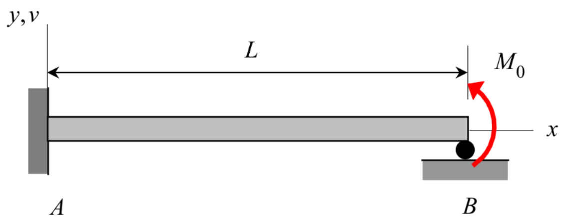
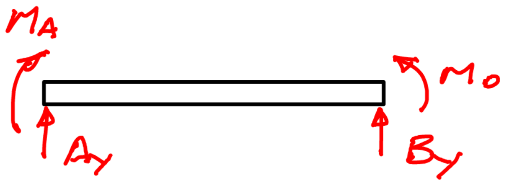
Example 11.10

The beam is made up of a material with an elastic modulus E and has a cross-sectional second area moment I, both of which are constant along the length of the beam. Determine the reactions at A and B.





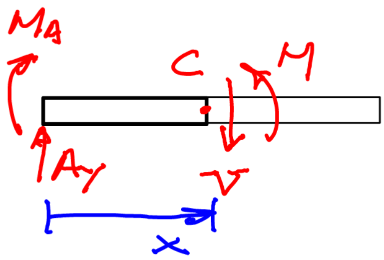
By integration

- Equilibrium

$$\sum M_A = -M_A + M_o + B_y L = 0 \Rightarrow -M_A + B_y L = -M_o \quad (1)$$

$$\sum F_y = A_y + B_y = 0 \Rightarrow A_y = -B_y \quad (2)$$

- Internal reactions



$$\sum M_c = M - M_A - A_y x = 0$$

$$\hookrightarrow M(x) = M_A + A_y x$$

- Deflection

$$\theta(x) = \cancel{\theta(0)}^{0} + \frac{1}{EI}\int_0^x M(x)\,dx$$

$$= \frac{1}{EI}\left(M_A x + \frac{1}{2}A_y x^2\right)$$

$$V(x) = \cancel{V(0)}^{0} + \int_0^x \theta(x)\,dx = \frac{1}{EI}\left[\frac{1}{2}M_A x^2 + \frac{1}{6}A_y x^3\right]$$

- Enforce BC

$$V(L) = 0 = \frac{1}{EA}\left[\frac{1}{2}M_A L^2 + \frac{1}{6}A_y L^3\right] \Rightarrow$$

$$\hookrightarrow M_A = -\frac{1}{3}A_y L = \frac{1}{3}B_y L \quad (3)$$

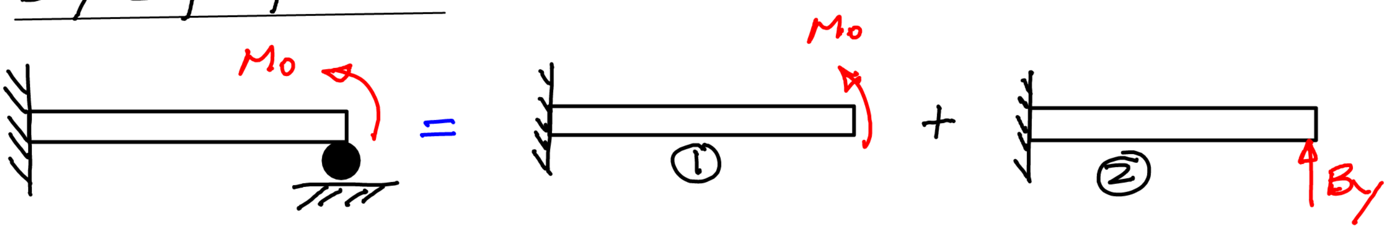
$$(1) \,\&\, (3) \Rightarrow \quad -\left(\frac{1}{3}B_y L\right) + B_y L = -M_o$$

$$\hookrightarrow \quad B_y = -\frac{3}{2}\frac{M_o}{L} \longleftarrow B_y$$

$$(2) \Rightarrow \quad A_y = -B_y = \frac{3}{2}\frac{M_o}{L} \longleftarrow A_y$$

$$(3) \Rightarrow \quad M_A = -\frac{1}{2}M_o \longleftarrow M_A$$

By superposition



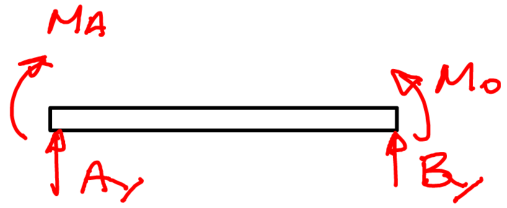
$$\begin{cases} v_1(x) = \dfrac{M_o x^2}{2EI} \\ v_2(x) = \dfrac{B_y x^2}{6EI}(3L-x) \end{cases} \Longrightarrow v(x) = v_1(x) + v_2(x)$$

Enforce BC

$$v(L) = 0 = \frac{M_o L^2}{2EI} + \frac{B_y L^2}{6EI}(2L)$$

$$\hookrightarrow B_y = -\frac{3}{2}\frac{M_o}{L} \longleftarrow B_y$$

Equilibrium



$$\sum M_A = -M_A + M_o + B_y L = 0$$

$$\hookrightarrow M_A = M_o + \left(-\frac{3}{2}\frac{M_o}{L}\right)L = -\frac{M_o}{2}$$

$$\sum F_y = A_y + B_y = 0 \Rightarrow A_y = -B_y = \frac{3}{2}\frac{M_o}{L} \longleftarrow A_y$$