Filled

ME 274 Lecture 42

Course Overview

Eugenio “Henny” Frias-Miranda

05/01/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. Office hours are on ME2008. Located on second floor of renovated side of ME

- **I'll be holding OHs on Wed/Fri of finals week**
- **Ill be holding a special office hours Sunday, May 3rd 1-3PM in ME~~2XXX~~ 2206 (old OHs location)**

2. Quizzes, there will be a total of 9 quizzes this semester:
    - Extra credit quizzes 3
        - Help you make up an unexcused attendance (ie. you forgot to email me before the lecture you missed)
        - **EC Quiz 3: H.6.M. H.6.N (Due Friday the 1st of May)**
    - Quiz 08 – Find a video in the following link below. In your own handwriting, talk about why you enjoyed the video and its relevance to the ME274 coursework. 3-5 sentences.
        - https://www.purdue.edu/freeform/me274/course-material/visualizing-mechanics/
        - **Due Friday, May 1st**
        - Remote/online quiz. Can be done on white sheet of paper. Just make sure you complete it on your own.
    - Quiz 09 - Thank you for filling out QR codes at end of lecture. I will be making a simple multiple-choice quiz asking:
        - "Did you fill out at least one post-lecture feedback survey? Y/N".
        - **Due Friday, May 1st**
        - Remote/online quiz. No handout, just make sure you fill it out.

3. **Semester Course Evals (make sure to fill out!).** Submit On Gradescope.

4. Final Exam details: https://www.purdue.edu/freeform/me274/exams-spring-2026/

- Seating Assignments: https://www.purdue.edu/freeform/me274/wp-content/uploads/sites/15/2026/04/charter.pdf

Equation Sheet

Kinematics

$$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} = v\hat{e}_t = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + \vec{\omega}\times\vec{r}_{B/A} = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega}\times\vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + \vec{\alpha}\times\vec{r}_{B/A} - \omega^2\vec{r}_{B/A} = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha}\times\vec{r}_{B/A} + 2\vec{\omega}\times\left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{B/A}\right)$$

$$\frac{dv}{dt} = v\frac{dv}{ds}$$

Kinetics

$$\Sigma\vec{F} = m\vec{a}_G\ ;\quad \Sigma\vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A}\times\vec{a}_A$$

$$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$$

$$V_{gr} = mgh$$

$$V_{sp} = \frac{1}{2}k\Delta^2$$

$$T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A\cdot\left(\vec{\omega}\times\vec{r}_{G/A}\right)$$

$$U_{1\to 2}^{(nc)} = \int_1^2\left(\Sigma\vec{F}\cdot\hat{e}_t\right)ds = \int_1^2\left(F_x dx + F_y dy\right)$$

Kinetics (*continued*)

$$e = -\left(\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}\right)$$

$$\int_1^2 \Sigma\vec{F}_{ext}\,dt = \left(\sum_i m_i\vec{v}_i\right)_2 - \left(\sum_i m_i\vec{v}_i\right)_1$$

$$\int_1^2 \Sigma\vec{M}_A\,dt = \left(\sum_i \left(\vec{H}_A\right)_i\right)_2 - \left(\sum_i \left(\vec{H}_A\right)_i\right)_1$$

$$\left(\vec{H}_A\right)_i = \vec{r}_{i/A}\times\left(m_i\vec{v}_A\right) = I_A\vec{\omega}$$

Vibrations

$$M\ddot{x} + C\dot{x} + Kx = F(t) = F_0 sin\omega t\ \ \boldsymbol{OR}\ \ F_0 cos\omega t$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{1}{M}F(t)$$

$$x_c(t) = e^{-\zeta\omega t}\left(C\ cos\omega_d t + S\ sin\omega_d t\right)\ ;\quad 0 \le \zeta < 1$$

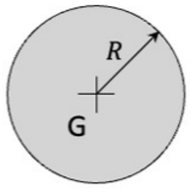
$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

