ME 274: Basic Mechanics II

Lecture 41: Vibrations – review

PURDUE UNIVERSITY | School of Mechanical Engineering

Announcements:

- Fill out the course Eval. and submit proof on Gradescope for bonus points – deadline 5/3 (Sunday)
- Submit any questions you have for Friday's review class over Gradescope
- What format do we want our review class? → high level overview of concepts
  → solving example problems
  → conceptual problems

Final Exam Info:

- Tuesday 5/5, 7-9 pm in UC 114 – seating chart will be posted
- Inform me of conflicts ASAP
- Review session 5/3 @ 7pm over Zoom – see Exams tab on course blog for the link
- Content: 3 long-form problems on material after exam 2 (lectures 27-40), multiple choice covering the entire course
  - Exam is intended to test your ability to apply concepts, not memorize → any eqn you will need will be given on the equsht.

The final section of the book is on damped forced vibrations,

I encourage you to look through on your own

Damped forced response will not be on the exam.

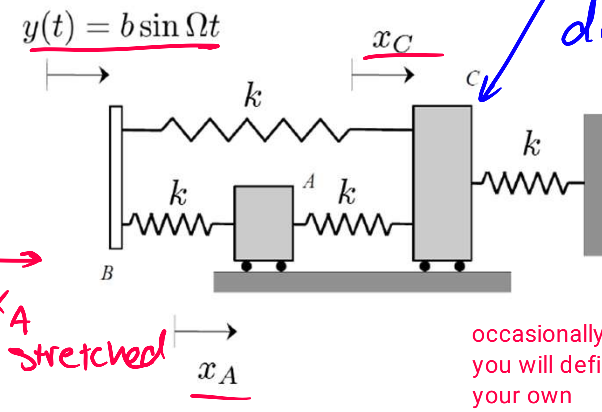
Note → damped free response CAN be tested how is displacement measured? from eq? from x=0? θ?

How to approach solving a vibrations problem – 4 step plan

look @ relative displacem

1. Draw your free body diagram(s) → FBD for each body
    - For vibes problems, generally keep your system small
    - Identify what coordinate(s) are being used to describe your system
    - * For spring forces: ← common pt. of confusion
        - Make an assumption about which end is being displaced further
        - Determine if your spring is in tension or compression
        - Draw your forces accordingly

$x_c > x_A$ spring stretched



occasionally you will define your own coord.

2. Kinetics $\Sigma F = ma$, $\Sigma M = I\alpha$ → useful to have kinetics diagram
    - Write the Newton-Euler equations relevant to your FBD(s) → in rotating systems, be careful w/ I
3. Kinematics
    - Take note of which coordinate is asked for in your problem statement
    - Relate your coordinates using rigid body equations, rolling w/o slip conditions, etc.
    - Calculate derivatives (ex. $\dot{x} = r\dot{\theta}$) → all of our problems are SDOF → you should be able to write in terms of a single coord.
4. Solve – see next slide

once we have done all this, we can find our EOM

& start our vibration analysis

deciding if our system is free vibration, forced vibration

damped, undamped, base exitation

If your coefficients are not all the same sign, Go figure out where the problem is I → dont just switch the sign

Homogeneous Dif. Eq.
no external forcing

remember M, C, K have to be the same sign

How to approach solving a vibrations problem – Free Vibration Response

4.) Solve - Use the results of your kinetic and kinematic analysis to derive the EOM: $\boldsymbol{M\ddot{x} + C\dot{x} + Kx = 0}$

Standard form: $\ddot{x} + \frac{C}{M}\dot{x} + \frac{K}{M} = 0$, $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$

