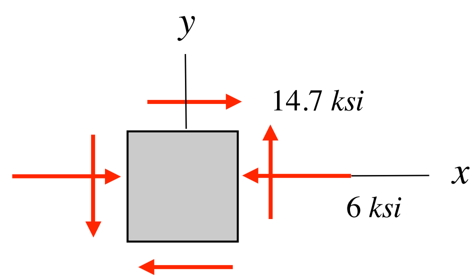
Example 15.2

Consider the state of stress shown below in a component made up of a ductile material with a shear strength of $\sigma_Y = 36 ksi$. Does the maximum shear stress theory predict failure for the material? Does the maximum distortional energy theory predict failure of the material?



SOLUTION

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{-6+0}{2} = -3\ ksi$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-6-0}{2}\right)^2 + 14.7^2}$$

$$= 15\ ksi$$

$\therefore$

$$\begin{cases} \sigma_{p1} = \sigma_{ave} + R = -3 + 15 = 12\ ksi \\ \sigma_{p2} = \sigma_{ave} - R = -3 - 15 = -18\ ksi \end{cases}$$

$\therefore$

$$\sigma_1 = \max(0, \sigma_{p1}) = 12\ ksi$$

$$\sigma_3 = \min(0, \sigma_{p2}) = -18\ ksi$$

Maximum Shear Stress (MSS)

$$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} = \frac{12+18}{2} = 15\ ksi$$

$$\frac{\sigma_Y}{2} = \frac{36}{2} = 18\ ksi \Rightarrow \tau_{max,abs} < \frac{\sigma_Y}{2}$$ (SAFE by MSS)

Maximum Distortional Energy (MDE)

$$\sigma_M = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2}$$

$$= \sqrt{12^2 - (12)(-18) + (-18)^2}$$

$$= 26.2 \text{ ksi}$$

Since:

$$\sigma_M < \sigma_Y \Rightarrow \text{SAFE by MDE}$$