$$x_p(t) = Asin\omega t + Bcos\omega t$$

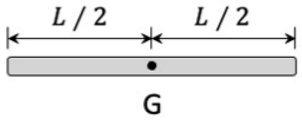
Other

$$I_A = I_G + md^2$$

$$I_A = mk_A^2$$

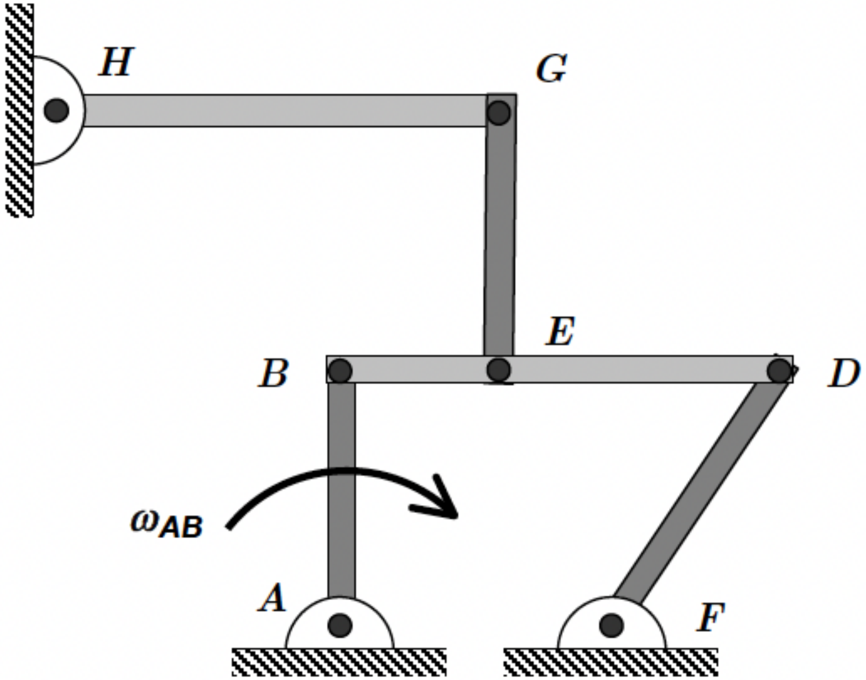


$$I_G = \frac{1}{2}mR^2$$



$$I_G = \frac{1}{12}mL^2$$

Link AB is rotating in the clockwise direction. What is the sense of rotation for the other links of this mechanism?



[pg. 108]

p. 108

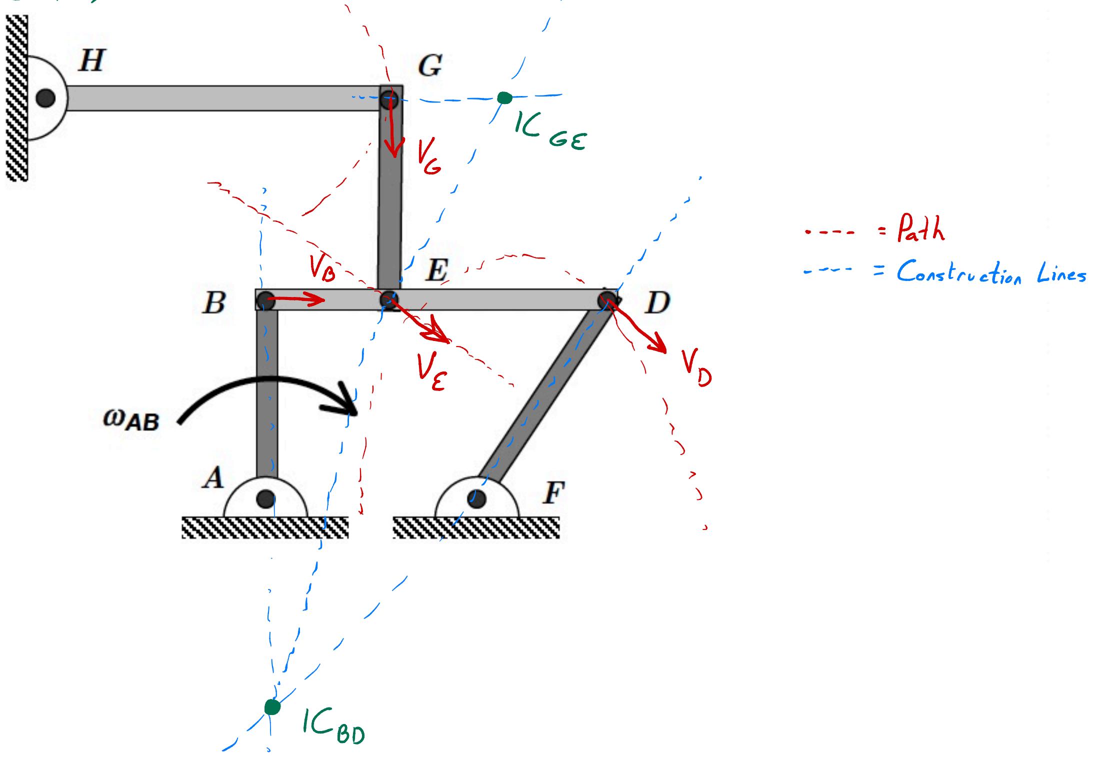
Link AB is rotating in the clockwise direction. What is the sense of rotation for the other links of this mechanism?

BD => CW

DF => CW

GE => CCW

GH => CW



For GE: Draw construction line from IC_{BD} to E to know the path of V_E

5. From this, we can find:

- magnitude of the angular velocity of the body, ω, from:

$$\omega = \frac{|\vec{v}_A|}{|\vec{r}_{A/C}|}$$

where $|\vec{r}_{A/C}|$ is the distance from C to A. Recall from above that we have assumed that we know the speed of point A.

