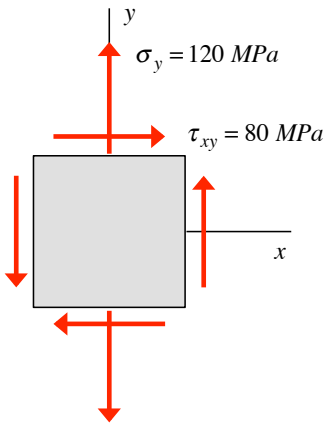
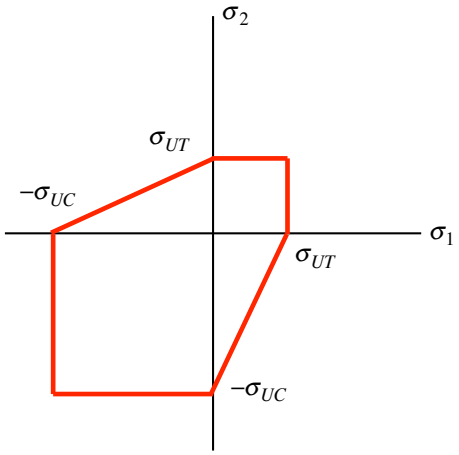
Example 15.7

Consider the state of stress shown below that exists at a location within a component made up of a *brittle* material, where this brittle material has tensile and compressive ultimate strengths of $\sigma_{UT} = 170\ MPa$ and $\sigma_{UC} = 850\ MPa$, respectively. Has the material failed at this location in the component?





SOLUTION

$$\begin{cases} \sigma_{ave} = \dfrac{\sigma_Y}{2} = 60\ MPa \\ R = \sqrt{\left(\dfrac{\sigma_Y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{60^2 + 80^2} = 100\ MPa \end{cases}$$

$$\begin{cases} \sigma_{p_1} = \sigma_{ave} + R = 60 + 100 = 160\ MPa \\ \sigma_{p_2} = \sigma_{ave} - R = 60 - 100 = -40\ MPa \end{cases} \quad 4^{th}\ quadrant$$

At failure boundary:

$$-40 = -850 + \frac{850}{170}\sigma_{p_1}$$

$$\hookrightarrow \sigma_{p_1}^* = 162$$

Since $\sigma_{p_1} < \sigma_{p_1}^* \Rightarrow$ SAFE by this failure theory