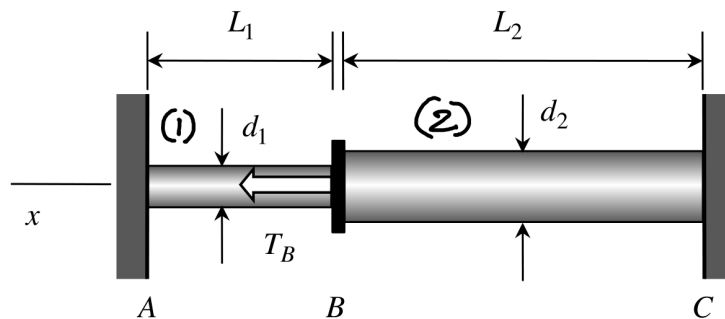


Example 8.5

A stepped steel shaft AC ($G = 12 \times 10^6$ psi) is subjected to an external torque of T_B at B and is fixed to rigid supports at ends A and C, as shown in the figure below.

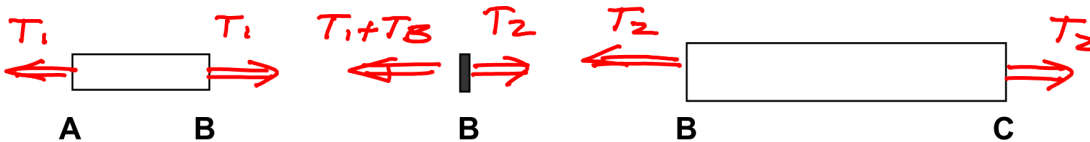
- Determine the torques T_1 and T_2 carried by segments (1) and (2), respectively.
- Determine the maximum shear stress in each segment.
- Determine the angle of rotation ϕ_B at joint B.

$$T_B = 4000 \text{ lb} \cdot \text{in.}, d_1 = 1.0 \text{ in.}, L_1 = 10.0 \text{ in.}, d_2 = 2.0 \text{ in.}, \\ L_2 = 20.0 \text{ in.}$$



Solution

Equilibrium



$$\underline{B}: \sum M = T_2 - T_1 - T_B = 0 \quad (1)$$

INDETERMINATE !!

Rotation

$$\Delta\phi_1 = \frac{T_1 L_1}{G J_1} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} J_1 = \frac{1}{2} \pi \left(\frac{d_1}{2}\right)^4 = \frac{\pi}{32} d_1^4 \quad (2)$$

$$\Delta\phi_2 = \frac{T_2 L_2}{G J_2} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} J_2 = \frac{1}{2} \pi \left(\frac{d_2}{2}\right)^4 = \frac{\pi}{32} d_2^4 \quad (3)$$

Compatibility

$$\phi_A = 0 \quad ; \text{ wall}$$

$$\phi_B = \phi_A + \Delta\phi_1$$

$$\phi_c = \phi_B + \Delta\phi_2 = \Delta\phi_1 + \Delta\phi_2 = 0 \quad ; \text{ wall} \quad (4)$$

Solve

$$(2) - (4): \quad \frac{T_1 L_1}{G J_1} + \frac{T_2 L_2}{G J_2} = 0$$

$$\hookrightarrow T_1 = - \frac{L_2}{L_1} \frac{J_1}{J_2} T_2 \quad (5)$$

(1) & (5):

$$T_2 - \left(- \frac{L_2}{L_1} \frac{J_1}{J_2} T_2 \right) = + T_B$$

$$\hookrightarrow \begin{cases} T_2 = \frac{T_B}{1 + \frac{L_2 J_1}{L_1 J_2}} \\ T_1 = - T_2 = \underline{\hspace{2cm}} \end{cases}$$

$$(\tau_1)_{\max} = \frac{T_1 \rho_{\max}}{J_1} = d_1/2$$

$$= \frac{16}{\pi d_1^3} T_1$$

$$(\tau_2)_{\max} = \frac{T_2 \rho_{\max}}{J_2} = \frac{16}{\pi d_2^3} T_2$$

$$\phi_B = \Delta\phi_1 = \frac{T_1 L_1}{G J_1} = \underline{\hspace{2cm}}$$

