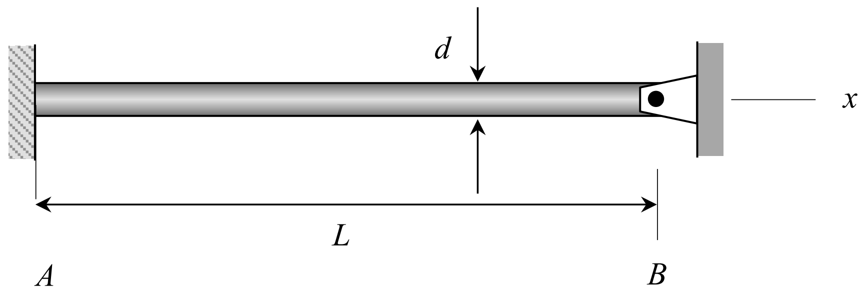
Example 18.4

A straight, slender rod is fixed to a rigid support at end A and pinned to a rigid support at end B. At the reference temperature, T_0, the rod is perfectly stress-free.

a) Derive a formula that expresses the uniform increase in temperature ΔT_{cr} required to cause elastic buckling of the compression member.

b) Determine the value of ΔT_{cr} required to cause elastic buckling of an aluminum rod with a diameter of $d = 20\ mm$ and a length of $L = 1m$. The coefficient of thermal expansion, Young's modulus and yield strength for aluminum are $\alpha = 23\times10^{-6}/°C$, $E = 10.6\times10^3 ksi$ and $\sigma_Y = 60\ ksi$.



SOLUTION

Equilibrium and compatibility

$$e = 0 = \frac{(-P)L}{EA} + \alpha\Delta TL \quad \Rightarrow \quad P = \alpha\Delta TEA$$

Calculate parameters:

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}{\pi\left(\frac{d}{2}\right)^2}} = \frac{d}{4} = radius\ of\ gyration$$

$$L_{eff} = \text{"}effective\ length\text{"}\ of\ the\ member = 0.7L\ \left(pinned - fixed\ BCs\right)$$

Therefore:

$$\frac{L_{eff}}{r} = 2.8\frac{L}{d} \quad ; \quad dependent\ on\ only\ geometry$$

$$\left(\frac{L_{eff}}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} \quad ; \quad \textit{dependent on only material properties}$$

Compare and choose:

- If $L_{eff}/r > \left(L_{eff}/r\right)_c$, then the <u>*Euler*</u> theory applies:

$$P_{cr} = \pi^2 \frac{EI}{L_{eff}^2} = \left[\frac{\pi^2}{\left(L_{eff}/r\right)^2}\right] EA$$

- If $L_{eff}/r < \left(L_{eff}/r\right)_c$, then the <u>*Johnson*</u> theory applies:

$$P_{cr} = \sigma_{cr} A = \left[1 - \frac{\left(L_{eff}/r\right)^2}{\left(L_{eff}/r\right)_c^2}\right] \sigma_Y A$$

Solve

Set $P = P_{cr}$:

$$P = \alpha \Delta T E A = P_{cr} \quad \Rightarrow$$

$$\Delta T = \frac{P_{cr}}{\alpha EA}$$

$$= \frac{1}{\alpha}\left[\frac{\pi^2}{\left(L_{eff}/r\right)^2}\right] \quad ; \quad \textit{if Euler buckling}$$

$$= \frac{1}{\alpha}\left[1 - \frac{\left(L_{eff}/r\right)^2}{\left(L_{eff}/r\right)_c^2}\right]\frac{\sigma_Y}{E} \quad ; \quad \textit{if Johnson buckling}$$