

Not Filled

ME 274 Lecture 42

Course Overview

Eugenio “Henny” Frias-Miranda

05/01/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. Office hours are on ME2008. Located on second floor of renovated side of ME
 - **I'll be holding OHs on Wed/Fri of finals week**
 - **Ill be holding a special office hours Sunday, May 3rd 1-3PM in ME²²⁰⁶~~2XXX~~ (old OHs location)**
2. Quizzes, there will be a total of 9 quizzes this semester:
 - Extra credit quizzes 3
 - Help you make up an unexcused attendance (ie. you forgot to email me before the lecture you missed)
 - **EC Quiz 3: H.6.M. H.6.N (Due Friday the 1st of May)**
 - Quiz 08 – Find a video in the following link below. In your own handwriting, talk about why you enjoyed the video and its relevance to the ME274 coursework. 3-5 sentences.
 - <https://www.purdue.edu/freeform/me274/course-material/visualizing-mechanics/>
 - **Due Friday, May 1st**
 - Remote/online quiz. Can be done on white sheet of paper. Just make sure you complete it on your own.
 - Quiz 09 - Thank you for filling out QR codes at end of lecture. I will be making a simple multiple-choice quiz asking:
 - “Did you fill out at least one post-lecture feedback survey? Y/N”.
 - **Due Friday, May 1st**
 - Remote/online quiz. No handout, just make sure you fill it out.
3. **Semester Course Evals (make sure to fill out!).** Submit On Gradescope.
4. Final Exam details: <https://www.purdue.edu/freeform/me274/exams-spring-2026/>
 - Seating Assignments: <https://www.purdue.edu/freeform/me274/wp-content/uploads/sites/15/2026/04/charter.pdf>

Kinematics

$$\begin{aligned}\vec{v} &= \dot{x}\hat{i} + \dot{y}\hat{j} \\ &= v\hat{e}_t \\ &= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\end{aligned}$$

$$\begin{aligned}\vec{a} &= \ddot{x}\hat{i} + \ddot{y}\hat{j} \\ &= \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n \\ &= (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta\end{aligned}$$

$$\begin{aligned}\vec{v}_B &= \vec{v}_A + \vec{v}_{B/A} \\ &= \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A} \\ &= \vec{v}_A + (\vec{v}_{B/A})_{rel} + \vec{\omega} \times \vec{r}_{B/A}\end{aligned}$$

$$\begin{aligned}\vec{a}_B &= \vec{a}_A + \vec{a}_{B/A} \\ &= \vec{a}_A + \vec{a} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A} \\ &= \vec{a}_A + (\vec{a}_{B/A})_{rel} + \vec{a} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} \\ &\quad + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})\end{aligned}$$

$$\frac{dv}{dt} = v \frac{dv}{ds}$$

Kinetics

$$\sum \vec{F} = m\vec{a}_G; \quad \sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$$

$$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$$

$$V_{gr} = mgh$$

$$V_{sp} = \frac{1}{2} k \Delta^2$$

$$T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2 + m \vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$$

$$U_{1 \rightarrow 2}^{(nc)} = \int_1^2 (\sum \vec{F} \cdot \hat{e}_t) ds = \int_1^2 (F_x dx + F_y dy)$$

Kinetics (continued)

$$e = - \left(\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}} \right)$$

$$\int_1^2 \sum \vec{F}_{ext} dt = \left(\sum_i m_i \vec{v}_i \right)_2 - \left(\sum_i m_i \vec{v}_i \right)_1$$

$$\int_1^2 \sum \vec{M}_A dt = \left(\sum_i (\vec{H}_A)_i \right)_2 - \left(\sum_i (\vec{H}_A)_i \right)_1$$

$$\begin{aligned}(\vec{H}_A)_i &= \vec{r}_{i/A} \times (m_i \vec{v}_i) \\ &= I_A \vec{\omega}\end{aligned}$$

Vibrations

$$M\ddot{x} + C\dot{x} + Kx = F(t) = F_0 \sin \omega t \quad \text{OR} \quad F_0 \cos \omega t$$

$$\ddot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x = \frac{1}{M} F(t)$$

$$x_c(t) = e^{-\zeta \omega_n t} (C \cos \omega_d t + S \sin \omega_d t); \quad 0 \leq \zeta < 1$$

