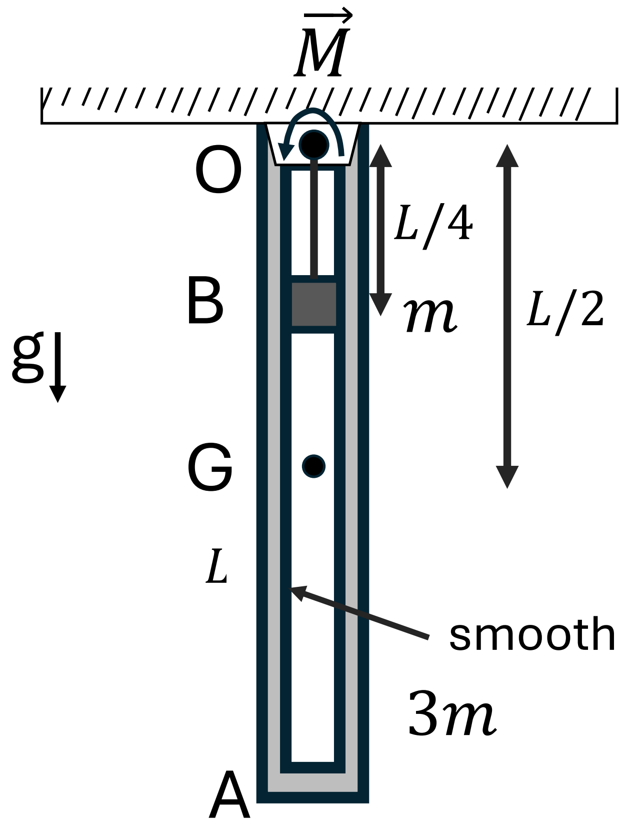
ME 274 - Final Exam - Spring 2026

Problem 1



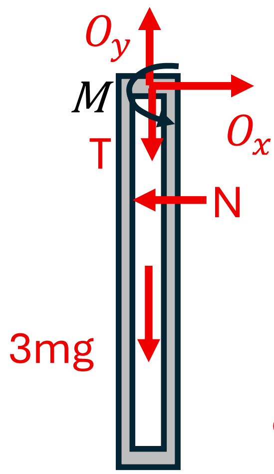
A thin uniform hollow bar (OA) with mass $3m$ and radius of gyration at its center of mass (k_G) contains a block (B) in its hollow section with mass m. Bar OA is pinned at O and is in a vertical orientation with block B positioned a distance L/4 away from end O. A rope connects end O to block B to keep the block from sliding inside the slot. Assume the inside surface of the bar to be smooth. A moment is applied at point O ($\vec{M}$) in the CCW direction to get this assembly **to start rotating from rest**. Follow the steps below to find the normal force acting on the block and the angular acceleration of the bar ($\vec{\alpha}_{OA}$) at the instant the moment is applied.

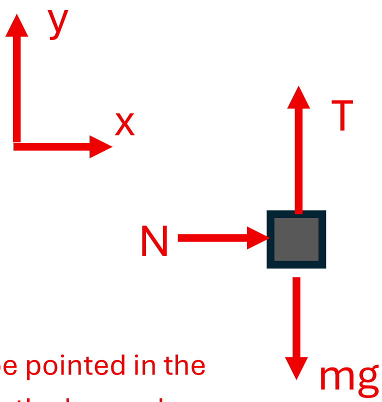
Solution:

Step 1: Circle the method you will be using to solve this problem

(a) Newton-Euler
(b) Work/Energy
(c) Linear Impulse Momentum
(d) Angular Impulse Momentum

Step 2: Draw free body diagrams for bar OA and block B. Please clearly define the coordinate system you choose for this problem.





Note that N can be pointed in the other direction for the bar and block. They just need to be pointed in ***opposite*** directions.

Step 3: Write out the *kinetics* equations you are using.

Writing out Euler for the bar using point O (CCW positive rotation)

$$\sum M_O = I_O \alpha_{OA} = M - N(L/4) \text{ [equation 1]}$$

Writing out Newton's law for the block in the x direction. (Note that the y direction is not needed here)

$$\sum F_x = N = ma_{Bx} \quad [\text{equation } 2]$$

Step 4: Write out the *kinematics* equations you will are using.

