ht^3$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{ht^3/12}{ht}} = \frac{t}{2\sqrt{3}}$$

$$\frac{L_{eff}}{r} = \frac{0.7L}{t/2\sqrt{3}} = 1.4\sqrt{3}\frac{L}{t} = 1.4\sqrt{3}\frac{L}{L/30} = 42\sqrt{3}$$

- For buckling about the z-axis, we have fixed (bottom) and free (top) BCs. Therefore: $L_{eff} = 2L$. The second aread moment, radius of gyration and slenderness ratio for this buckling are:

$$I = \frac{1}{12} th^3$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{th^3/12}{ht}} = \frac{h}{2\sqrt{3}}$$

$$\frac{L_{eff}}{r} = \frac{2L}{h/2\sqrt{3}} = 4\sqrt{3}\frac{L}{h} = 4\sqrt{3}\frac{L}{L/10} = 40\sqrt{3}$$

Since buckling about the x-axis has the largest slenderness ratio, buckling will occur about the x-axis. Therefore, we will use: $\frac{L_{eff}}{r} = 42\sqrt{3}$.

(The rest of the buckling analysis as is standard.)

Calculate parameters:

$$\frac{L_{eff}}{r} = 42\sqrt{3} \quad ; \quad \textit{dependent on only geometry}$$

$$\left(\frac{L_{eff}}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} \quad ; \quad \textit{dependent on only material properties}$$

Compare and choose:

- If $L_{eff}/r > \left(L_{eff}/r\right)_c$, then the <u>*Euler*</u> theory applies:

$$P_{cr} = \pi^2 \frac{EI}{L_{eff}^2} = \left[\frac{\pi^2}{\left(L_{eff}/r\right)^2}\right] EA$$

- If $L_{eff}/r < \left(L_{eff}/r\right)_c$, then the <u>*Johnson*</u> theory applies:

$$P_{cr} = \sigma_{cr} A = \left[1 - \frac{\left(L_{eff}/r\right)^2}{\left(L_{eff}/r\right)_c^2}\right] \sigma_Y A$$