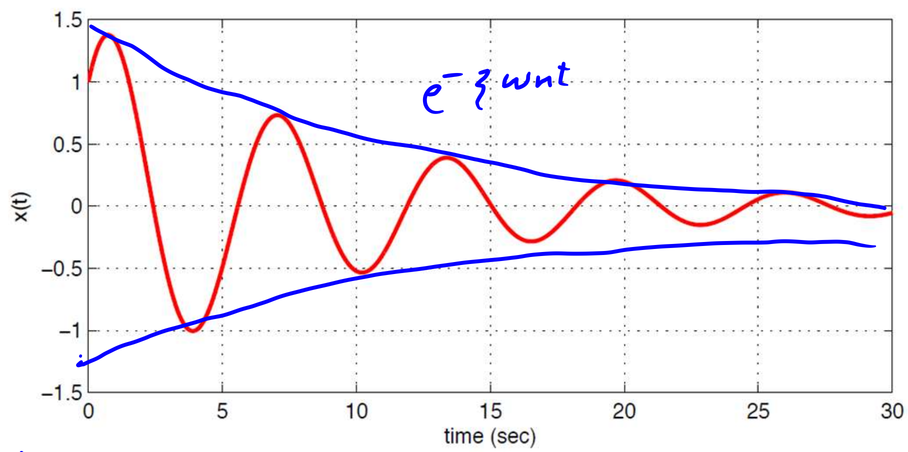
What are you asked to solve for? (Free vibration response)

- Natural frequency: $\omega_n = \sqrt{\frac{K}{M}}$ → how quickly our system wants to osci late when released
- Damping ratio: $\zeta = \frac{C}{2\sqrt{KM}}$ → tells us how our response will behave

  $\zeta = 0$ → oscilates forever $0 < \zeta < 1$ oscilates + decays
- Damped natural frequency: $\omega_d = \omega_n\sqrt{1-\zeta^2}$
- Period: undamped - $T = \frac{2\pi}{\omega_n}$, damped $T = \frac{2\pi}{\omega_d}$
- Complimentary solution:
  - Undamped system - $x_c(t) = C\cos\omega_n t + S\sin\omega_n t$
  - Underdamped system – $x_c(t) = e^{-\zeta\omega_n t}[C\cos\omega_d t + S\sin\omega_d\, t]$

  C & S come from our initial conditions

$\zeta = 1$ → critically damped → goes to eq asap
$\zeta > 1$ → overdamped → goes to eq slowly



Wn → property of the system → depends on mass, stiffness

ω → property of the input

How to approach solving a vibrations problem – Forced Response

→ externally & continuously driving motion applied force, applied moment base displacemen

4.) **Solve** - Use the results of your kinetic and kinematic analysis to derive the EOM: $\boldsymbol{M\ddot{x} + C\dot{x} + Kx = F(t)}$

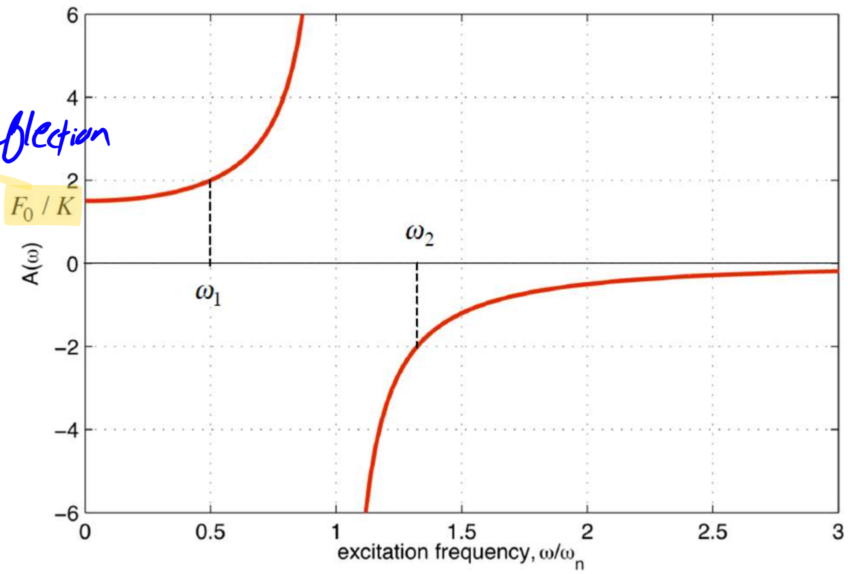
→ we only have a sin/cos Forcing ← assume $F = F_0 \sin \omega t$