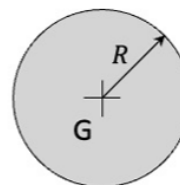
$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$x_p(t) = A \sin \omega t + B \cos \omega t$$

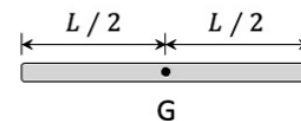
Other

$$I_A = I_G + m d^2$$

$$I_A = m k_A^2$$

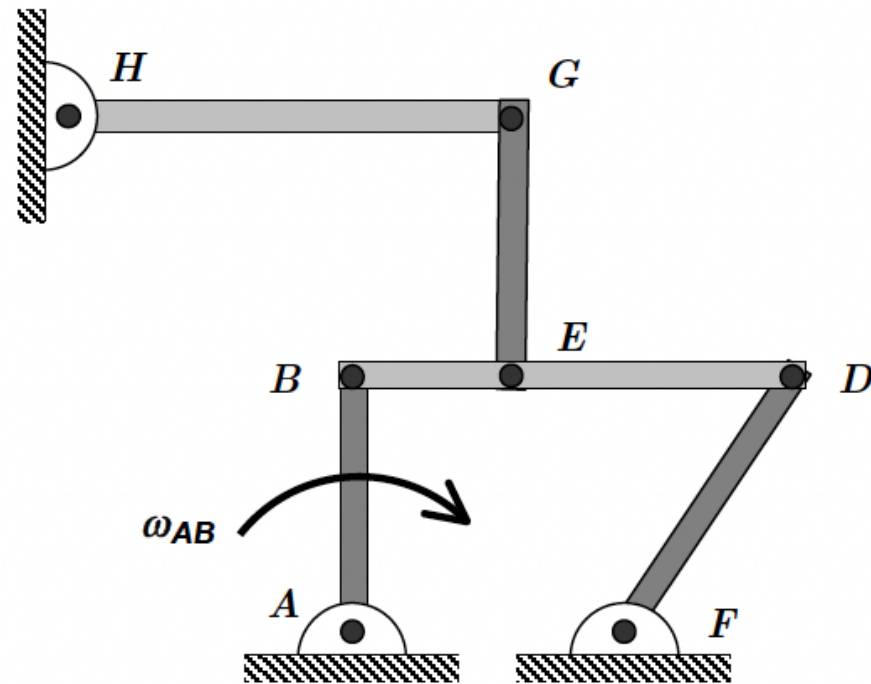


$$I_G = \frac{1}{2} m R^2$$



$$I_G = \frac{1}{12} m L^2$$

Link AB is rotating in the clockwise direction. What is the sense of rotation for the other links of this mechanism?



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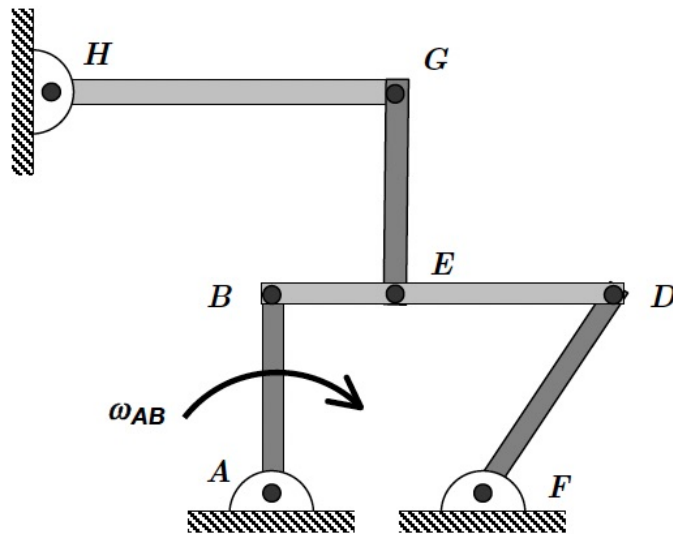
Link AB is rotating in the clockwise direction. What is the sense of rotation for the other links of this mechanism?