- the direction of the angular velocity from examining the figure. Since C is the instant center of the body, the body is instantaneously rotating about C. Knowing the direction of $\vec{v}_A$, we can determine if the body is rotating clockwise or counterclockwise about C.

6. The velocity of ANY point D on the body is perpendicular to the line connecting C and D. The speed of D is found from:

$$|\vec{v}_D| = |\vec{\omega}|\ |\vec{r}_{D/C}|$$

The direction of is found from recalling again that the body is instantaneously rotating about point C.

Compare: V_B & V_D? $V_B < V_D$

V_E & V_B? $V_B < V_E$

V_E & V_G? $V_E > V_G$

Kinetics Summary Handout

Kinetics Table (page 352 of the lecture book)

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\, dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A\vec{\omega}$

Notes on the *four-step* method:

1. ***FBD(s)***
   - Newton/Euler (*N/E*): typically individual FBDs
   - Work/energy (*W/E*), linear impulse momentum (*LIM*) and angular impulse/momentum (*AIM*): typically make it "BIG" (include all moving bodies)
2. ***Kinetics*** (see suggestions in the table to the right as to which method(s) to use)
   - Choose coordinate system(s) based on the *"Given"* and *"Find"*, including consideration of any motion constraints in the system.
   - *N/E*: point G in the Newton equation must be the center of mass for the body
   - *W/E*: mechanical energy is conserved if $U_{1\to 2}^{(nc)} = 0$ (no non-conservative work being done on your system)
   - *LIM*: linear momentum in the *x*-direction is conserved if $\sum F_x = 0$ (no net force in the *x*-direction for your system)
   - *AIM*: angular momentum in the *z*-direction is conserved if $\sum M_{Oz} = 0$ (no net moment in the *z*-direction about point O for your system).For particles, O must be a fixed point. For rigid bodies, O can be either a fixed point or the center of mass.
3. ***Kinematics***
   - *N/E*: typically need *acceleration* kinematics
   - *W/E, LIM and AIM:* typically need *velocity* kinematics
4. ***Solve***

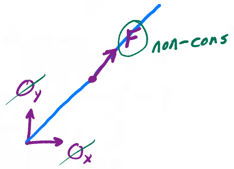
Rapid Fire Questions from Chapter 5

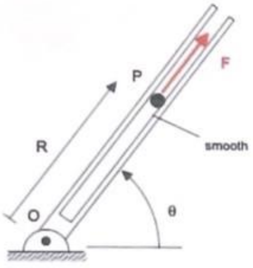
Choose method(s) of solution and draw FBD(s)
(See details of problems on Daily Schedule page)

Su09 – E2 - P1

Determine change in angular velocity and radial velocity. ω ? $V\hat{e}_r$?

BIG FBD:





HORIZONTAL PLANE

AIM : $\Sigma M_o = 0$

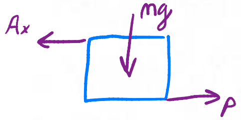
W/E : $\int(\vec{F}\cdot e_t)\,ds = U_{1-2}^{NC}$

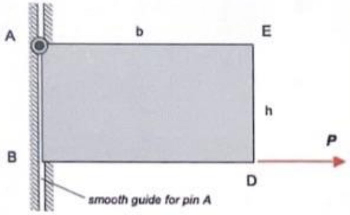
Su09 – E2 – P2

Determine angular accelration of plate on release.

α ?

⇒ N/E ; only method that deals w/ acceleration





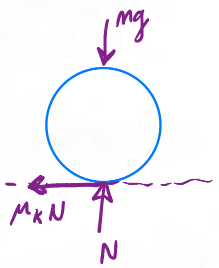
HORIZONTAL PLANE

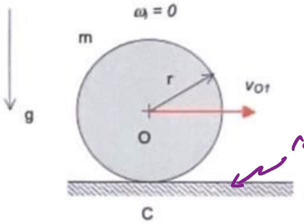
Su09 – F – P4

Time when slipping ceases.

has to do w/ velocity

⇒ LIM & AIM





rough

motion of C

Su10 – E2 - P1

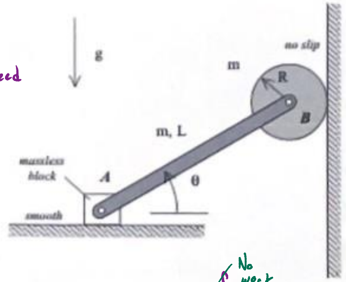
Angular velocity of bar when $\theta = 0$.

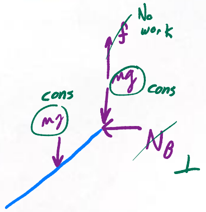
ω ?

chang in position → change in speed

⇒ W/E

↳ FBD → BIG!



