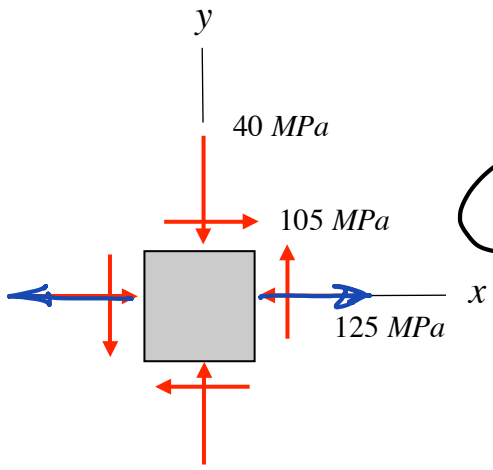
Example 15.1

Consider the state of stress shown below in a component made up of a ductile material with a shear strength of $\sigma_Y = 250 MPa$. Does the maximum shear stress theory predict failure for the material? Does the maximum distortional energy theory predict failure of the material?



Note: Sign change for σ_x

SOLUTION

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{125 - 40}{2} = 42.5 MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{125+40}{2}\right)^2 + 105^2}$$

$$= 133.5 MPa$$

$\therefore$

$$\begin{cases} \sigma_{p1} = \sigma_{ave} + R = 42.5 + 133.5 = 176.0 MPa \\ \sigma_{p2} = \sigma_{ave} - R = 42.5 - 133.5 = -91.1 MPa \end{cases}$$

$$\begin{cases} \sigma_1 = \max(0, \sigma_{p1}) = 176.0 MPa \\ \sigma_3 = \min(0, \sigma_{p2}) = -91.1 MPa \end{cases}$$

Maximum Shear Stress Theory (MSS)

$$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} = \frac{176.0 + 91.1}{2} = 133.5 MPa$$

$$\frac{\sigma_Y}{2} = \frac{250}{2} = 125 MPa \Rightarrow \tau_{max,abs} > \frac{\sigma_Y}{2}$$ (FAIL by MSS)

Maximum Distortional Energy (MDE)

$$\sigma_M = \sqrt{\sigma_{P1}^2 - \sigma_{P1}\sigma_{P2} + \sigma_{P2}^2}$$

$$= \sqrt{176.0^2 - (176.0)(-91.1) + (-91.1)^2}$$

$$= 235.2 \text{ MPa}$$

Since:

$$\sigma_M < \sigma_Y \Rightarrow \text{SAFE by MDE}$$

Note: Since σ_{P1} & σ_{P2} are of opposite signs, the state of stress lies in the 4th quadrant. This state of stress is outside of MSS boundary but inside MDE boundary