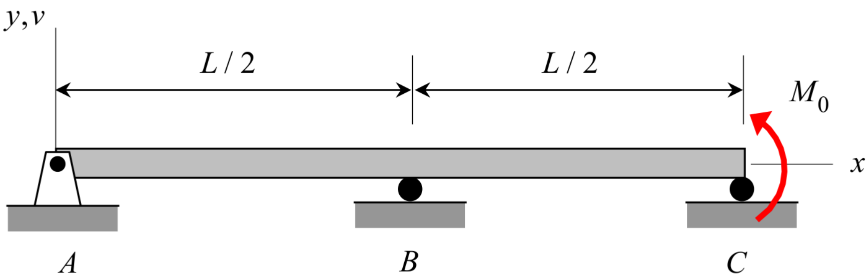
Example 11.9

The beam is made up of a material with an elastic modulus E that is constant throughout the beam. The beam has a square cross section of dimensions $h \times h$.

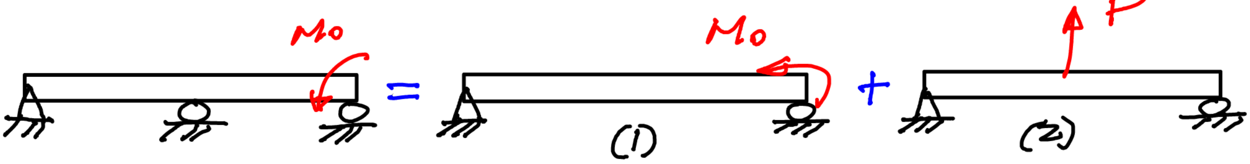
a) Determine the reactions at A, B and C.

b) Determine the bending moment at B.

c) Determine the maximum (magnitude) normal stress at B.



Use superposition:



Where:

$$v_1(x) = -\frac{M_0 x}{6EIL}(L^2 - x^2)$$

$$v_2(x) = \frac{Px}{48EI}(3L^2 - 4x^2)$$

&

$$v(x) = v_1(x) + v_2(x)$$

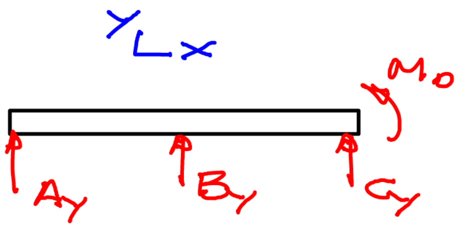
Enforce BC at B:

$$v\left(\frac{L}{2}\right) = v_1\left(\frac{L}{2}\right) + v_2\left(\frac{L}{2}\right)$$

$$0 = -\frac{M_0\left(\frac{L}{2}\right)}{6EIL}\left[L^2 - \frac{L^2}{4}\right] + \frac{P\left(\frac{L}{2}\right)}{48EI}\left[3L^2 - 4\frac{L^2}{4}\right]$$

$$= -\frac{M_0 L^2}{8EI} + \frac{PL^3}{96EI} \Rightarrow P = 12\frac{M_0}{L}$$

Reactions



From above: $B_y = P = 12\frac{M_0}{L}$

$$\Sigma M_A = B_y\left(\frac{L}{2}\right) + C_y(L) = 0 \Rightarrow$$

$$C_y = -\frac{1}{2}B_y = -6\frac{M_0}{L}$$

$$\Sigma F_y = A_y + B_y + C_y = 0 \Rightarrow A_y = -B_y - C_y = -6\frac{M_0}{L}$$

Bending moment @ B

$$M(x) = EI\frac{d^2v}{dx^2} = EI\frac{d^2v_1}{dx^2} + EI\frac{d^2v_2}{dx^2}$$

$$= EI\left[-\frac{M_0}{6LEI}(-6x)\right] + EI\left[\frac{P}{48EI}(-24x)\right]$$

$$= M_0\frac{x}{L} - \frac{1}{2}Px = \left(\frac{M_0}{L} - 12\frac{M_0}{L}\right)x = -11\,M_0\frac{x}{L}$$

$$M_B = M\left(\frac{L}{2}\right) = -\frac{11}{2}M_0$$

Flexural stress at B

$$|\sigma|_{max} = \frac{|M_B|(h/2)}{\frac{1}{12}h^4} = 33\frac{M_0}{h^3}$$