ME 274: Basic Mechanics II

Final Review

PURDUE UNIVERSITY | School of Mechanical Engineering

Announcements:

- Fill out the course Eval. and submit proof on Gradescope for bonus points – deadline 5/3 (Sunday)

Final Exam Info:

- Tuesday 5/5, 7-9 pm in UC 114 – seating chart will be posted
- Inform me of conflicts ASAP
- Review session 5/3 @ 7pm over Zoom – see Exams tab on course blog for the link
- Content: 3 long-form problems on material after exam 2 (lectures 27-40), multiple choice covering the entire course

What should I study?

- Homework solutions (assignment 27 – 39)
- Lecture book example problems (see course blog for soln. videos)
- Weekly Joys sample exams (see final exam for longform problems, work conceptual problems from all exams)
- Conceptual problems at the end of chapters
- Additional past exam problems posted in the Exam tab
- “Rapid fire” review – see “daily schedule” week 13 on course blog

Topics that can be in the Long-form problems - Rigid Body Kinetics

- Know how to apply the **4-step problem solving method**
- Newton Euler

$$\Sigma\vec{F} = m\vec{a}_G\ ;\quad \Sigma\vec{M}_A = \boxed{I_A\vec{\alpha}} + m\vec{r}_{G/A} \times \vec{a}_A$$

$$I_A = I_G + md^2$$

$$I_A = mk_A^2$$

$\frac{1}{2}mr^2$

- Work Energy

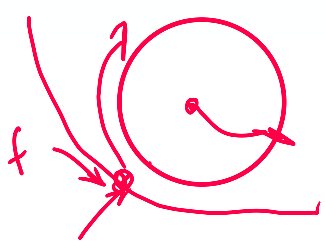
$$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$$

$$V_{gr} = mgh$$

$$V_{sp} = \frac{1}{2}k\Delta^2$$

$$T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$$

$$U_{1\to2}^{(nc)} = \int_1^2 (\Sigma\vec{F} \cdot \hat{e}_t)ds = \int_1^2 (F_x dx + F_y dy)$$



- Know how to determine when to use each method!!! – Look at what is being asked for and what is being conserved

- Impulse Momentum

$$e = -\left(\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}\right)$$

$$\int_1^2 \Sigma\vec{F}_{ext}\, dt = \left(\sum_i m_i\vec{v}_i\right)_2 - \left(\sum_i m_i\vec{v}_i\right)_1$$

$$\int_1^2 \Sigma\vec{M}_A\, dt = \left(\sum_i (\vec{H}_A)_i\right)_2 - \left(\sum_i (\vec{H}_A)_i\right)_1$$

$$(\vec{H}_A)_i = \vec{r}_{i/A} \times (m_i\vec{v}_A)$$

$$= I_A\vec{\omega}$$

Topics that can be in the Long-form problems - Vibrations

Free vibrations

$M\ddot{x} + C\dot{x} + Kx = 0$

- Know how the **process for finding the EOM** $x_c(t) = e^{-\zeta\omega t}(C\ cos\omega_d t + S\ sin\omega_d t)\ ;\quad 0 \le \zeta < 1$
- Know how the **identify characteristics of the system** $(\omega_n, \omega_d, \zeta)$ from your EOM
- Know how to calculate the complimentary solution
- Know how to apply the initial conditions

$\ddot{x} + \frac{C}{M}\dot{x} + \frac{K}{M}x = 0$, $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$

- Know how to **interpret the response plot**

$\hookrightarrow \omega_n$, ω_d , $T = \frac{2\pi}{\omega_n, \omega_d}$ $\leftarrow$

Forced vibrations

- Know how the **process for finding the EOM** $M\ddot{x} + C\dot{x} + Kx = F(t) = F_0 sin\omega t$ ***OR*** $F_0 cos\omega t$
- Know how to **find the particular solution** $x_p(t) = Asin\omega t + Bcos\omega t$ $\leftarrow$
- Know how to tell if the response is **in-phase or out-of-phase**

$x_p = Asin\omega t$ $\qquad \frac{\omega}{\omega_n}$

4C – Spring 25

Given: In System I, a particle P of mass *m* is free to slide without friction along a lightweight bar that rotates in a **horizontal plane** about a vertical shaft at point O. A spring of stiffness *k* and unstretched length R_O connects P to O. Initially, the spring is compressed to half its natural length, and the bar rotates with an angular speed ω_1. Upon release, the particle moves outward, and when the spring reaches its unstretched length, the bar's rotational speed is ω_2.

In System II, the initial conditions are the same, with the bar rotating at the same initial angular speed ω_1. However, two springs are attached to the particle: one compressed to half its natural length and the other stretched to 1.5 times its natural length. After release, the particle moves until both springs reach their unstretched lengths.

ang. mom.

$$\underline{m r_1^2 \omega_1} = m r_2^2 \omega_2$$

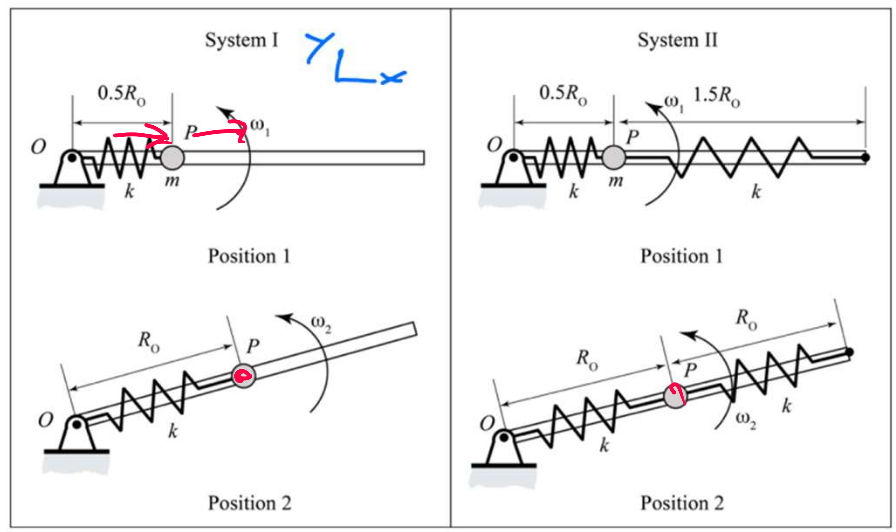
Find: Let $(\omega_2)_I$ and $(\omega_2)_{II}$ represent the final rotational speeds in Systems I and II, respectively. Circle the correct response:

a) $(\omega_2)_I < (\omega_2)_{II}$

b) $(\omega_2)_I = (\omega_2)_{II}$

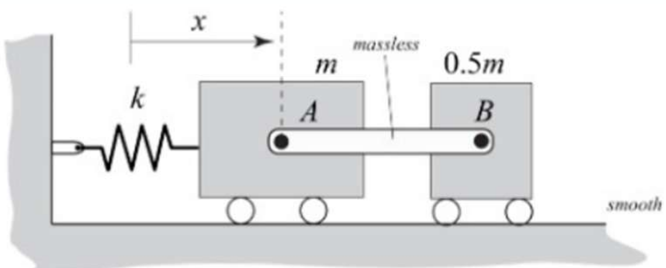
c) $(\omega_2)_I > (\omega_2)_{II}$

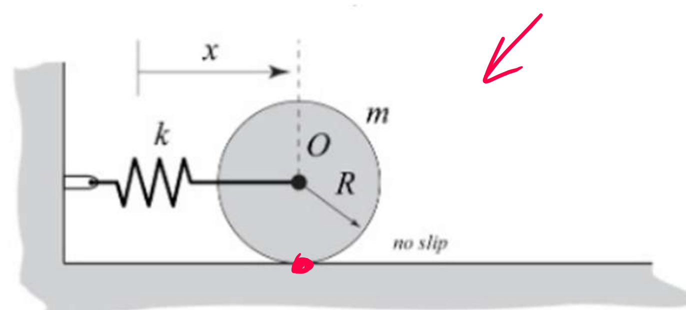
d) More information is needed on the two systems to answer this question.

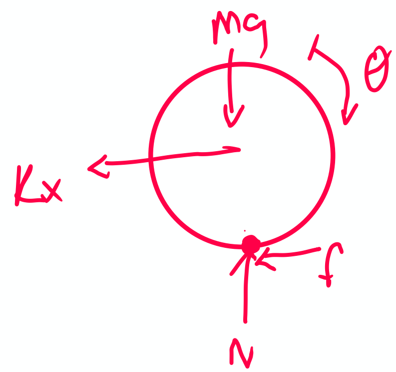


Horizontal Plane

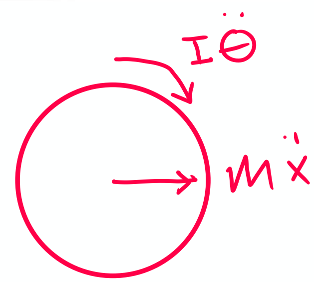
Consider Systems 1 and 2 shown below. System 1 is made up of a spring and block A with mass m rigidly connected to another block B with mass $0.5m$, and the blocks are moving in pure translation along a smooth horizontal surface. System 2 is made up of a spring and a homogeneous disk of mass m and outer radius R, with the center of the disk at O, and rolls without slipping on a horizontal surface. Each system has the same total mass m and same spring stiffness k. Let ω_{n1} and ω_{n2} represent the natural frequencies of Systems 1 and 2, respectively. Circle the answer below that most accurately represents the natural frequencies for the two systems:







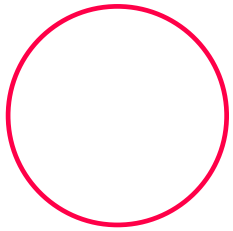
$x = r\theta$

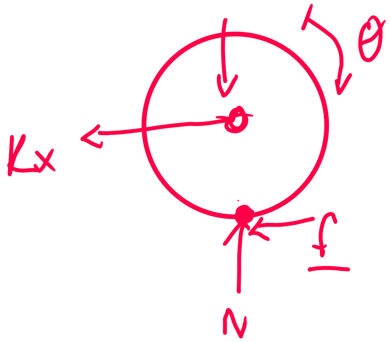


$$\Sigma M_c = -(kx\,r) = I_c\ddot{\theta}$$

$$I_c = \left(\tfrac{1}{2}mr^2 + mR^2\right) = \tfrac{3}{2}mR^2$$

$$\tfrac{3}{2}m\ddot{x} + kx = 0$$





$x = r\theta$

$$\Sigma F_x = m\ddot{x} = -kx - f$$

$$m\ddot{x} + kx + f = 0$$

$$\Sigma M_o = I\ddot{\theta} = fr \rightarrow fr = I_G \frac{\ddot{\theta}}{r}$$

$$f = \frac{1}{2} m r^2 \left(\frac{\ddot{x}}{r}\right)\frac{1}{r}$$

$$= \frac{1}{2} m\ddot{x}$$

$$m\ddot{x} + kx + \frac{1}{2} m\ddot{x} = 0$$

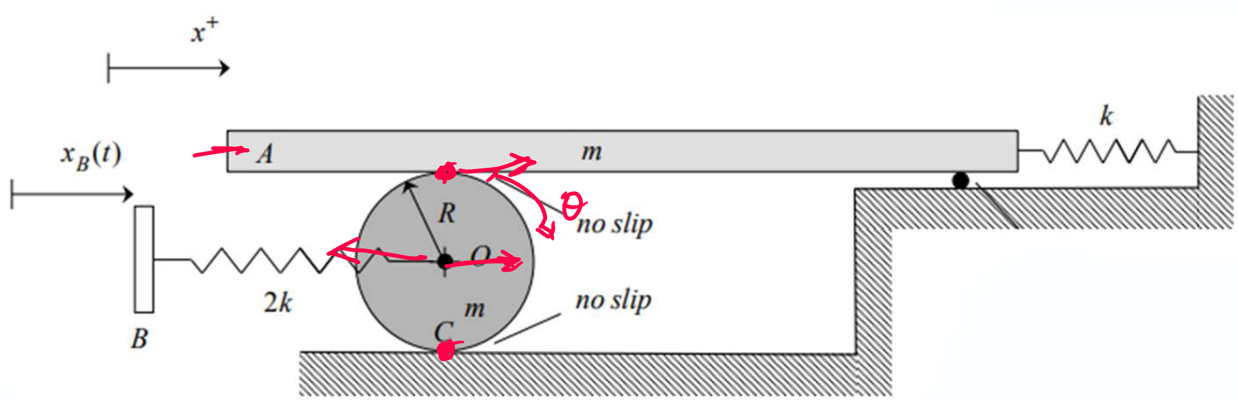
$$\frac{3}{2} m\ddot{x} + kx = 0$$

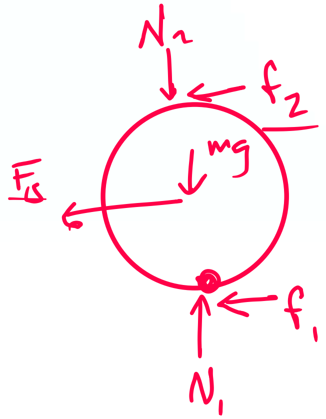
ME 274 – Spring 2018
Final Examination – Problem 2

Given: A homogeneous disk of mass m and outer radius R is able to roll without slipping on a rough horizontal surface. A bar A (of mass m) is in a no-slip contact with the top of the disk with the other end of A being supported by a smooth roller. A spring of stiffness k is connected between the right end of bar A and ground. A second spring (of stiffness $2k$) is connected between the center O of the disk and a movable base B. Base B is given a prescribed motion of $x_B(t) = b \sin \Omega t$. Let x represent the motion of the bar A, where $x = x_B = 0$ when the springs are unstretched.

Find: For this problem:

(a) Draw individual free body diagrams of the disk and of the bar A.
(b) Derive the differential equation of motion (EOM) for the system in terms of the coordinate x.
(c) What is the natural frequency ω_n of the system?
(d) Determine the particular solution $x_P(t)$ of your EOM.
(e) Based on your particular solution above, is the motion of the bar in phase or 180° out of phase with the motion of the base B when $\Omega = \sqrt{k/m}$?

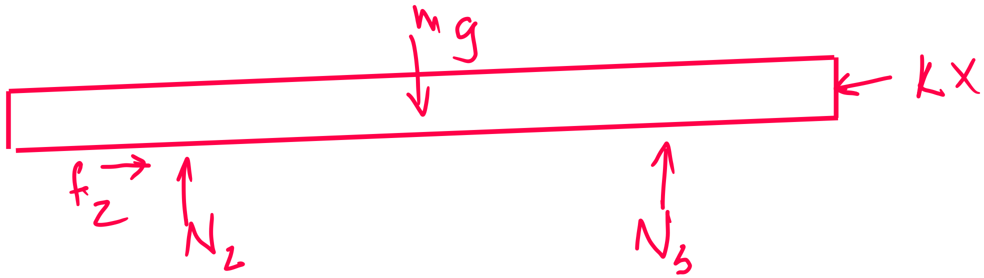




$x_0 = r\theta$

$x_0 > x_B \rightarrow F(r\theta - x_B)$

$x_0 < x_B \rightarrow F($

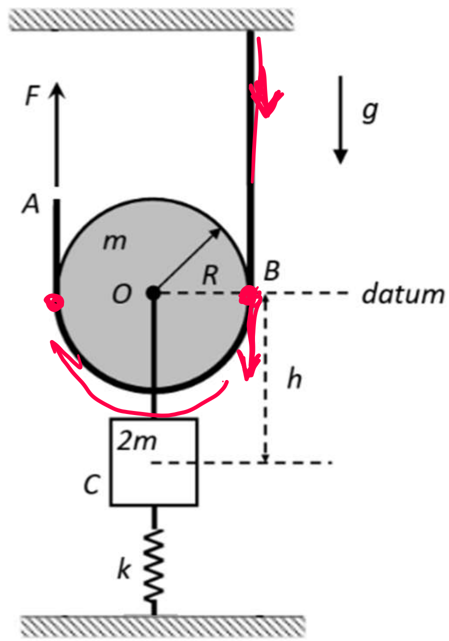


bar: $\sum F_x = -kx + f_2 = m\ddot{x}$

disk: $\circlearrowright + \sum M_c = +F_s r + f_2 2r = I_c \ddot{\theta}$

Problem 2 _(20 points)_

Given: A homogeneous pulley (having a mass m and outer radius R) is supported by a cable. A block C with mass $2m$ is connected to the pulley with a rope at point O. The height difference between the pulley and block is h. The block is connected to the spring with a stiffness of k and negligible mass. The system is released from rest with the spring initially unstretched (State 1) with a constant force F applied to end of the cable. Assume that the cable does not slip on the pulley. The figure shows the system at State 1.



Find: Follow the solution steps below to determine the angular velocity ω of the pulley after point O has moved **up** a distance of d (State 2). Express your answer in terms of, at most m, h, d, R, k and the gravitational constant g. Write your answer as a vector.