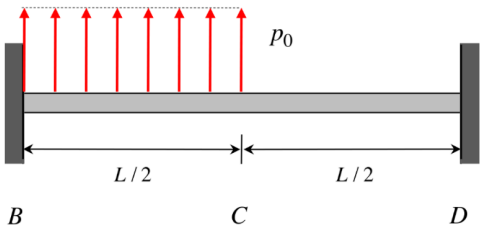
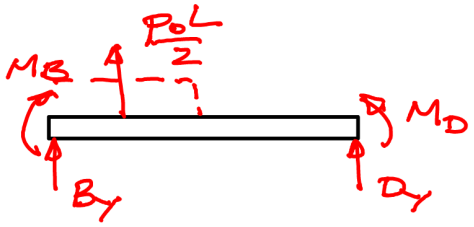
Example 11.20

Determine the reactions acting on the beam loaded as below using superposition.





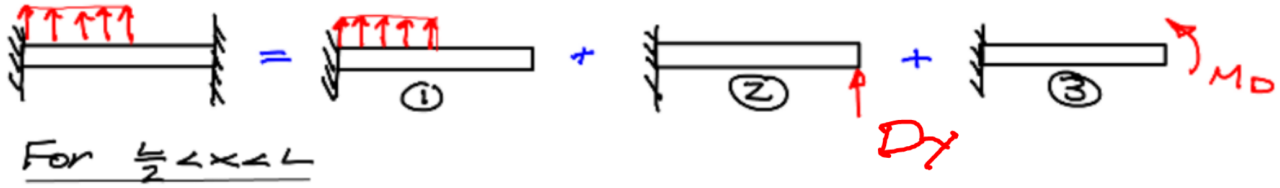
- Equilibrium

$$\Sigma M_B = -M_B + \frac{P_0 L}{2}\left(\frac{L}{4}\right) + D_y L + M_D = 0$$

$$\hookrightarrow M_B = M_D + D_y L + \frac{P_0 L^2}{8} \quad (1)$$

$$\Sigma F_y = B_y + D_y + \frac{P_0 L}{2} = 0 \quad (2)$$

- Load / Deflection



For $\frac{L}{2} < x < L$

$$V_1(x) = \frac{P_0 x^2}{24EI}\left[\frac{3}{2}L^2 - 2Lx + x^2\right]$$

$$V_2(x) = \frac{D_y x^2}{6EI}[3L - x]$$

$$V_3(x) = \frac{M_D x^2}{2EI}$$

$\therefore$

$$V(x) = V_1(x) + V_2(x) + V_3(x)$$

$$= \frac{P_0 x^2}{24EI}\left[\frac{3}{2}L^2 - 2Lx + x^2\right] + \frac{D_y x^2}{6EI}[3L - x] + \frac{M_D x^2}{2EI}$$

$$\theta(x) = \frac{dV}{dx} = \frac{P_0}{24EI}[3L^2 x - 6Lx + 4x^3] + \frac{D_y}{6EI}[6Lx - 3x^2] + \frac{M_D x}{EI}$$

- Compatibility

Enforcing BC's at $x = L$

$$V(L) = 0 = -\frac{P_0 L^4}{48EI} + \frac{D_y L^3}{3EI} + \frac{M_D L^2}{2EI} \Rightarrow M_D + \frac{2}{3}D_y L = \frac{P_0 L^2}{24} \quad (3)$$

$$\theta(L) = 0 = \frac{P_0 L^3}{24EI} + \frac{D_y L^2}{2EI} + \frac{M_D L}{EI} \Rightarrow M_D + \frac{D_y L}{2} = -\frac{P_0 L^2}{24} \quad (4)$$

- Solve

Subtract (4) from (3): $D_y = \frac{p_o L}{2}$

(4) $\Rightarrow$ $M_D = -\frac{p_o L^2}{24} - \frac{D_y L}{2} = -\frac{13}{24} p_o L^2$

(2) $\Rightarrow$ $B_y = -p_o L$

(1) $\Rightarrow$ $M_B = -\frac{13}{24} p_o L^2 + \frac{p_o L^2}{2} + \frac{p_o L^2}{8} = \frac{p_o L^2}{12}$