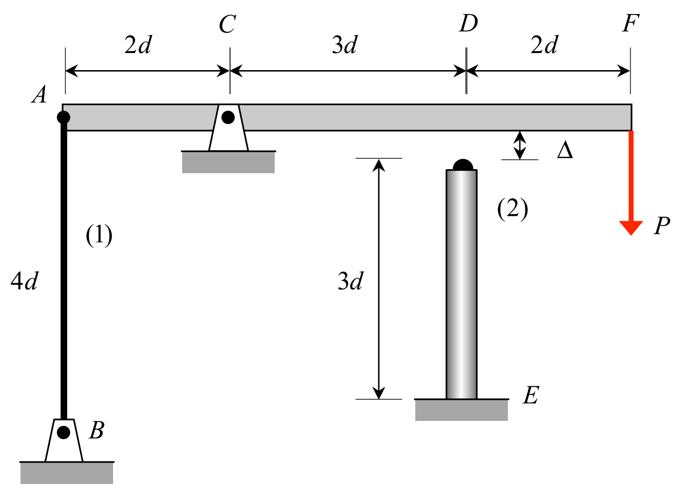
Example 2.2

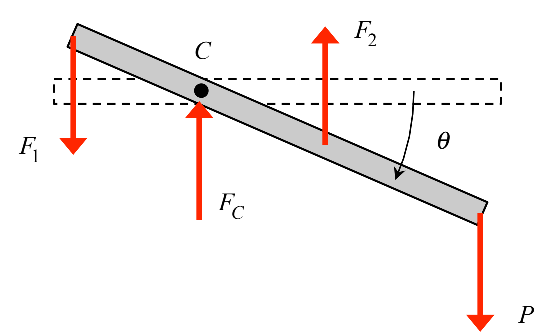
For small loads P, the rotation of the rigid beam AF is controlled by the stretching of rod AB. For larger loads, the beam comes into contact with the top of column DE, and further resistance to rotation is shared by the rod and the column. Assume that the clockwise angle θ through which beam AF rotates is small enough to assume that points on the beam essentially move vertically.

a) A load P is applied that is just sufficient to close the Δ gap between the beam and the column. What is the strain ε_1 in rod AB for this value of P?

b) If the load P in a) is doubled, what is the corresponding strain ε_2 in column DE?



SOLUTION



Equilibrium

From the FBD of beam AF above:

$$\sum M_C = F_1(2d) + F_2(3d) - P(5d) = 0 \;\Rightarrow\; 2F_1 + 3F_2 = 5P$$

Kinematics

$$\theta \approx tan^{-1}\left(\frac{\Delta_1}{2d}\right) \approx \frac{\Delta_1}{2d}$$

$$\theta \approx tan^{-1}\left(\frac{\Delta_2}{3d}\right) \approx \frac{\Delta_2}{3d}$$

Therefore:

$$\frac{\Delta_1}{2d} = \frac{\Delta_2}{3d} \quad \Rightarrow \quad \Delta_1 = \frac{2}{3}\Delta_2$$

Part a):

Since $\Delta_2 = \Delta = initial\ gap$, then:

$$\varepsilon_1 = \frac{\Delta_1}{4d} = \frac{\Delta}{6d}$$

And, since member (2) is not compressed at this position:

$$F_2 = 0 \quad \Rightarrow \quad 5P_a = 2F_1 \quad \Rightarrow \quad P_a = \frac{2}{5}F_1 = \frac{2}{5}\sigma_1 A_1 = \frac{2}{5}E_1 A_1 \varepsilon_1 = \frac{E_1 A_1 \Delta}{15d}$$

Part b):

Since $P_b = 2P_a \ \left(given\right)$, we have: $P_b = \dfrac{2E_1 A_1 \Delta}{15d}$

Substituting this into the equilibrium equation above gives:

$$5P_b = 2F_1 + 3F_2$$

$$\frac{2E_1 A_1 \Delta}{3d} = 2\sigma_1 A_1 + 3\sigma_2 A_2$$

$$= 2E_1 A_1 \varepsilon_1 + 3E_2 A_2 \varepsilon_2$$

$$= 2E_1 A_1\left(\frac{\Delta_1}{4d}\right) + 3E_2 A_2\left(\frac{\Delta_2 - \Delta}{3d}\right)$$

$$= \frac{E_1 A_1}{2d}\left(\frac{2\Delta_2}{3}\right) + \frac{E_2 A_2}{d}\left(\Delta_2 - \Delta\right) \quad \Rightarrow$$

$$\left(\frac{2}{3}E_1 A_1 + E_2 A_2\right)\frac{\Delta}{d} = \left(\frac{1}{3}E_1 A_1 + E_2 A_2\right)\frac{\Delta_2}{d} \quad \Rightarrow$$

$$\Delta_2 = \left(\frac{2E_1 A_1 / 3 + E_2 A_2}{E_1 A_1 / 3 + E_2 A_2}\right)\Delta$$

And:

$$\varepsilon_2 = \frac{\Delta_2 - \Delta}{3d} = \left(\frac{2E_1 A_1 / 3 + E_2 A_2}{E_1 A_1 / 3 + E_2 A_2} - 1\right)\frac{\Delta}{3d} = \left(\frac{E_1 A_1 / 3}{E_1 A_1 / 3 + E_2 A_2}\right)\frac{\Delta}{3d} \quad \left(compression\right)$$