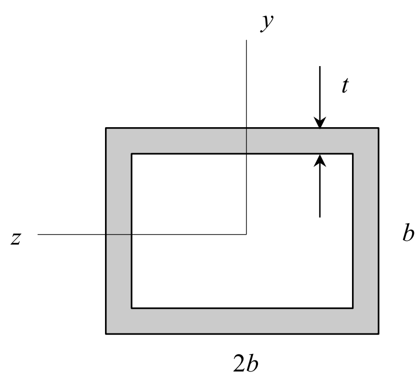
Example 18.3

A tubular steel column, with the cross section shown below and a length of L, is subjected to an axial load of P. The material of the column has a Young's modulus of E and a yield strength of σ_Y. If the column has fixed-free end conditions, what is the factor of safety for buckling?



SOLUTION

Calculate parameters:

Will consider buckling about the z-axis since the second area moment about that axis is the smallest.

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{1}{12}\left[(2b)b^3 - (2b-2t)(b-2t)^3\right]}{(2b)b - 2(2b-2t)(b-2t)}}$$

$$= \sqrt{\frac{2b^4 - 2(b-t)(b-2t)^3}{24\left[b^2 - 2(b-t)(b-2t)\right]}} = \text{radius of gyration}$$

$$L_{eff} = \text{"effective length" of the member} = 2L \quad (\text{fixed} - \text{free BCs})$$

Therefore:

$$\frac{L_{eff}}{r} = 2\left[\frac{24\left[b^2 - 2(b-t)(b-2t)\right]}{2b^4 - 2(b-t)(b-2t)^3}\right]^{1/2} L \quad ; \text{ dependent on only geometry}$$

$$\left(\frac{L_{eff}}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} \quad ; \text{ dependent on only material properties}$$

Compare and choose:

- If $L_{eff}/r > \left(L_{eff}/r\right)_c$, then the <u>*Euler*</u> theory applies: $P_{cr} = \pi^2 \dfrac{EI}{L_{eff}^2}$
- If $L_{eff}/r < \left(L_{eff}/r\right)_c$, then the <u>*Johnson*</u> theory applies:

$$P_{cr} = \sigma_{cr} A = \left[1 - \frac{\left(L_{eff}/r\right)^2}{\left(L_{eff}/r\right)_c^2}\right]\sigma_Y A$$

Factor of safety

$$FS = \frac{P_{cr}}{P}$$