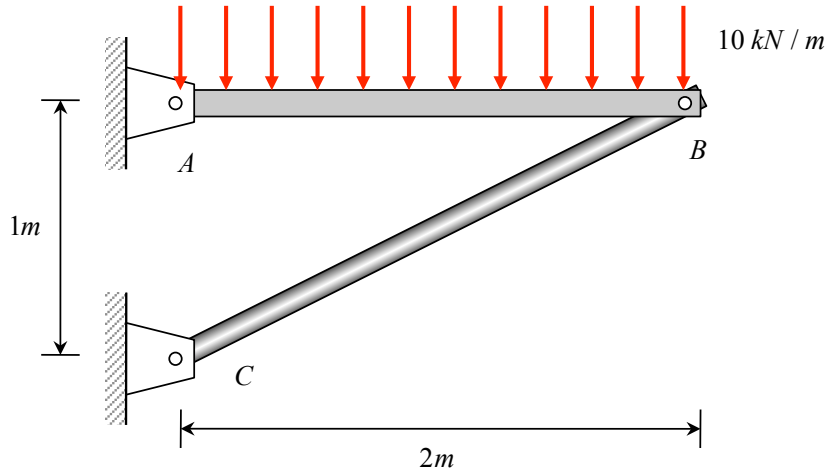
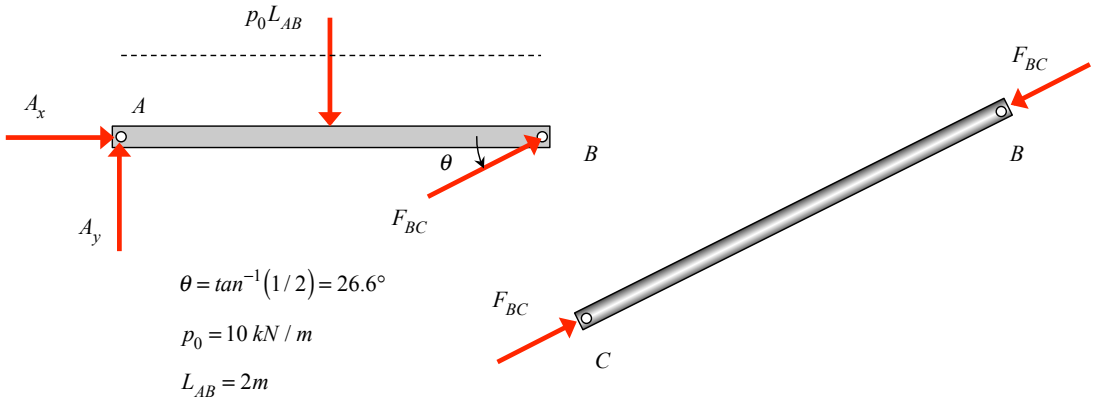
Example 18.2

The steel compression strut BC of the frame ABC is a steel tube with an outer diameter of d = 48 mm and a wall thickness of t = 5 mm. Determine the factor of safety against elastic buckling if a distributed load of 10 kN/m is applied to the horizontal frame member AB as shown. Let E = 210 GPa and σ_Y = 340 GPa.



SOLUTION

Equilibrium



$$\sum M_A = (F_{BC} sin\theta) L_{AB} - (p_0 L_{AB})(L_{AB}/2) = 0 \quad \Rightarrow \quad F_{BC} = \frac{p_0 L_{AB}^2}{2 sin\theta}$$

Calculate parameters:

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}\left[\left(\frac{d}{2}\right)^4 - \left(\frac{d}{2} - t\right)^4\right]}{\pi\left[\left(\frac{d}{2}\right)^2 - \left(\frac{d}{2} - t\right)^2\right]}} = radius\ of\ gyration$$

$$L_{eff} = \text{"}effective\ length\text{"}\ of\ the\ member = L_{BC}\ \left(pinned - pinned\ BCs\right)$$

where $L_{BC} = \sqrt{5}\ m$.

Therefore:

$$\frac{L_{eff}}{r} = 2\left[\frac{\left(\frac{d}{2}\right)^2 - \left(\frac{d}{2} - t\right)^2}{\left(\frac{d}{2}\right)^4 - \left(\frac{d}{2} - t\right)^4}\right]^{1/2} L_{BC} \quad ; \quad dependent\ on\ only\ geometry$$

$$\left(\frac{L_{eff}}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} \quad ; \quad dependent\ on\ only\ material\ properties$$

Compare and choose:

- If $L_{eff} / r > \left(L_{eff} / r\right)_c$, then the <u>*Euler*</u> theory applies: $P_{cr} = \pi^2 \frac{EI}{L_{eff}^2}$
- If $L_{eff} / r < \left(L_{eff} / r\right)_c$, then the <u>*Johnson*</u> theory applies:

$$P_{cr} = \sigma_{cr} A = \left[1 - \frac{\left(L_{eff} / r\right)^2}{\left(L_{eff} / r\right)_c^2}\right]\sigma_Y A$$

Factor of safety

$$FS = \frac{P_{cr}}{F_{BC}} = \frac{2P_{cr} sin\theta}{p_0 L_{AB}^2}$$