BD $\Rightarrow$

DF $\Rightarrow$

GE $\Rightarrow$

GH $\Rightarrow$



..... = Path

..... = Construction Lines

5. From this, we can find:

- magnitude of the angular velocity of the body, ω , from:

$$\omega = \frac{|\vec{v}_A|}{|\vec{r}_{A/C}|}$$

where $|\vec{r}_{A/C}|$ is the distance from C to A. Recall from above that we have assumed that we know the speed of point A.

- the direction of the angular velocity from examining the figure. Since C is the instant center of the body, the body is instantaneously rotating about C. Knowing the direction of $\vec{v}_A$, we can determine if the body is rotating clockwise or counterclockwise about C.

6. The velocity of ANY point D on the body is perpendicular to the line connecting C and D. The speed of D is found from:

$$|\vec{v}_D| = |\vec{\omega}| |\vec{r}_{D/C}|$$

The direction of is found from recalling again that the body is instantaneously rotating about point C.

Compare: v_B & v_D ? v_B v_D
 v_E & v_B ? v_B v_E
 v_E & v_G ? v_E v_G

Kinetics Summary Handout

Kinetics Table (page 352 of the lecture book)

Method	Body model	Fundamental equations
Newton-Euler (relating forces to accelerations)	particle	$\sum \vec{F} = m\vec{a}$
	rigid body (G = c.m. and A = any point on body)	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy (relating change in speed to change in position)	particle	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body (G = c.m. and A = any point on body)	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum (relating change in velocity to change in time)	particle	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body (G = c.m.)	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum (relating change in angular velocity to change in time)	particle (O = fixed point)	$\int_{t_1}^{t_2} \sum \vec{M}_O dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body (A = fixed point or c.m.)	$\int_{t_1}^{t_2} \sum \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A \vec{\omega}$

Notes on the *four-step* method:

1. FBD(s)

- Newton/Euler (N/E): typically individual FBDs
- Work/energy (W/E), linear impulse momentum (LIM) and angular impulse/momentum (AIM): typically make it "BIG" (include all moving bodies)

2. Kinetics (see suggestions in the table to the right as to which method(s) to use)

- Choose coordinate system(s) based on the "Given" and "Find", including consideration of any motion constraints in the system.
- N/E: point G in the Newton equation must be the center of mass for the body
- W/E: mechanical energy is conserved if $U_{1 \rightarrow 2}^{(nc)} = 0$ (no non-conservative work being done on your system)
- LIM: linear momentum in the x-direction is conserved if $\sum F_x = 0$ (no net force in the x-direction for your system)
- AIM: angular momentum in the z-direction is conserved if $\sum M_{Oz} = 0$ (no net moment in the z-direction about point O for your system). For particles, O must be a fixed point. For rigid bodies, O can be either a fixed point or the center of mass.

3. Kinematics

- N/E: typically need *acceleration* kinematics
- W/E, LIM and AIM: typically need *velocity* kinematics

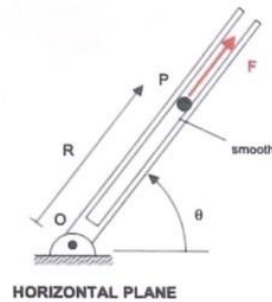
4. Solve

Rapid Fire Questions from Chapter 5

Choose method(s) of solution and draw FBD(s)
(See details of problems on Daily Schedule page)

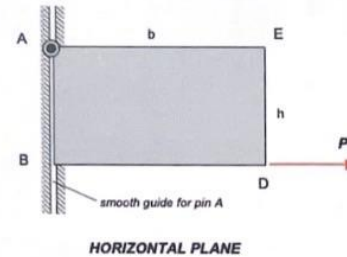
Su09 – E2 – P1

Determine change in angular velocity and radial velocity.



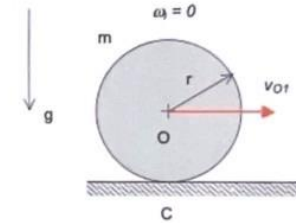
Su09 – E2 – P2

Determine angular acceleration of plate on release.



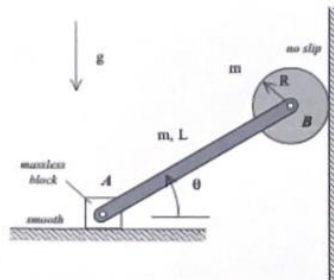
Su09 – F – P4

Time when slipping ceases.



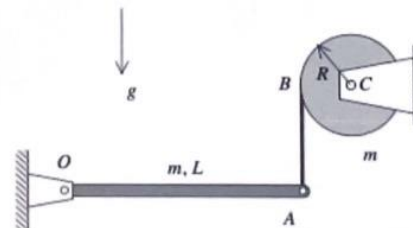
Su10 – E2 – P1

Angular velocity of bar when $\theta = 0$.



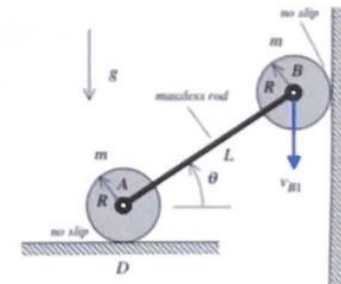
Su10 – E2 – P3

Determine angular acceleration of disk on release.



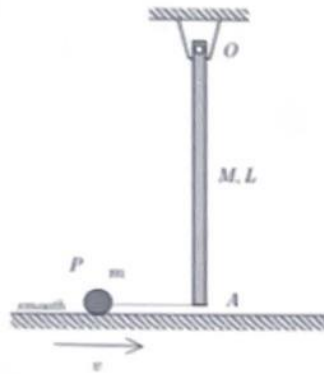
Su10 – F – P1

Angular velocity of disk B when $\theta = 0$.

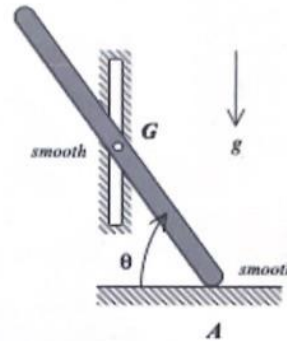


Su10 – F – P2

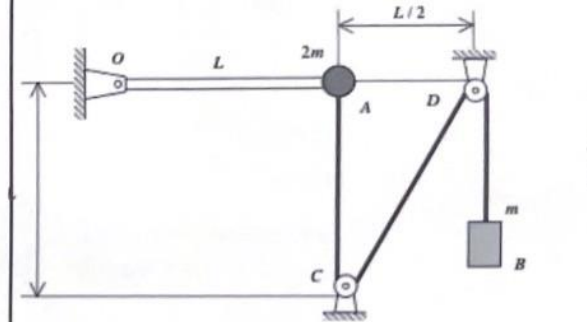
Determine angular speed after impact.

**Su11 – E2 – P1**

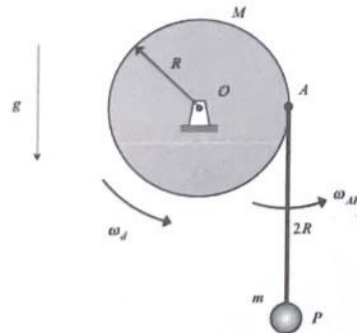
Determine angular acceleration on release.

**Su11 – E2 – P3**

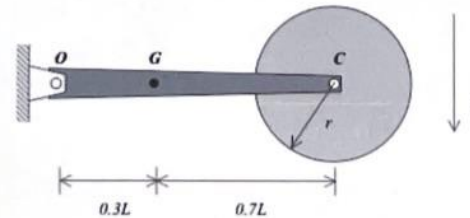
Angular velocity of bar after rotating 90°.

**Su012– E2 – P1**

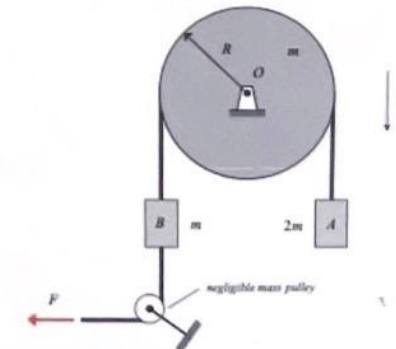
Determine angular acceleration of disk on release.

**Su12– F – P2**

Angular speed of bar after rotating 90°.

