***Example 18.1***

Determine the critical load $P_{cr}$ of a steel pipe column that has a length of L with having a tubular cross section of inner radius $r_i$ and thickness t. The material has a Young's modulus of E and yield strength of $\sigma_Y$. Use pinned-fixed boundary conditions.

*SOLUTION*

***Calculate parameters***:

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}\left[\left(r_i+t\right)^4 - r_i^4\right]}{\pi\left[\left(r_i+t\right)^2 - r_i^2\right]}} = \frac{1}{2}\sqrt{\frac{\left(r_i+t\right)^4 - r_i^4}{\left(r_i+t\right)^2 - r_i^2}} = radius\ of\ gyration$$

$$L_{eff} = \text{"}effective\ length\text{"}\ of\ column = 0.7L\ \left(pinned - fixed\ column\right)$$

Therefore:

$$\frac{L_{eff}}{r} = 1.4\left[\frac{\left(r_i+t\right)^2 - r_i^2}{\left(r_i+t\right)^4 - r_i^4}\right]^{1/2} L \quad ;\ dependent\ on\ only\ geometry$$

$$\left(\frac{L_{eff}}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} \quad ;\ dependent\ on\ only\ material\ properties$$

***Compare and choose***:

- If $L_{eff}/r > \left(L_{eff}/r\right)_c$, then the <u>*Euler*</u> theory applies: $P_{cr} = \pi^2 \frac{EI}{L_{eff}^2}$
- If $L_{eff}/r < \left(L_{eff}/r\right)_c$, then the <u>*Johnson*</u> theory applies:

$$P_{cr} = A\sigma_{cr} = \pi\left[\left(r_i+t\right)^2 - r_i^2\right]\left[1 - \frac{\left(L_{eff}/r\right)^2}{\left(L_{eff}/r\right)_c^2}\right]\sigma_Y$$