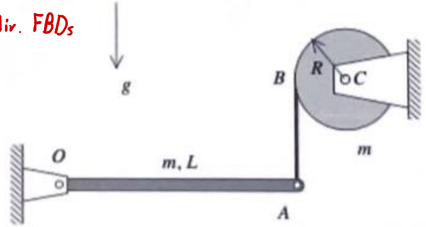


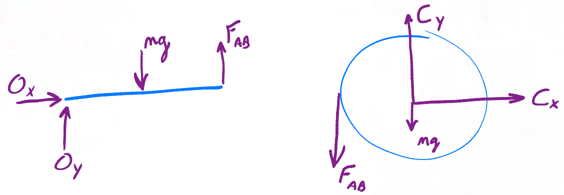
Su10 – E2 – P3

Determine angular acceleration of disk on release.

α ? → N/E bc only 1 deals w/ $\vec{a}$

indir. FBDs





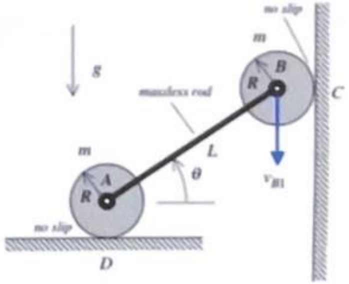
Su10 – F – P1

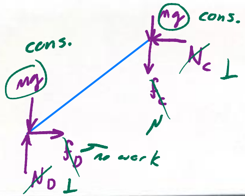
Angular velocity of disk B when $\theta = 0$. ω ?

change in ω for change in position

W/E

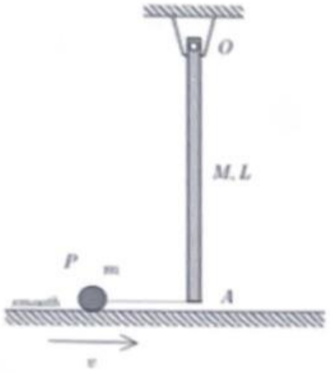
↳ BIG FBD



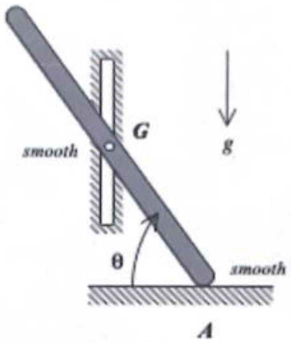


[pg. chp 5 Rapid fire Q's on website]

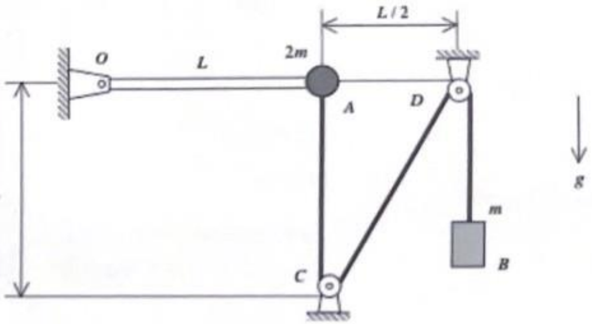
Su10 – F – P2
Determine angular speed after impact.



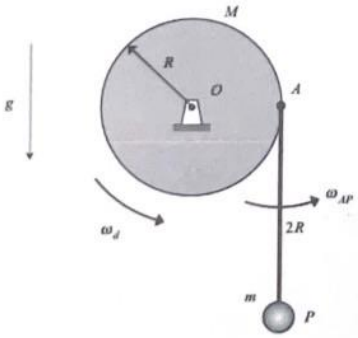
Su11 – E2 – P1
Determine angular acceleration on release.



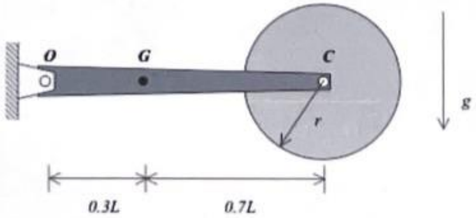
Su11 – E2 – P3
Angular velocity of bar after rotating 90°.



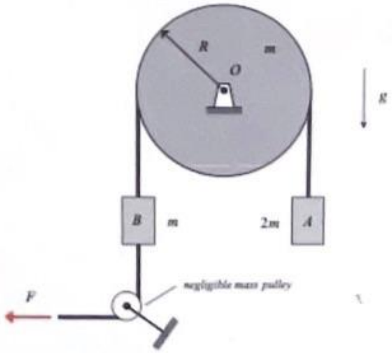
Su012– E2 – P1
Determine angular acceleration of disk on release.



Su12– F – P2
Angular speed of bar after rotating 90°.



Su15 – F – P1
Determine angular acceleration of disk.