**Su15 – F – P1**

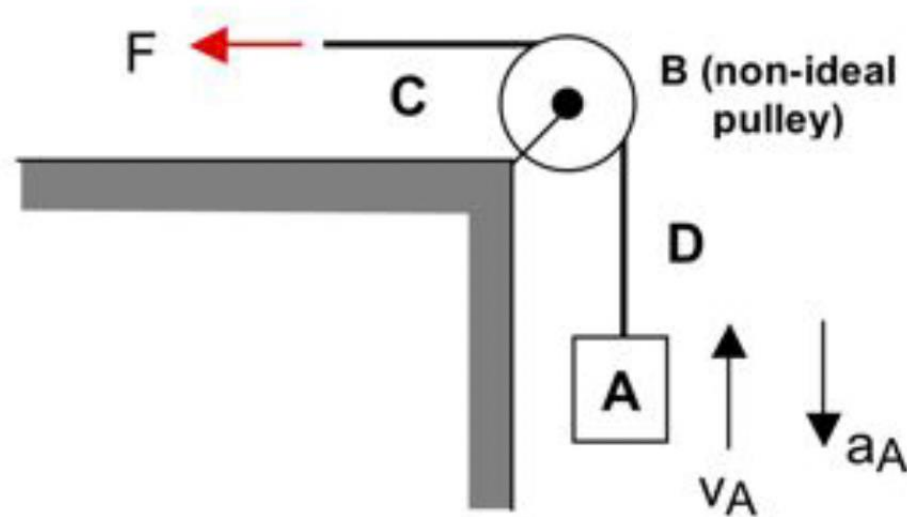
Determine angular acceleration of disk.



Question C5.5

The force F is lifting block A; however, the speed of A is decreasing. The pulley is non-ideal (it has a non-zero mass moment of inertia). Circle the correct description below.

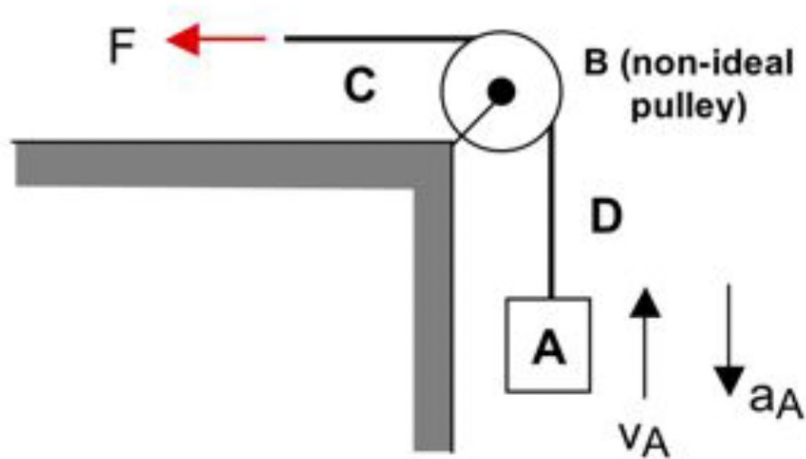
- (a) The tension in section C of the cable is larger than the tension in section D.
- (b) The tension in section C of the cable is smaller than the tension in section D.
- (c) The tension in section C of the cable is the same as the tension in section D.



Question C5.5 p.358

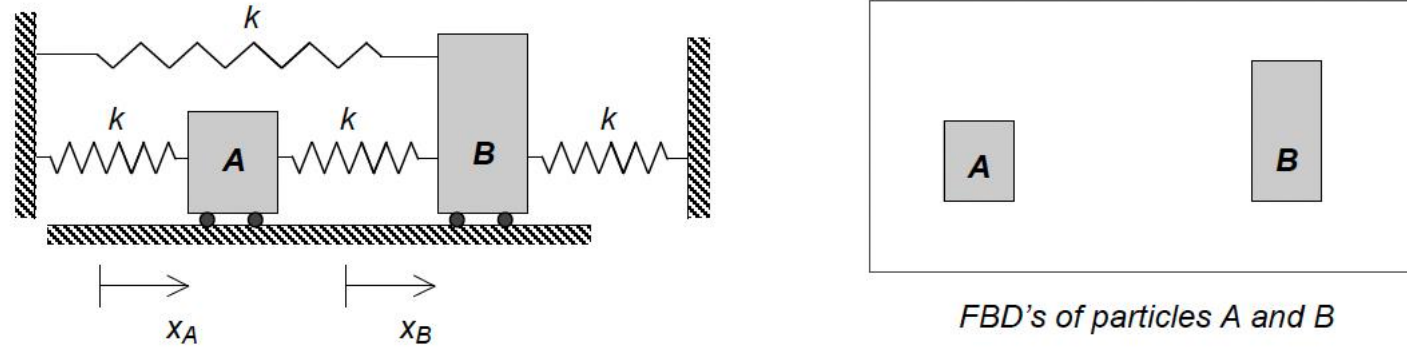
The force F is lifting block A; however, the speed of A is decreasing. The pulley is non-ideal (it has a non-zero mass moment of inertia). Circle the correct description below.

