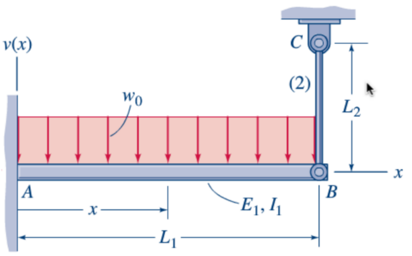
Example 14.8

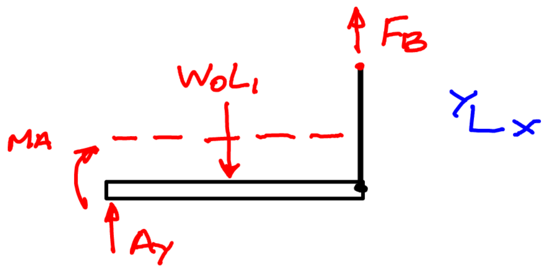
For the following examples, ***set up*** the problem for determining the requested deflections using Castigliano's method:

- draw appropriate FBDs;
- determine internal results for each section;
- set up the integrals for calculating the required deflections;
- explain how Castigliano's method is used to solve. Discuss the application of dummy forces (when needed) and how to handle redundant forces for indeterminate structures.

Problem A

Find the vertical deflection of point B on the beam. Let E_2 and A_2 be the Young's modulus and cross-sectional area, respectively, of member (2).



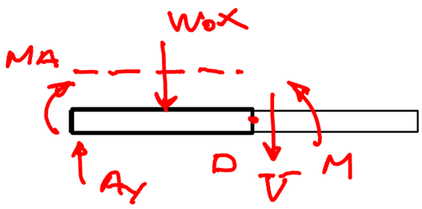


External reactions

$$\sum M_A = -M_A - (W_oL_1)\left(\frac{L_1}{2}\right) + F_B L_1 = 0 \Rightarrow M_A = -\tfrac{1}{2}W_oL_1^2 + F_B L_1 \qquad (1)$$

$$\sum F_y = A_y - W_oL_1 + F_B = 0 \qquad \Rightarrow A_y = W_oL_1 - F_B \qquad (2)$$

Internal reactions



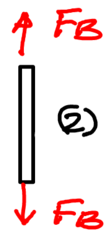
- $\sum M_D = -M_A - A_y x + W_o x\left(\frac{x}{2}\right) + M = 0$

  $\hookrightarrow M(x) = M_A + A_y x - \tfrac{1}{2}W_o x^2 \qquad (3)$

  $\sum F_y = A_y - W_o x - V = 0$

  $\hookrightarrow V(x) = A_y - W_o x \qquad (4)$

- For element (2): See FBD



Strain energy ($\leftarrow$ system = (1)+(2))

$$U = \underbrace{\frac{1}{2E_1I_1}\int_0^{L_1} M^2(x)\,dx}_{\text{flexural from (1)}} + \underbrace{\frac{f_s}{2G_1A_1}\int_0^{L_1} V^2(x)\,dx}_{\text{shear from (1)}} + \underbrace{\frac{F_B^2L_2}{2E_2A_2}}_{\text{axial from (2)}}$$

Castigliano's Theorem

$$\delta_c = \frac{\partial U}{\partial F_B} = \frac{1}{E_1I_1}\int_0^{L_1} M\frac{\partial M}{\partial F_B}dx + \frac{f_s}{G_1A_1}\int_0^{L_1} V\frac{\partial V}{\partial F_B}dx + \frac{F_BL_2}{E_2A_2}$$

w/ $\quad M = M_A + A_y x - \tfrac{1}{2}W_o x^2 = -\tfrac{1}{2}W_oL_1^2 + F_BL_1 + (W_oL_1 - F_B)x$

$$- \tfrac{1}{2}W_o x^2$$

$$V = A_y - w_o x = (w_o L_1 - F_B)x - w_o x$$

$$\therefore \quad \frac{\partial M}{\partial F_B} = L_1 - x$$

$$\frac{\partial V}{\partial F_B} = -x$$

$$\therefore \quad 0 = \frac{1}{E_1 I_1}\int_0^{L_1}\left[-\tfrac{1}{2}w_o L_1^2 + F_B L_1 + (w_o L_1 - F_B)x\right](L_1 - x)\,dx$$

$$+ \frac{f_s}{G_1 A_1}\int_0^{L_1}\left[(w_o L_1 - F_B)x - w_o x\right][-x]\,dx + \frac{F_B L_2}{E_2 A_2}$$

Integrate and solve for F_B.

Recall $\delta_B = \dfrac{F_B L_2}{E_2 A_2}$ = elongation of rod (2)

Also: $v_B = -\delta_B = -\dfrac{F_B L_2}{E_2 A_2}$