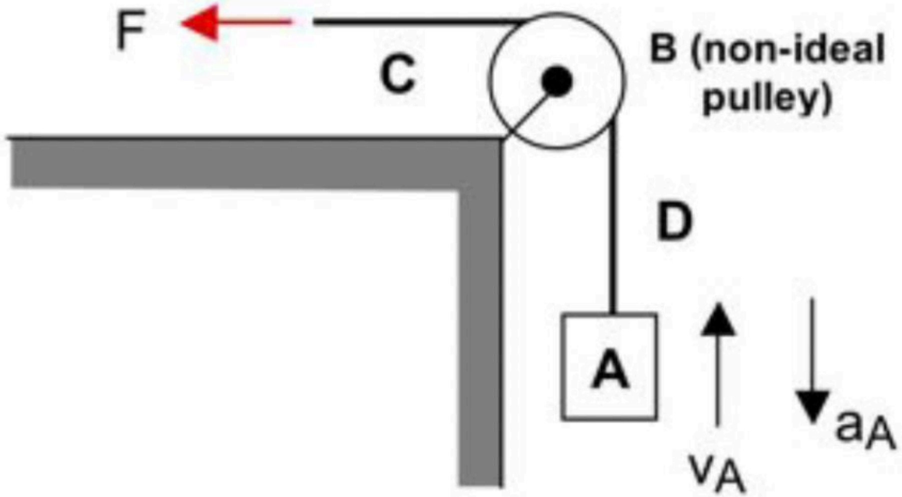
[pg. chp 5 Rapid fire Q's on website]

Question C5.5
The force F is lifting block A; however, the speed of A is decreasing. The pulley is non-ideal (it has a non-zero mass moment of inertia). Circle the correct description below.

(a) The tension in section C of the cable is larger than the tension in section D.

(b) The tension in section C of the cable is smaller than the tension in section D.

(c) The tension in section C of the cable is the same as the tension in section D.



[pg. 358]

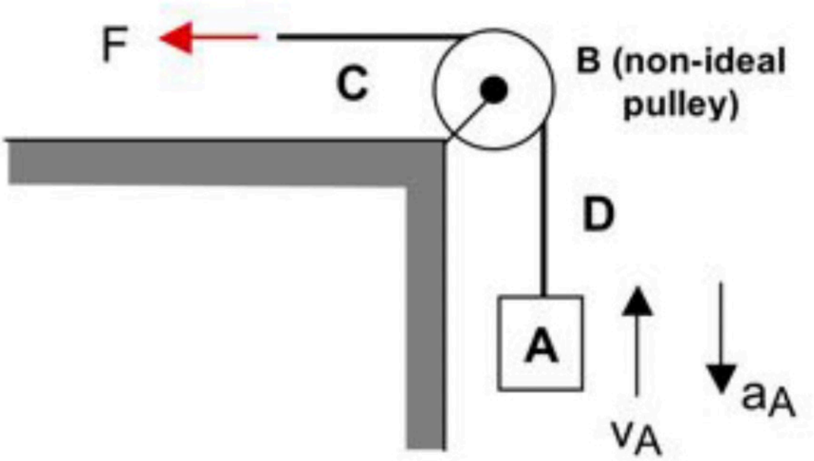
Question C5.5 p. 358

The force F is lifting block A; however, the speed of A is decreasing. The pulley is non-ideal (it has a non-zero mass moment of inertia). Circle the correct description below.

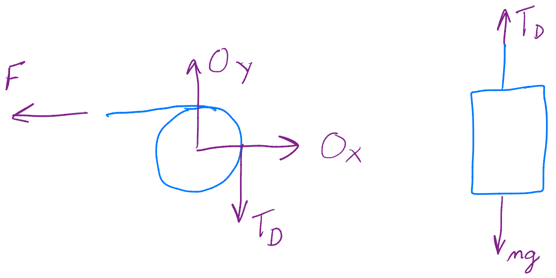
(a) The tension in section C of the cable is larger than the tension in section D.

(b) The tension in section C of the cable is smaller than the tension in section D.

(c) The tension in section C of the cable is the same as the tension in section D.