- (a) The tension in section C of the cable is larger than the tension in section D.
- (b) The tension in section C of the cable is smaller than the tension in section D.
- (c) The tension in section C of the cable is the same as the tension in section D.



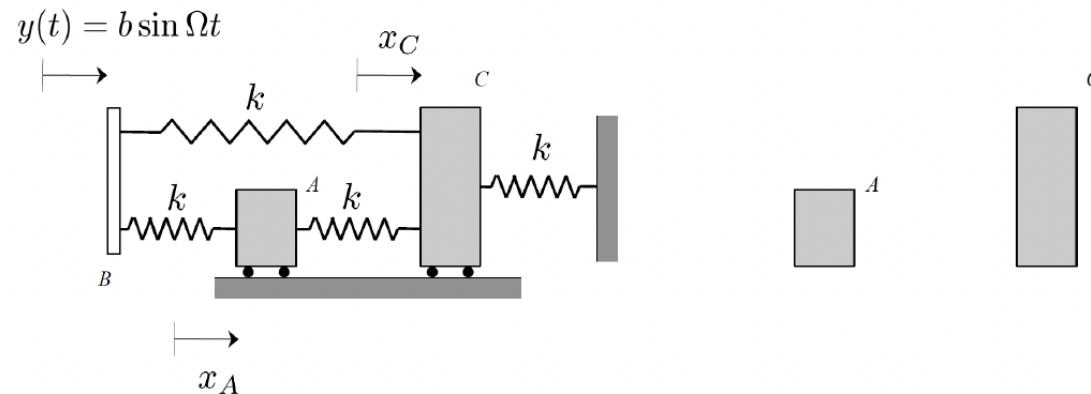
Question C6.1

The positions of particles A and B are described by the coordinates x_A and x_B shown below. All springs are unstretched when $x_A = x_B = 0$. In the free body diagrams shown below, draw and label (in terms of k , x_A and x_B) the spring force vectors on particles A and B.



Question C6.6

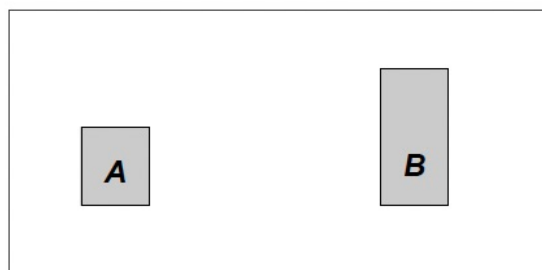
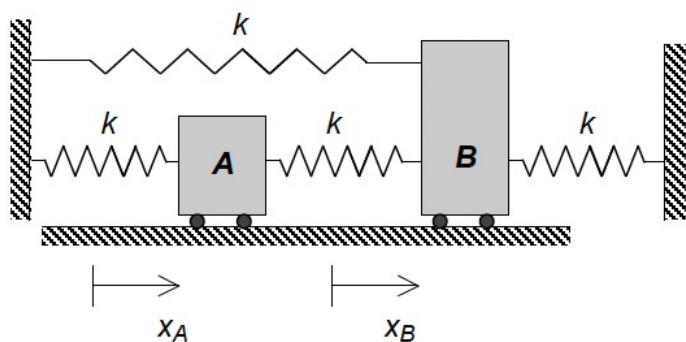
The positions of particles A and C are described by the coordinates x_A and x_C shown below. The base B on the left side is given prescribed motion of $y(t) = b \sin \Omega t$. All springs are unstretched when $x_A = x_C = y = 0$. In the free body diagrams shown below, draw and label [in terms of k , $y(t)$, x_A and x_C] the spring force vectors on particles A and C.



Question C6.1

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The positions of particles A and B are described by the coordinates x_A and x_B shown below. All springs are unstretched when $x_A = x_B = 0$. In the free body diagrams shown below, draw and label (in terms of k , x_A and x_B) the spring force vectors on particles A and B.

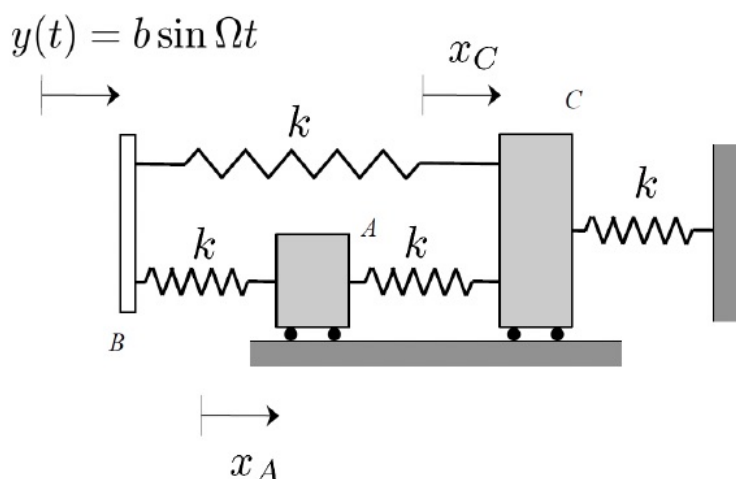


FBD's of particles A and B

Question C6.6

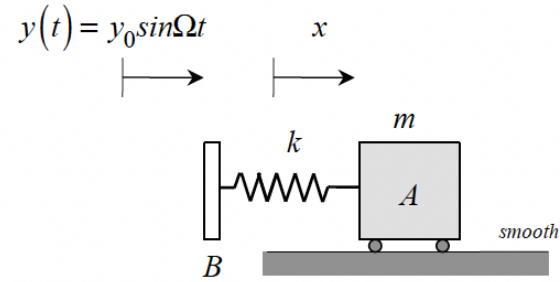
p. 439

The positions of particles A and C are described by the coordinates x_A and x_C shown below. The base B on the left side is given prescribed motion of $y(t) = b \sin \Omega t$. All springs are unstretched when $x_A = x_C = y = 0$. In the free body diagrams shown below, draw and label [in terms of k , $y(t)$, x_A and x_C] the spring force vectors on particles A and C.

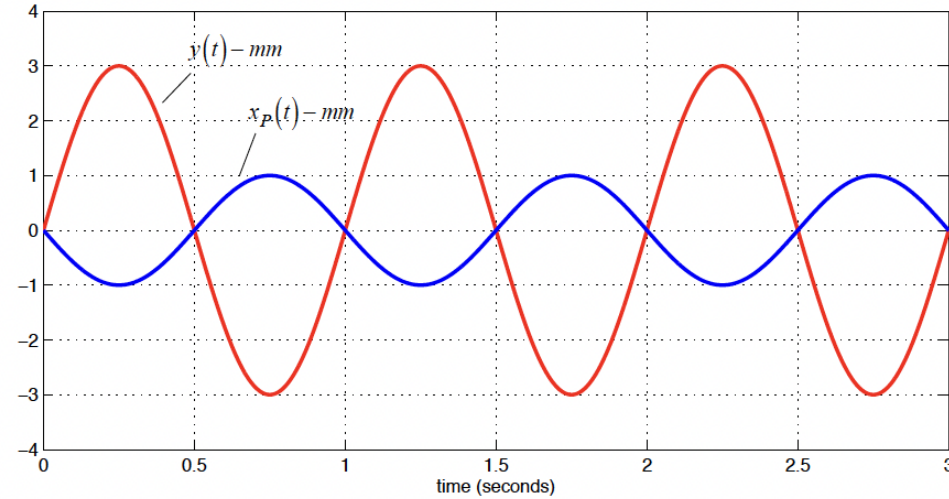


Question C6.13

The undamped, single-degree-of-freedom system shown below is made up of block A (of mass m) and a spring of stiffness k . The spring is connected between A and base B, with B given a prescribed displacement of $y(t) = y_0 \sin \Omega t$.



Let $x_P(t)$ represent the particular solution of the EOM for this system. Time histories for $x_P(t)$ and $y(t)$ are shown below.



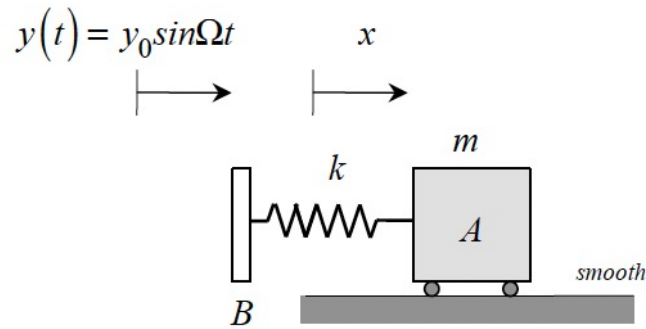
From the plot, provide estimates for:

- The excitation amplitude y_0 .
- The excitation frequency Ω .
- The natural frequency ω_n of the system.

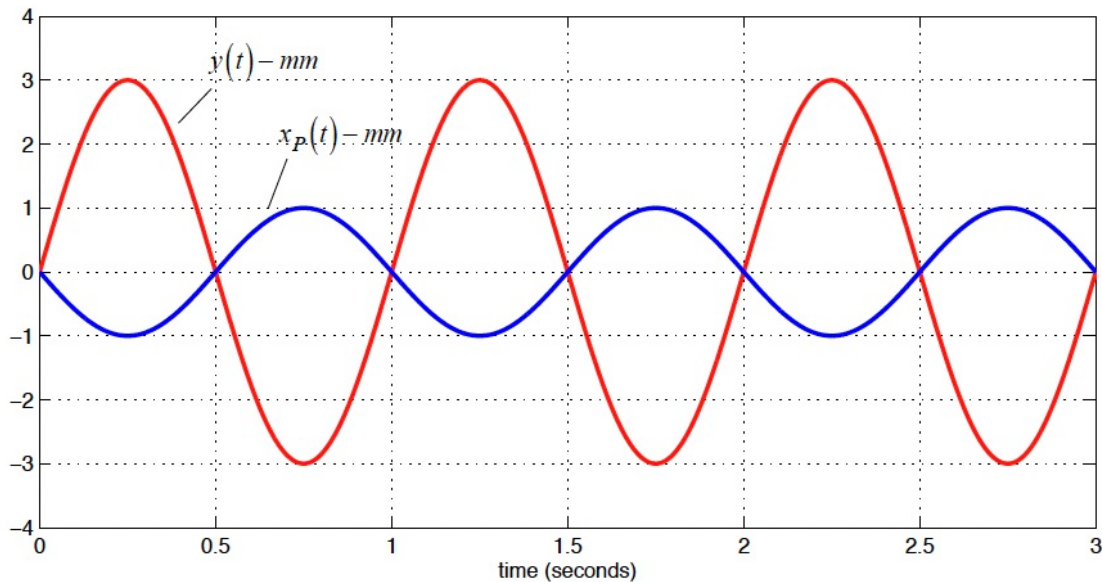
Question C6.13

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The undamped, single-degree-of-freedom system shown below is made up of block A (of mass m) and a spring of stiffness k . The spring is connected between A and base B, with B given a prescribed displacement of $y(t) = y_0 \sin \Omega t$.



Let $x_P(t)$ represent the particular solution of the EOM for this system. Time histories for $x_P(t)$ and $y(t)$ are shown below.



From the plot, provide estimates for:

- The excitation amplitude y_0 .
- The excitation frequency Ω .
- The natural frequency ω_n of the system.

1. Thank you so much for the wonderful semester!
2. Good luck with finals!!
3. It will be fine 😊
4. Class picture in front of classroom (if we have time)

Lec 42 Short
Feedback Form:

