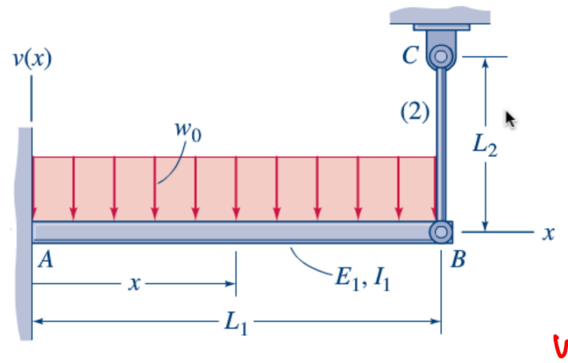
Example 14.8

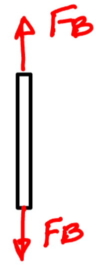
For the following examples, ***set up*** the problem for determining the requested deflections using Castigliano's method:

- draw appropriate FBDs;
- determine internal results for each section;
- set up the integrals for calculating the required deflections;
- explain how Castigliano's method is used to solve. Discuss the application of dummy forces (when needed) and how to handle redundant forces for indeterminate structures.

Problem A

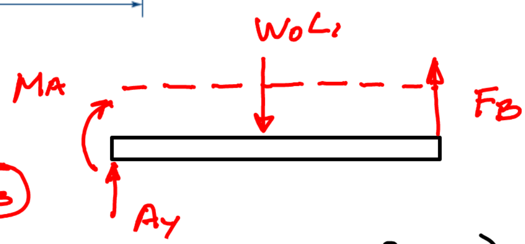
Find the vertical deflection of point B on the beam. Let E_2 and A_2 be the Young's modulus and cross-sectional area, respectively, of member (2).





$$M_A = -\frac{w_0 L_1^2}{2} + F_B L_2$$

External reactions



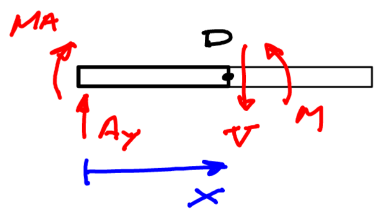
(1) $\sum M_A = -M_A - w_0 L_1\left(\frac{L_1}{2}\right) + F_B L_1 = 0$

(2) $\sum F_y = A_y - w_0 L_1 + F_B = 0$

IN determinate! $\hookrightarrow$ $A_y = w_0 L_1 - F_B$

Also have: $\delta_B = \oplus \frac{F_B L_2}{EA}$ (Need "+" since $F_B > 0 \Rightarrow \delta_B > 0$)

Internal reactions



$$\sum M_D = M - A_y x - M_A = 0$$

$$\hookrightarrow M(x) = A_y x + M_A$$

$$\sum F_y = A_y - V = 0 \Rightarrow V(x) = A_y$$

$$\therefore \; U = \frac{1}{2E_1I_1}\int_0^{L_1} M^2 dx + \frac{f_s}{2GA_1}\int_0^{L_1} V^2 dx$$

$$v_B = \frac{\partial U}{\partial F_B} = \frac{1}{E_1I_1}\int_0^{L_1} M \frac{\partial M}{\partial F_B} dx + \frac{f_s}{GA_1}\int_0^{L_1} V \frac{\partial V}{\partial F_B} dx$$

w/ $M = A_y x + M_A$

$$\frac{\partial M}{\partial F_B} = \frac{\partial A_y}{\partial F_B} x + \frac{\partial M_A}{\partial F_B} = -x + L_1$$

$$V = A_y$$

$$\frac{\partial V}{\partial F_B} = \frac{\partial A_y}{\partial F_B} = -1$$

$\therefore$

$$V_B = \frac{1}{E_1 I_1} \int_0^{L_1} (A_y x + M_A)(-x + L_1)\, dx$$

$$+ \frac{f_s}{G_1 A_1} \int_0^{L_1} (-A_y)\, dx \qquad (3)$$

Need "−" since $\delta_B > 0$
$\Longrightarrow$ B moves down

Also: $V_B = -\delta_B = -\dfrac{F_B L_2}{EA}$ (4)

Equate (3) & (4): $$-\frac{F_B L_2}{EA} = \frac{1}{E_1 I_1} \int_0^{L_1} \left[M_A L_2 + (A_y L_1 - M_A) x - A_y x^2 \right] dx - \frac{f_s}{G_1 A_1} A_y L_1$$

Solve (1) − (3) for M_A, A_y, F_B

Then $$V_B = -\frac{F_B L_2}{EA} = \ldots$$