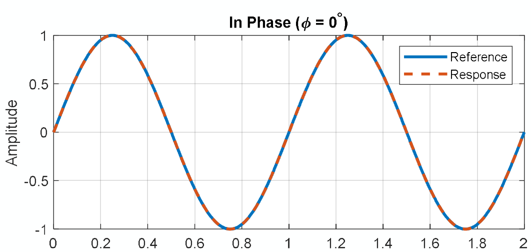
ωt ← different from ω_n

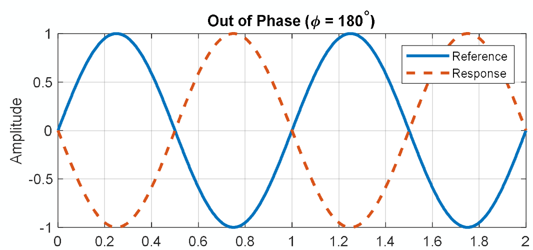
What are you asked to solve for? (Forced Response)

- Anything that was asked for with regards to the Free Response stays the same
- Particular solution: $x_p(t) = A\sin\omega t + B\cos\omega t$
- Amplitude: $|A| = \left|\dfrac{\frac{F_0}{K}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right|$
- In-phase or out-of-phase: $\dfrac{\omega}{\omega_n} > or < 1?$

Static deflection







ex. limits on amplitude

ex. max amplitude

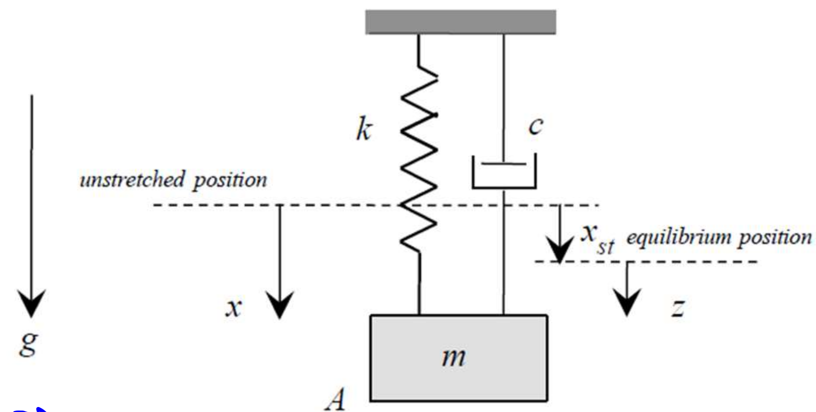
Question C6.7 testing: coord choice & equilibrium

Consider the spring-mass-dashpot system shown below. Let x represent the position of block A as measured from its position when the spring is unstretched. Let z represent the position of block A as measured from its position when the system is in static equilibrium; that is, $x = x_{st} + z$.

(a) Derive the equation of motion (EOM) of the system in terms of the coordinate x.

(b) Derive the EOM of the system in terms of the coordinate z.

(c) Compare the EOMs from (a) and (b). How do they differ?

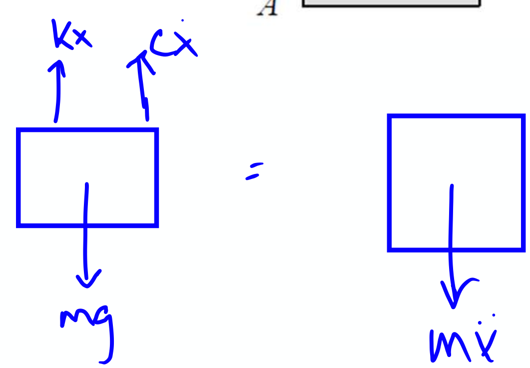


2) Kinetics

$$\sum F_x = m\ddot{x} = mg - kx - c\dot{x}$$

EOM: $m\ddot{x} + c\dot{x} + kx = mg$

FBD



in terms of z:

at st. eq. → $\ddot{x} = 0$, $\dot{x} = 0$ → $kx_{st} = mg$, $x_{st} = \underset{\text{const}}{\frac{mg}{k}}$

$$z = x - x_{st}, \quad \dot{z} = \dot{x}, \quad \ddot{z} = \ddot{x}$$

EOM: $m\ddot{z} + c\dot{z} + k(z + x_{st}) = mg$

$$m\ddot{z} + c\dot{z} + kz + \cancel{k}\frac{mg}{\cancel{k}} = \cancel{mg}$$

$$m\ddot{z} + c\dot{z} + kz = 0$$

Question C6.7 testing: coord choice & equilibrium

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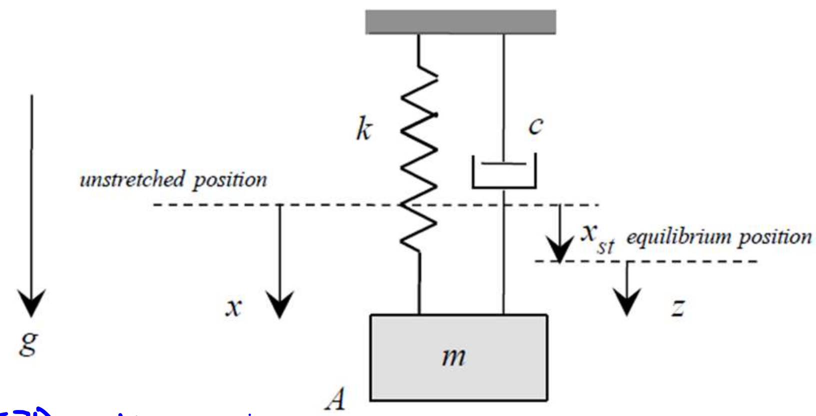
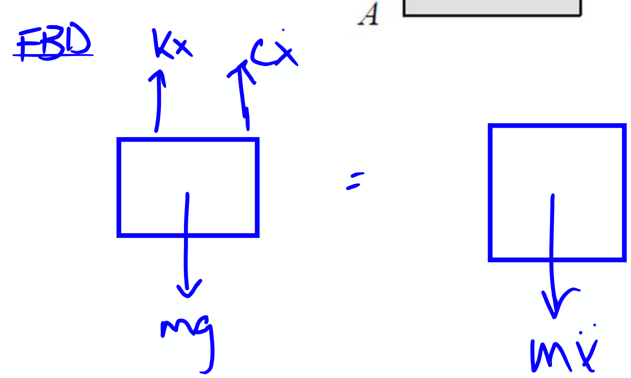


2) kinetics

$$\Sigma F_x = m\ddot{x} = mg - kx - c\dot{x}$$

EOM: $m\ddot{x} + c\dot{x} + kx = mg$



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$$z = x - x_{st}, \quad \dot{z} = \dot{x}, \quad \ddot{z} = \ddot{x}$$

EOM: $m\ddot{z} + c\dot{z} + k(z + x_{st}) = mg$

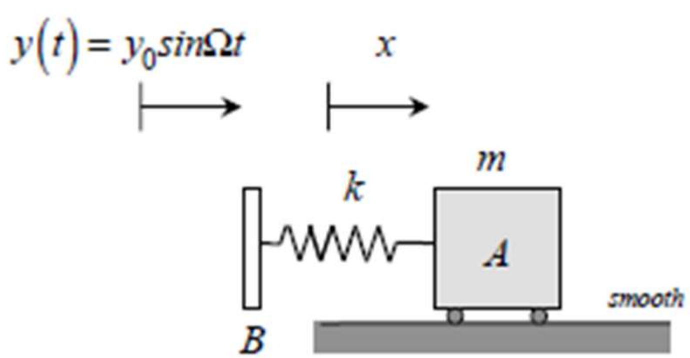
$$m\ddot{z} + c\dot{z} + kz + \cancel{k}\frac{mg}{\cancel{k}} = \cancel{mg}$$

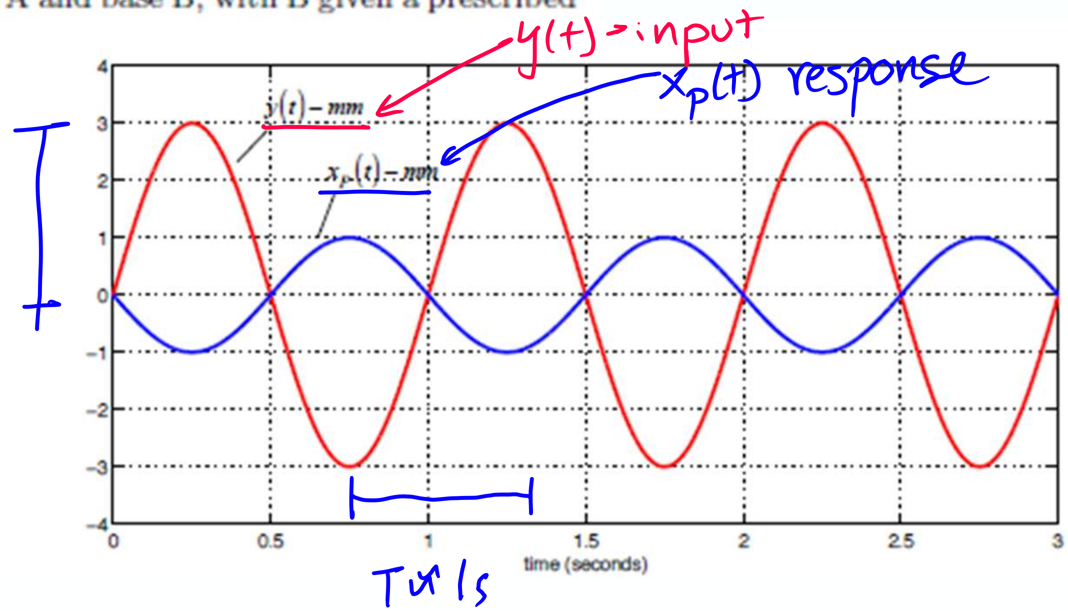
$$m\ddot{z} + c\dot{z} + kz = 0$$

↔ simplifies motion & lets us focus on displacement around eq.

Question C6.13

The undamped, single-degree-of-freedom system shown below is made up of block A (of mass m) and a spring of stiffness k. The spring is connected between A and base B, with B given a prescribed displacement of $y(t) = y_0 \sin \Omega t$.





From the plot, provide estimates for:

(a) The excitation amplitude y_0. → amplitude of y(t) curve 3mm

(b) The excitation frequency Ω. $T = 1s \rightarrow \frac{2\pi}{T} = \Omega \rightarrow \Omega = 2\pi$ rad/s

(c) The natural frequency ω_n of the system. → out of phase $\omega/\omega_n > 1$

$$A = \frac{F_0/K}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \frac{y_0}{1-\left(\frac{\Omega}{\omega_n}\right)^2}$$

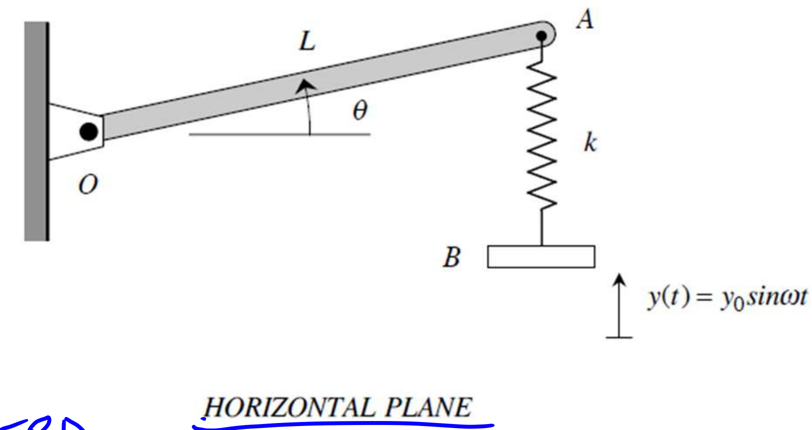
$$1 = \frac{3}{1-\left(\frac{\Omega}{\omega_n}\right)^2} \qquad \left(\frac{\Omega}{\omega_n}\right)^2 = 4$$

$$\omega_n = \frac{\Omega}{2} = \pi \text{ rad/s}$$

Example 6.C.11

Given: A thin homogeneous bar of length L and mass m is pinned to ground at its left end O. A spring of stiffness k is connected between end A of the bar and a moveable base B, as shown. The base B is given a prescribed motion of $y(t) = y_0 \sin \omega t$.

Find: Find the differential equation of motion for the system in terms of the rotation angle θ. Assume small oscillations.

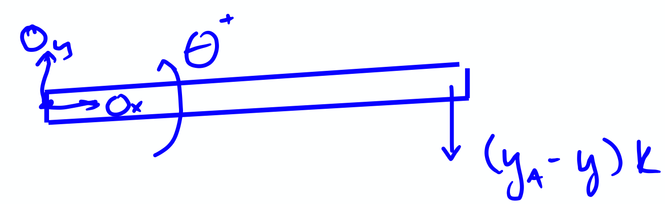


2) N-E $\quad \Sigma M_o = I_o \ddot{\theta} = -Lk(y_A - y)$

$I_o \rightarrow$ p.a.t. $\quad I_o = I_G + mr^2 = \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{3} mL^2$

EOM: $\quad \frac{1}{3} mL^2 \ddot{\theta} + kLy_A = kLy_0 \sin\omega t$

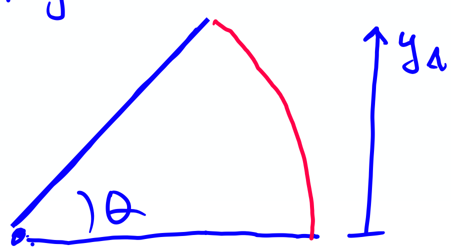
FBD



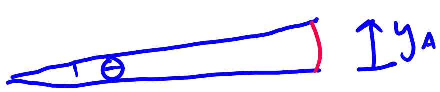
3) kinematics

"assume small oscillations" *

big oscillations



small oscillations



$y_A = L\sin\theta \leftarrow$ for small ∠s, $\sin\theta \approx \theta \Rightarrow y_A = L\theta$

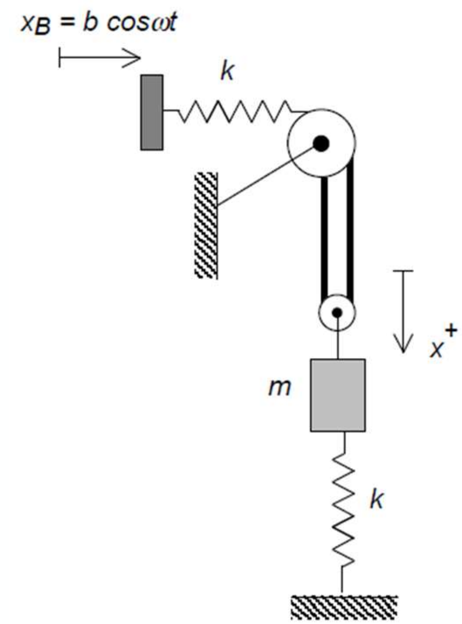
EOM: $\frac{1}{3} m L^2 \ddot{\theta} + k L^2 \theta = k y_o L \sin \omega t$

Example 6.C.10

Given: The system shown below.

Find: Determine:

(a) The differential equation of motion for the system shown; and
(b) The resonant frequency of the system.



Kinematics

$$\Sigma F_x = -kx - 2T + mg = m\ddot{x}$$

$$T = k(2x - x_B) \leftarrow \text{ask}$$

$$\Rightarrow -kx - 2k(2x - x_B) + mg = m\ddot{x}$$

$$m\ddot{x} + 5kx = 2kx_B + mg$$

$$= 2kb\cos wt + mg$$

$$m\ddot{x} + 5kx = 0$$

$$\ddot{x} + \underbrace{\frac{5k}{m}}_{\hookrightarrow\, \omega_n^2} x = 0 \qquad \omega_n = \sqrt{\frac{5k}{m}}$$

1) FBD