① FBD



② Newton on y

$\sum F_y = T_D - mg = -ma_A$

Not useful

③

$T_c = F$

④ Pulley Euler on O

$< 0_{(cw)}$

$\sum M_o = FR - T_D R = I_o \alpha$

$\Rightarrow F = T_D + I_o \alpha^{<0}$

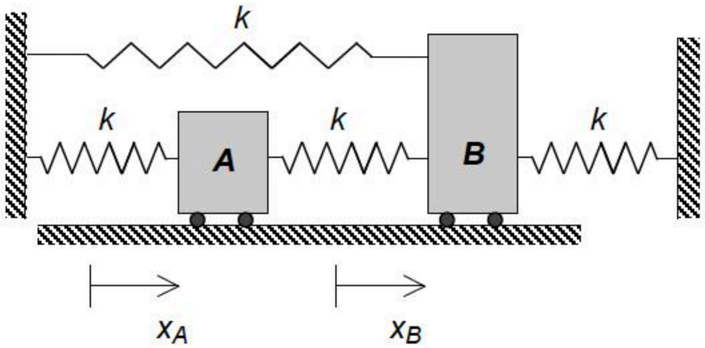
$\Rightarrow F - T_D < 0$

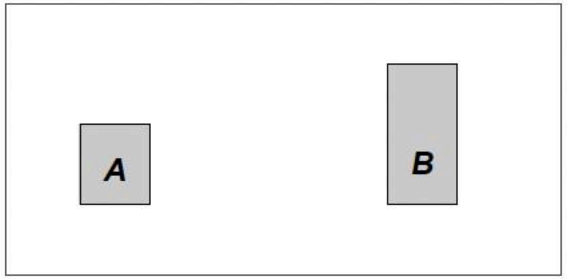
$\Rightarrow F < T_D$

$\Rightarrow T_c < T_D$

Question C6.1

The positions of particles A and B are described by the coordinates x_A and x_B shown below. All springs are unstretched when $x_A = x_B = 0$. In the free body diagrams shown below, draw and label (in terms of k, x_A and x_B) the spring force vectors on particles A and B.

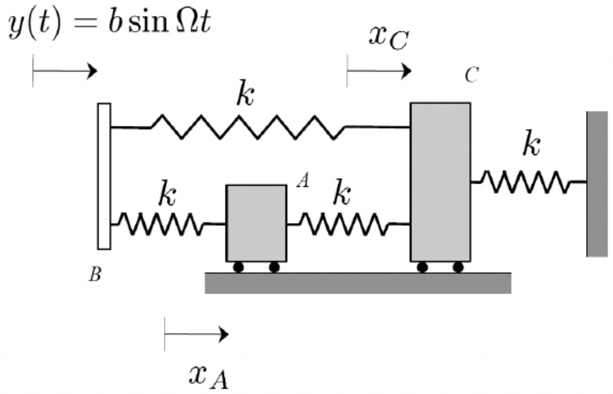


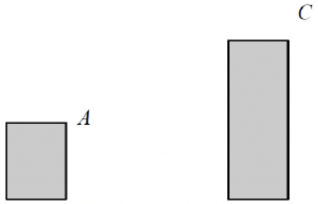


FBD's of particles A and B

Question C6.6

The positions of particles A and C are described by the coordinates x_A and x_C shown below. The base B on the left side is given prescribed motion of $y(t) = b \sin \Omega t$. All springs are unstretched when $x_A = x_C = y = 0$. In the free body diagrams shown below, draw and label [in terms of k, $y(t)$, x_A and x_C] the spring force vectors on particles A and C.

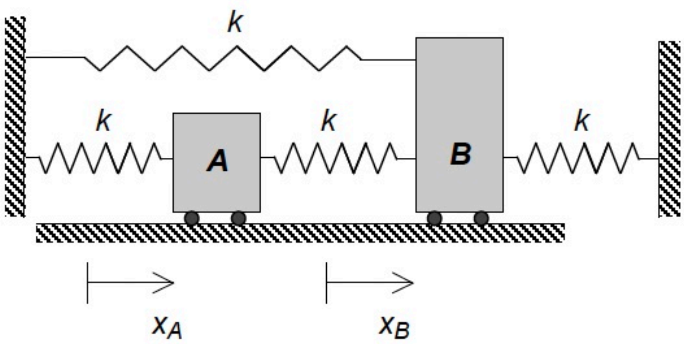


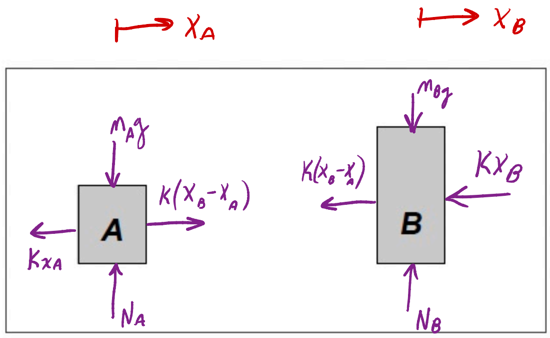


[pg. 434 and 439]

Question C6.1 p. 434

The positions of particles A and B are described by the coordinates x_A and x_B shown below. All springs are unstretched when $x_A = x_B = 0$. In the free body diagrams shown below, draw and label (in terms of k, x_A and x_B) the spring force vectors on particles A and B.

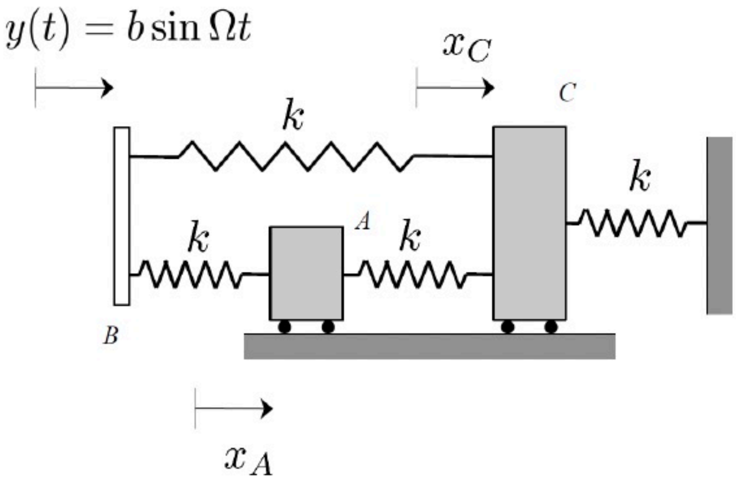


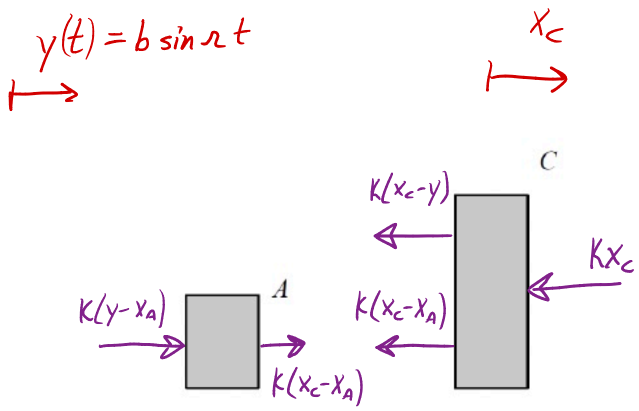


FBD's of particles A and B

Question C6.6 p. 439

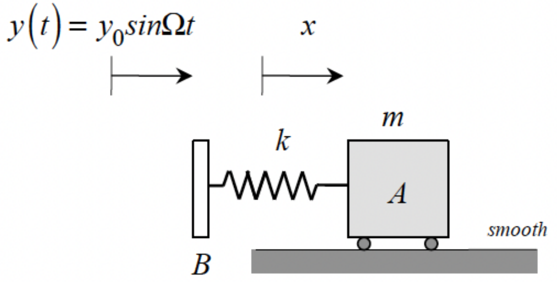
The positions of particles A and C are described by the coordinates x_A and x_C shown below. The base B on the left side is given prescribed motion of $y(t) = b \sin \Omega t$. All springs are unstretched when $x_A = x_C = y = 0$. In the free body diagrams shown below, draw and label [in terms of k, $y(t)$, x_A and x_C] the spring force vectors on particles A and C.



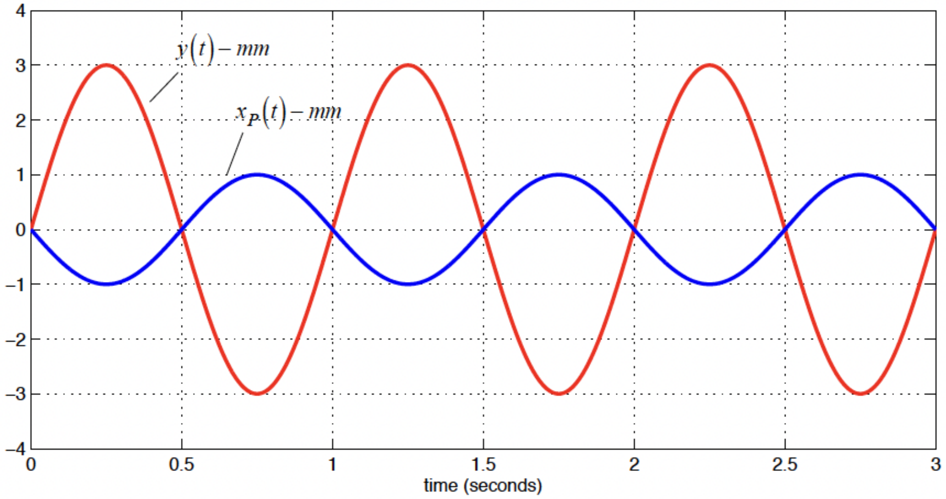


Question C6.13

The undamped, single-degree-of-freedom system shown below is made up of block A (of mass m) and a spring of stiffness k. The spring is connected between A and base B, with B given a prescribed displacement of $y(t) = y_0 \sin \Omega t$.



Let $x_P(t)$ represent the particular solution of the EOM for this system. Time histories for $x_P(t)$ and $y(t)$ are shown below.



From the plot, provide estimates for:

(a) The excitation amplitude y_0.

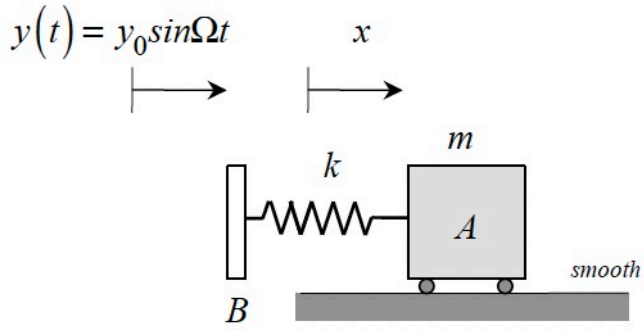
(b) The excitation frequency Ω.

(c) The natural frequency ω_n of the system.