Need to relate the acceleration of point O to point B (center of mass for block) on the block using the rotating reference frame equations (ch. 3) .

$$\vec{a}_B = \cancel{\vec{a}_O} + \cancel{(\vec{a}_{O|B})_{rel}} + \vec{\alpha}_{OA} \times \vec{r}_{O/B} - \cancel{\omega^2 \vec{r}_{O|B}} \quad \rightarrow \quad a_{Bx}\hat{\imath} + a_{By}\hat{\jmath} = \alpha_{OA}(L/4)\hat{\imath}$$

Using the x direction in the equation on the RHS we find

$$a_{Bx} = \alpha_{OA}(L/4) \quad [\text{equation } 3]$$

<u>Step 5:</u> Solve for the normal force acting on the block and the angular acceleration of the bar ($\vec{\alpha}_{OA}$) using the values below.

$$m = 2\ kg,\ k_G = \ 1\ meters,\ M = 7\ Newton - meters,\ L = 4\ meters.$$

First the moment of intertia for the bar about O.

$$I_O = k_G^2 * 3m + 3m\left(\frac{L}{2}\right)^2 = 3m\left(k_G^2 + \frac{L^2}{4}\right) = 3(2)(1+4) = 30\ kg - m^2$$

Plugging equation 3 into equation 2 we find...

$$N = m\alpha_{OA}\left(\frac{L}{4}\right)\ \text{[Equation 4]}$$

Plugging this equation for N into equation 1 we find...

$$I_O\alpha_{OA} = M - m\alpha_{OA}\left(\frac{L}{4}\right)^2$$

Rearranging a little

$$M = \alpha_{OA}\left[I_O + \frac{mL^2}{16}\right]$$

Just a little more math we find

$$\alpha_{OA} = \frac{M}{\left[I_O + \frac{mL^2}{16}\right]} = \frac{7\ kg - m^2/s^2}{[30+2]kg - m^2} = \frac{7}{32}\ s^{-2}$$

Using this number, we can plug it into equation 4 to find a value for the normal force.

$$N = m\alpha_{OA}\left(\frac{L}{4}\right) = (2\ kg)\left(\frac{7}{32}\ s^{-2}\right)\left(\frac{4\ m}{4}\right) = \frac{7}{16}\frac{kg - m}{s^2} = \frac{7}{16}\ Newtons$$

Step 6: Is the block in contact with the left or right side of the slot?

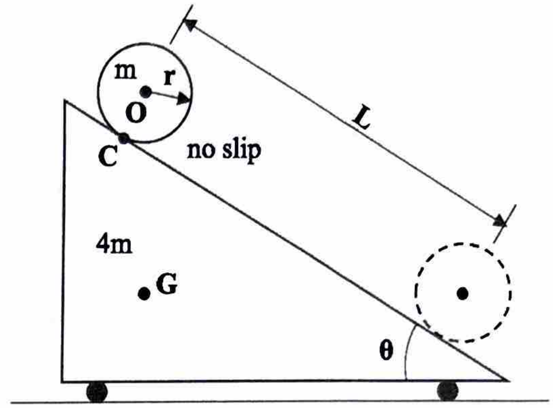
The value of N is positive, and we assumed it was to the right in the FBD for the block. Since the side of the slot can only “push” on the block to cause N, the block must be in contact with the left side.

ME 274 – Final Exam **Name** *(print)*____________________________________

May 5, 2026 *(Last)* *(First)*

PROBLEM NO. 2 (20 points)

Given: A homogeneous disk with mass m is released from rest on top of a stationary triangular cart with mass $4m$. The disk rolls without slipping down the cart, and the surface between the cart and ground is smooth. The cart's surface has an incline of θ measured from the horizontal. The cart's center of mass (CM) is G, and the disk's CM is O.



Find: The velocity of the center of mass of the disk after it has rolled a distance L down the incline. At that instant, the centers of mass of the disk and the cart are at the same height. Please write your final answer for the velocity in vector form.

Solution:

STEP A: Circle the method(s) below that you will use to solve the problem:

(a) Newton/Euler

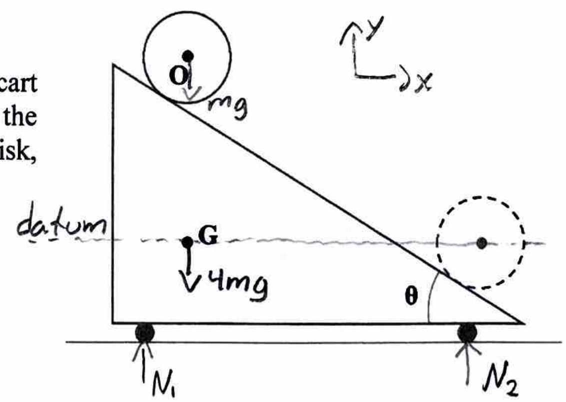
(b) Work/Energy

(c) Linear Impulse Momentum

(d) Angular Impulse Momentum

STEP B:

Draw the free body diagram (FBD) for the disk and cart system on the diagram to the right. On your FBD indicate the coordinate system you are using, states 1 and 2 of the disk, and your datum (if needed).



STEP C: Based on your FBD, write out the kinetics and kinematics equations that you will use. Note that more space is provided on the following page.

(Hint: The contact point between the disk and the cart is moving with the cart's velocity, i.e., $\vec{v}_C = \vec{v}_G$).

$$W/E: \cancelto{0}{T_1} + V_1 + \cancelto{0}{U_{1\to2}^{(nc)}} = T_2 + \cancelto{0}{V_2}$$

$$V_1 = mgL\sin\theta$$

$$T_2 = \frac{1}{2}mv_0^2 + \frac{1}{2}I_0\omega^2 + \frac{1}{2}(4m)v_G^2$$

$$I_0 = \frac{1}{2}mr^2$$

$$T_2 = \frac{1}{2}mv_0^2 + \frac{1}{4}mr^2\omega^2 + 2mv_G^2$$

LIM: $\Sigma F_x = 0 \to$ LM conserved in x

$$0 = \cancel{m}v_{0x} + 4\cancel{m}v_G$$

$$v_G = -\frac{1}{4}v_{0x}$$

ME 274 – Final Exam **Name (print)**______________________________

May 5, 2026 *(Last)* *(First)*

PROBLEM NO. 2 (continued)

STEP C: Kinetic and kinematic equations continued.

$$\vec{V}_O = \vec{V}_C + \vec{\omega} \times \vec{r}_{O/C}$$

$$\vec{V}_O = \vec{V}_G + (\omega \hat{k}) \times (r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j})$$

$$V_{Ox}\hat{i} + V_{Oy}\hat{j} = V_G\hat{i} + \omega r\sin\theta\,\hat{j} - \omega r\cos\theta\,\hat{i}$$

$$\hat{i}: V_{Ox} = V_G - \omega r\cos\theta \qquad \hat{j}: V_{Oy} = \omega r\sin\theta$$

$$V_{Ox} = -\frac{1}{4}V_{Ox} - \omega r\cos\theta$$

$$V_{Ox} = -\frac{4}{5}\omega r\cos\theta \longrightarrow V_O^2 = \omega^2 r^2\left(\frac{16}{25}\cos^2\theta + \sin^2\theta\right) = \frac{481}{625}\omega^2 r^2$$

$$V_G = \frac{1}{5}\omega r\cos\theta \longrightarrow V_G^2 = \frac{1}{25}\omega^2 r^2\cos^2\theta = \frac{16}{625}$$

STEP D: Solve for the velocity of the CM of the disk when it has traveled a distance L down the incline. Write your answer as a vector in terms of your established coordinates.
Use $m = 1$ kg, $L = 1.5$ m, $r = 0.1$ m, $\theta = 36.87^\circ$.

$$\cancel{m}gL\sin\theta = \frac{1}{2}\cancel{m}V_O^2 + \frac{1}{4}\cancel{m}r^2\omega^2 + 2\cancel{m}V_G^2$$

$$\frac{3}{5}gL = \frac{481}{1250}\omega^2 r^2 + \frac{1}{4}\omega^2 r^2 + \frac{32}{625}\omega^2 r^2$$

$$\sqrt{\omega^2} = \sqrt{\frac{\frac{3}{5}gL}{r^2\left(\frac{481}{1250} + \frac{1}{4} + \frac{32}{625}\right)}}, \quad g = 9.81\ m/s^2$$

$$|\omega| = 35.86\ rad/s$$

In kinematics, assumed $\circlearrowleft \omega^+$, but we know ω in opposite direction

$$\omega = -35.86\ rad/s$$

$$\vec{V}_O = -\frac{4}{5}\omega r\cos\theta\,\hat{i} + \omega r\sin\theta\,\hat{j}$$

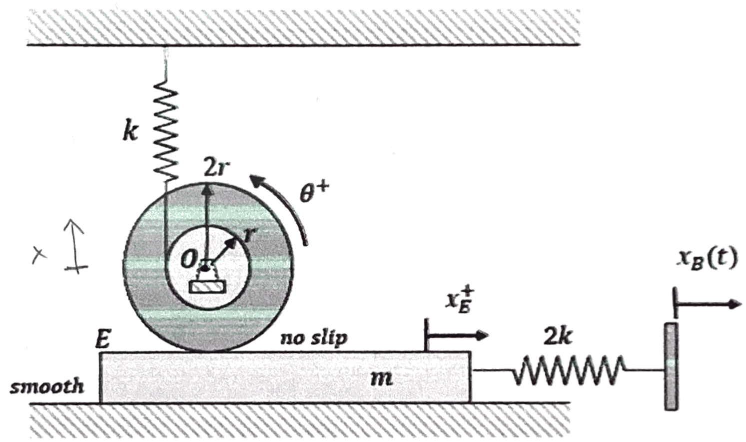
$$\boxed{\vec{V}_O = 2.296\,\hat{i} - 2.153\,\hat{j}\ m/s}$$

ME 274 – Final Exam Name (print) Solution

May 5, 2026 (Last) (First)

PROBLEM NO. 3 (20 points)

Given: Consider the system shown in the figure. A stepped disk of mass m is pinned at its center O and has a mass moment of inertia $I_O = \frac{1}{2}m(2r)^2$. The disk has an inner radius r and an outer radius $2r$. The disk rolls without slipping on block E of mass m, which can translate horizontally along a smooth surface. The block is attached to a moving base by a spring of stiffness $2k$, where the base motion is $x_B(t) = b\sin\omega t$. A second spring of stiffness k is attached to the inner radius of the disk and connected to a fixed wall. When the system is at rest, all the springs are unstretched. The rotation of the disk is defined so that counterclockwise rotation is positive.



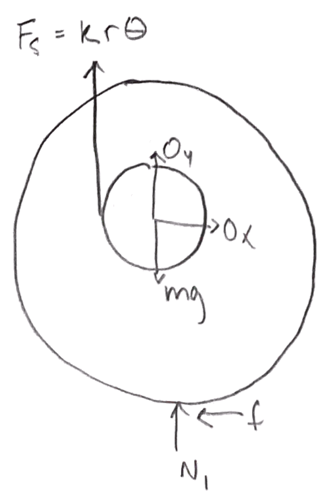
Find: For this problem, you are asked to follow the steps below to find the particular solution for the equation of motion of the system <u>in terms of the rotation angle θ</u>.

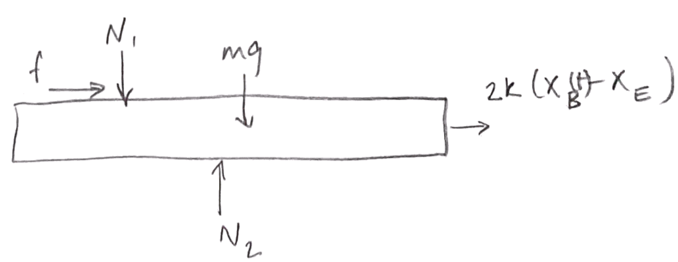
Solution:

Hint: Assume small angular displacements for the vertical spring; i.e., the motion of the spring attachment point on the inner radius may be approximated as a vertical displacement of $r\theta$.

Part (a)

Draw the free body diagram(s) necessary to derive the equation of motion for the system. On the FBDs, express the spring forces in terms of k, θ, x_E, and x_B.





PROBLEM NO. 3 (continued)

Part (b)

Write down the relevant *kinetics* equations for your FBD(s). (**See hint**)

disk:

$$\Sigma M_o = I_o \ddot{\theta} = -kr^2\theta - 2fr$$

$$\frac{1}{2}m(2r)^2\ddot{\theta} = -kr^2\theta - 2fr$$

$$2mr^2\ddot{\theta} = -kr^2\theta - 2fr$$

block:

$$\Sigma F_x = m\ddot{x}_E$$

$$m\ddot{x}_E = f + 2k(x_B - x_E)$$

Part (c)

Using the rolling without slipping condition, establish the *kinematic* relationship between the block displacement x_E and the disk rotation θ. Also relate their velocities and accelerations.

$$x_E = 2r\theta$$

$$\dot{x}_E = 2r\dot{\theta}$$

$$\ddot{x}_E = 2r\ddot{\theta}$$

Part (d)

Derive the equation of motion of the system in terms of $\theta, \ddot{\theta}, k, m, r, x_B(t)$.

$$m\ddot{x}_E = f + 2k(x_B - x_E)$$

$$f = m\ddot{x}_E - 2kx_B + 2kx_E$$

$$= 2mr\ddot{\theta} - 2kx_B + 4kr\theta$$

$$2m\cancel{r}^2\ddot{\theta} = -k\cancel{r}^2\theta - 2\cancel{r}(2mr\ddot{\theta} - 2kx_B + 4kr\theta)$$

$$2mr\ddot{\theta} = -kr\theta - 4mr\ddot{\theta} + 4kx_B - 8kr\theta$$

$$\boxed{6mr\ddot{\theta} + 9kr\theta = 4kx_B}$$

PROBLEM NO. 3 (continued)

Part (e)

Determine the natural frequency of the system.

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{9k\cancel{r}}{6m\cancel{r}}} = \sqrt{\frac{3k}{2m}}$$

Part (f)

Determine the steady-state particular solution $\theta_p(t)$ due to the base excitation: $x_B(t) = b\sin\omega t$.

$$\theta_p(t) = A\sin\omega t + B\cos\omega t$$

$$\ddot{\theta}_p(t) = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$$

$$6mr(-A\omega^2\sin\omega t - B\omega^2\cos\omega t) + 9kr(A\sin\omega t + B\cos\omega t) = 4kb\sin\omega t$$

$$\sin: \; -6Amr\omega^2 + 9krA = 4kb$$

$$\cos: \; -6mrB\omega^2 + 9krB = 0 \rightarrow B = 0$$

$$A = \frac{4kb}{r(9k - 6m\omega^2)}$$

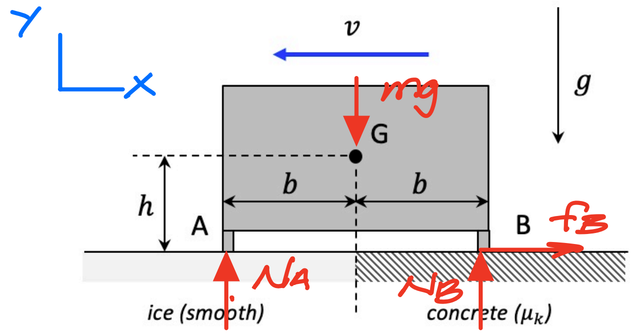
ME 274 – Final Exam **Name** *(print)* SOLUTION

May 5, 2026 *(Last)* *(First)*

PROBLEM NO. 4

Part 4A (2 points)

A crate is sliding to the left along a flat, horizontal concrete surface (coefficient of kinetic friction of μ_k). At some point the crate begins to slide upon a smooth icy surface. At the instant shown, the center of mass G is directly above the transition from concrete to ice. Let N_A and N_B be the normal forces on the crate at A and B at that instant. Circle the correct response below regarding the relative sizes of N_A and N_B:



a) $N_A < N_B$

b) $N_A = N_B$

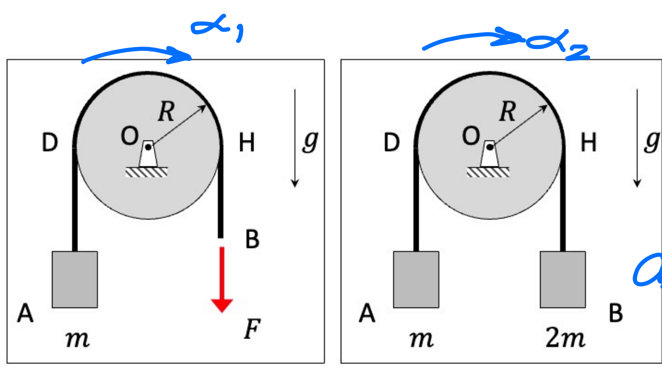
c) $\boxed{N_A > N_B}$

d) More information is needed to answer

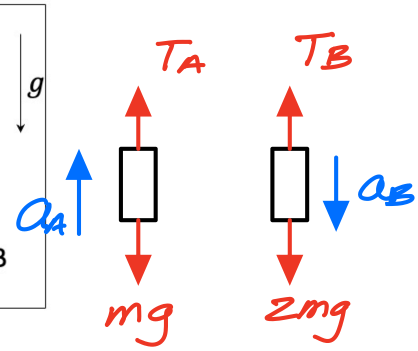
$$\Sigma M_G = -N_A b + N_B b + f_B h = 0$$

$$\hookrightarrow N_A = N_B + \frac{h}{b} f_B > N_B$$

Part 4B (2 points)



System 1 System 2



Consider the two cable-drum systems shown. For these systems, the drum has a mass moment of inertia of I_O about the centered pin support at O, and a cable is pulled over the drum. The cable does not slip on the drum.

- *For System 1*, a block of mass m is attached to end A of the cable, with a force $F = 2mg$ acting at end B.
- *For System 2*, a block of mass m is attached to end A of the cable, with a second block of mass $2m$ being attached at end B.

Let $|\alpha_1|$ and $|\alpha_2|$ represent the magnitudes of the angular acceleration of the drums for Systems 1 and 2, respectively. Circle the response below that correctly describes the relative sizes of $|\alpha_1|$ and $|\alpha_2|$:

a) $\boxed{|\alpha_1| > |\alpha_2|}$

b) $|\alpha_1| = |\alpha_2|$

c) $|\alpha_1| < |\alpha_2|$

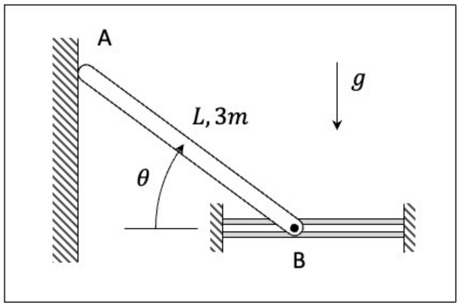
d) More information is needed to answer

System 2 has more rotational inertia $\Rightarrow |\alpha_1| > |\alpha_2|$

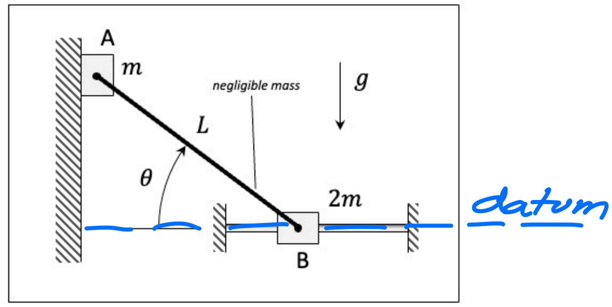
ME 274 – Final Exam **Name** *(print)* SOLUTION

May 5, 2026 *(Last)* *(First)*

Part 4C



System 1



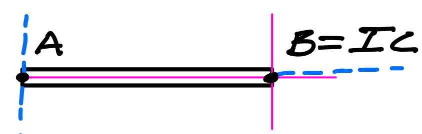
System 2

In System 1 shown, a thin homogeneous bar AB (having a mass of $3m$) is constrained to move along a vertical wall at end A, and is constrained to move along a horizontal guide at end B. In System 2, the thin bar is replaced by a bar AB having negligible mass to which particles A and B are attached, where A and B have masses of m and $2m$, respectively. All surfaces in each system are smooth. Both systems are released from rest with $\theta > 0$.

Part 4C.1 (2 points)
Let v_{A1} and v_{B1} represent the speeds of ends A and B of the bar, respectively, in System 1 when $\theta = 0$. Circle the response below that correctly describes the relative sizes of v_{A1} and v_{B1}:

a) $\boxed{v_{A1} > v_{B1}}$
b) $v_{A1} = v_{B1}$
c) $v_{A1} < v_{B1}$
d) More information is needed to answer

Since $B = IC \Rightarrow$
$V_{B1} = 0 \Rightarrow V_{A1} > V_{B1}$



$$mg\frac{L}{2}\sin\theta = \frac{1}{2}I_B\omega_1^2$$

$$\omega_1 = \sqrt{\frac{\cancel{m}gL\sin\theta}{\frac{1}{3}\cancel{m}L^{\cancel{2}}}} = \sqrt{3}\sqrt{\frac{g\sin\theta}{L}}$$

Part 4C.2 (2 points)
Let ω_1 and ω_2 represent the angular speeds of AB in Systems 1 and 2, respectively, when $\theta = 0$. Circle the response below that correctly describes the relative sizes of ω_1 and ω_2:

a) $\boxed{\omega_1 > \omega_2}$
b) $\omega_1 = \omega_2$
c) $\omega_1 < \omega_2$
d) More information is needed to answer

Since $B = IC \Rightarrow V_A = L\omega \Rightarrow$

$$\cancel{m}g\frac{\cancel{L}}{2}\sin\theta = \frac{1}{2}m(L\omega_2)^2 \Rightarrow$$

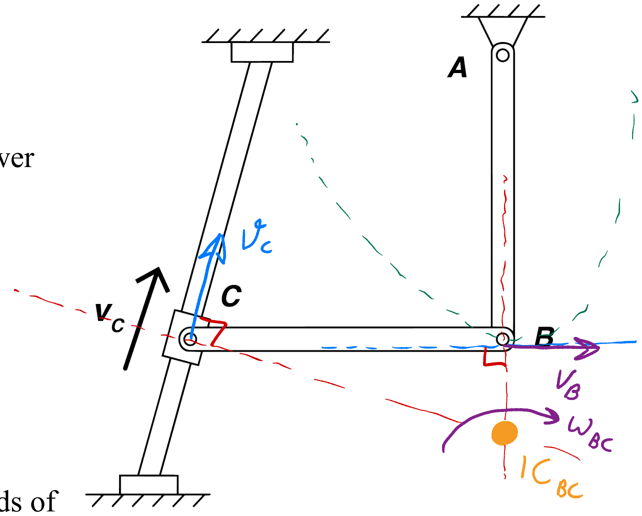
$$\omega_2 = \sqrt{\frac{g\sin\theta}{L}}$$

ME 274 – Final Exam **Name** *(print)*________________________________

May 5, 2026 *(Last)* *(First)*

Part 4D

Given: The collar has a velocity v_C in the direction shown.



Part 4D.1 (2 points)

Find: With the information given above, choose the best answer choice that describes the rotation of link BC:

a) Link BC has zero angular speed.

b) Link BC is rotating counterclockwise.

c) Link BC is rotating clockwise.

d) Cannot be determined/not enough information.

Part 4D.2 (2 points)

Find: Choose the correct response below comparing the speeds of points B and C:

a) $v_B > v_C$

b) $v_B = v_C$

c) $v_B < v_C$

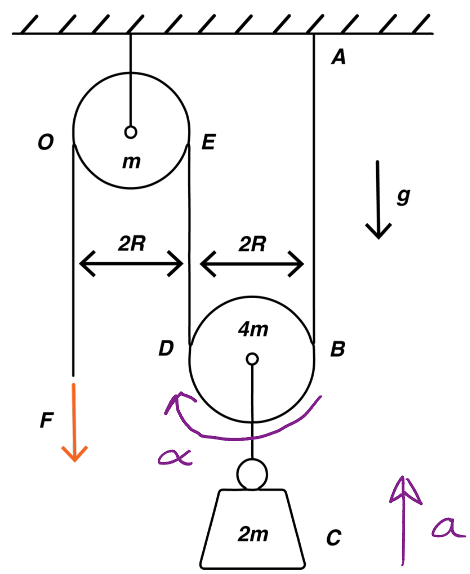
d) Cannot be determined/not enough information.

① $v_B = \omega_{BC}\, r_{IC/B}$ ②

$v_C = \omega_{BC}\, r_{IC/C}$ $r_{IC/B} < r_{IC/C}$ ∴ (c)

Part 4E (2 points)

Given: A disk is supported by a cable/pulley system, as shown, where the cables do not slip on the pulleys. Let T_{AB} and T_{DE} represent the tension in sections AB and DE of the cable, respectively. On release, it is known that C is accelerating upward.



Find: Choose the correct expression below about the relationship between T_{AB} and T_{DE}.

a) $T_{AB} > T_{DE}$

b) $T_{AB} = T_{DE}$

c) $T_{AB} < T_{DE}$

d) None of the above.

① we know C is accel. upward

② ∴ α is CW

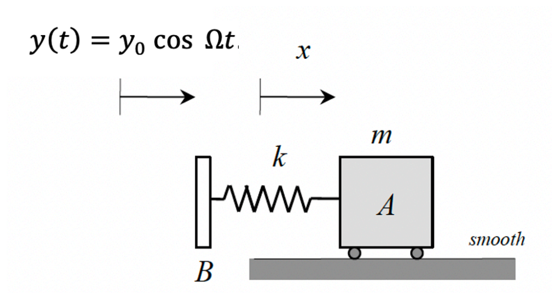
③ ∴ $T_{DE} > T_{AB}$

ME 274 – Final Exam **Name** *(print)*__________________________________

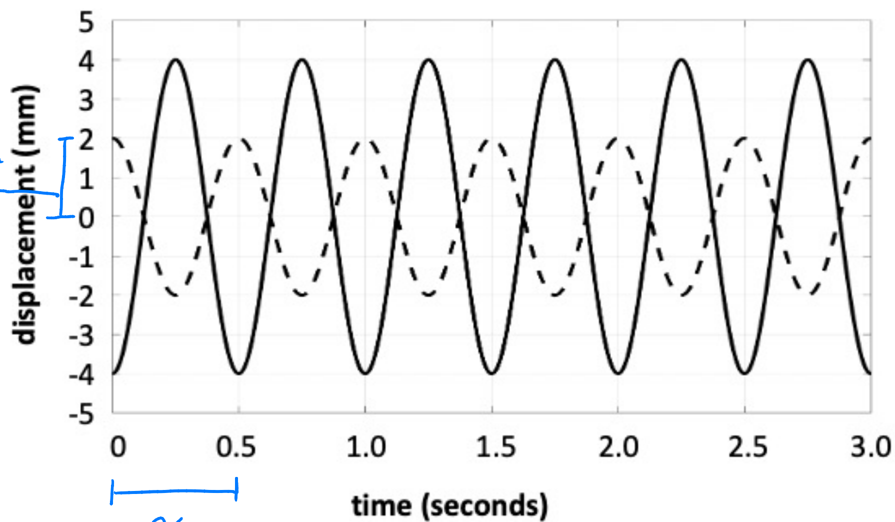
May 5, 2026 *(Last)* *(First)*

Part 4F (6 points)

Given: The undamped, single-degree-of-freedom system shown below is made up of block A (of mass m) and a spring of stiffness k. The spring is connected between A and base B, with B given a prescribed displacement of $y(t) = y_0 \cos \Omega t$. Let $x_P(t)$ represent the particular solution of the EOM for this system. Time histories for $x_P(t)$ (SOLID line) and $y(t)$ (DASHED line) are shown below.



Amplitude = 2mm



$\tau = 0.5s$

$y(t)$ = excitation = - - - -

$x_p(t)$ = part. soln. = ———

Find: From the figure above:

The excitation amplitude y_0 is equal to:

a) 1 mm
b) 2 mm
c) 4 mm
d) None of the above.

The excitation frequency Ω is equal to:

a) 2π rad/s
b) 3π rad/s
c) 4π rad/s
d) None of the above.

excitation freq

$\Omega = \frac{2\pi}{\tau} = \frac{2\pi}{0.5} = 4\pi$ rad/s

period

The natural frequency ω_n of the system is:

a) *greater than* the excitation frequency, Ω.
b) *less than* the excitation frequency, Ω.
c) *equal to* the excitation frequency, Ω.
d) More information is needed in order to answer this.

① Look @ plot, compare $x_p(t)$ & $y(t)$

② Notice they out of phase

③ $\therefore \Omega > \omega_n \rightarrow$ (b)