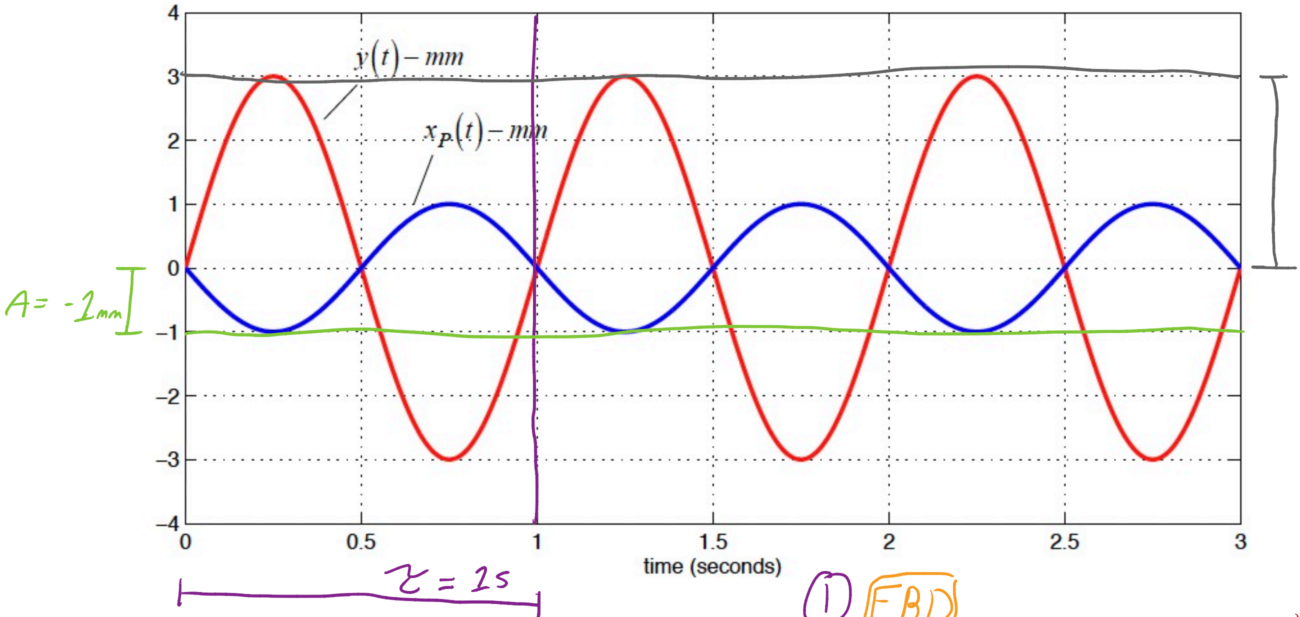
[pg. 446]

Question C6.13 p.446

The undamped, single-degree-of-freedom system shown below is made up of block A (of mass m) and a spring of stiffness k. The spring is connected between A and base B, with B given a prescribed displacement of $y(t) = y_0 \sin \Omega t$.



Let $x_P(t)$ represent the particular solution of the EOM for this system. Time histories for $x_P(t)$ and $y(t)$ are shown below.



④ Amplitude

$y_0 = 3mm$

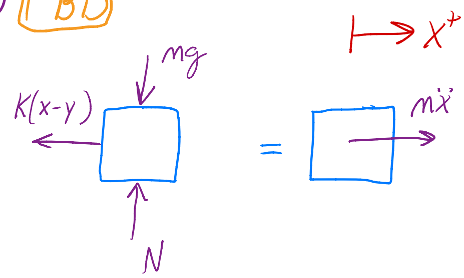
From the plot, provide estimates for:

(a) The excitation amplitude y_0.

(b) The excitation frequency Ω.

(c) The natural frequency ω_n of the system.

① FBD



② N/E

$\xrightarrow{+} \Sigma F_x = m\ddot{x}$

$-K(x-y) = m\ddot{x}$

$\Rightarrow m\ddot{x} + Kx = Ky$ General Form

③ Standard Form

$\Rightarrow \ddot{x} + \frac{K}{m}x = \frac{K}{m} y_0 \sin \Omega t$; $\div m$

⑤ Ω

$\tau = \frac{2\pi}{\Omega}$

$\Rightarrow \Omega = \frac{2\pi}{\tau}$

⑥ ω_n

$\ddot{x} + \frac{K}{m}x = \frac{K}{m} y_0 \sin \Omega t$; EOM Standard form

$\therefore \frac{K}{m} = \omega_n^2$

⑦ Solve for A using $x_p(t)$. plug back A into $x_p t$. Solve for ω_n

$\Rightarrow x_p(t) = A \sin \Omega t$; Amplitude $= |x_p(t)| = A$

1. Thank you so much for the wonderful semester!

2. Good luck with finals!!

3. It will be fine ☺

4. Class picture in front of classroom (if we have time)

Lec 42 Short Feedback Form: