

ME 274 – Spring 2026

Study resources

Final Exam

Working sample exam questions is a good practice to follow in your exam preparation. However, as you prepare for this midterm exam in ME 274, consider the fact that starting out with a solid understanding of the fundamental principles will take you much further in doing well in the course than simply getting practice with old exams.

The following is a suggested path for you to take in your exam preparation:

- Review the fundamentals. If you have not yet read the lecture book material for the course, you need to do that now. If you have done so, you do not need to go back and reread everything. Review the one-page summary sheets provided to you every class period on the Daily Schedule page of the blog. For your convenience, I am providing you with PDFs for all of these summary pages. Make note of anything on these summary pages with which you are not comfortable. Read more of those topics in the lecture book and/or ask your instructor (me) for help.
- Review your homework. Go back over your submitted and graded homework on Gradescope. Make note of things that you did not do well on with those homeworks and of any unresolved issues.
- Review old sample exams. Weekly Joys has many sample exams from previous semesters, including solutions. Focus on problems that cover topics with which you are having problems.
- Save back a sample exam. Work this sample exam under the pressure of the 2-hour time constraint. Quit at the end of 120 minutes. Do an honest appraisal of your work. Where did you have difficulties? Where did you spend most of your time? From this, determine where you might have weaknesses and work on those topics as the exam approaches. Do not wait until the exam to find out your weaknesses. It is too late then. Talk to your instructor and/or visit the tutorial room for help.

Study resources

Included in this packet is the following material:

- The set of PDFs of the daily course summaries for the course (these are also found on the course website on the Daily Schedule page).
- A set of sample ME 274 Final Exam questions from previous terms. These problems are intended to serve as a set of talking points between you and the instructor/TA as you prepare for your exam. *We will NOT be providing solutions for these problems.*

We encourage you to use this material in your exam preparation.

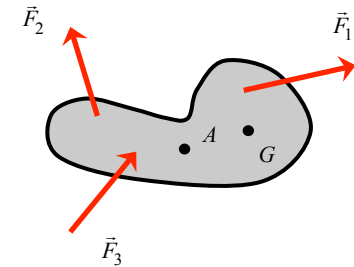
Summary: Newton/Euler Equations 1

FUNDAMENTAL equations:

$$(1) \quad \sum \vec{F} = m\vec{a}_G$$

$$(2) \quad \sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$$

SAME point "A"!



CRITICAL ISSUES:

- For NEWTON (1): G must be the center of mass of the body
- For EULER (2): A is ANY point on the body. The same point "A" must be used across the board in the equation – you cannot mix and match points A.

SIMPLIFICATION: If A is: EITHER the center of mass G OR a fixed point (zero acceleration) OR \vec{a}_A is parallel to $\vec{r}_{G/A}$, then the Euler equation (2) reduces to:

$$\sum \vec{M}_A = I_A \vec{\alpha} \quad \leftarrow \text{We will use this form of the equation most of the time}$$

TERMINOLOGY: I_A is known as the "mass moment of inertia" of the body about point A. The size of I_A is dependent on the location of A.

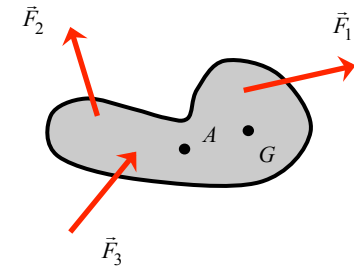
Summary: Newton/Euler Equations 3

FUNDAMENTAL equations:

$$(1) \quad \sum \vec{F} = m\vec{a}_G$$

$$(2) \quad \sum \vec{M}_A = I_A \vec{\alpha} \quad ; \quad A = \text{c.m. } \underline{\text{OR}} \text{ fixed point } \underline{\text{OR}} \vec{r}_{G/A} \parallel \vec{a}_A$$

SAME point "A"!



SOLUTION METHOD: the four-step plan

Kinetics: Four-Step Problem Solving Method

The suggested plan of action for solving kinetics problems:

1. **Free body diagram(s).** Draw appropriate free body diagrams (FBDs) for the problem. Your choice of FBDs is problem dependent. For some problems, you will draw an FBD for each body; for others, you will draw an FBD for the entire system. An integral part of your FBDs is your choice of coordinate systems. For each FBD, draw the unit vectors corresponding to your coordinate choice.
2. **Kinetics equations.** At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1.
3. **Kinematics.** Perform the needed kinematic analysis. A study of the equations in Step 2 above will guide you in deciding what kinematics are needed to find a solution to the problem.
4. **Solve.** Count the number of unknowns and the number of equations from above. If you do not have enough equations to solve for your unknowns, then you either: (i) need to draw more FBDs, OR (ii) need to do more kinematic analysis. When you have sufficient equations for the number of unknowns, solve for the desired unknowns from the above equations.

Draw INDIVIDUAL free-body diagrams for Newton/Euler.

Be sure to use the correct mass moment of inertia for your choice of point "A". Use PAT if necessary.

Typically the most difficult step. Recall the rigid body kinematics from Chapter 2.

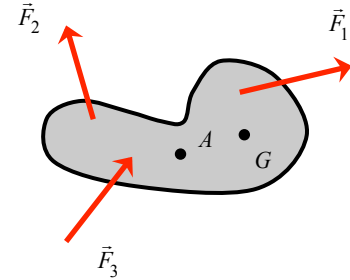
If you are short equations, go back to Step 3 – Kinematics.

Summary: Work/Energy Equation 1

FUNDAMENTAL equation: $T_2 + V_2 = T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)}$

with:

$$T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2 + m \vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$$



SPECIAL CHOICES FOR POINT "A": If A is EITHER the c.m. OR a fixed point, then the kinetic energy equation reduces to:

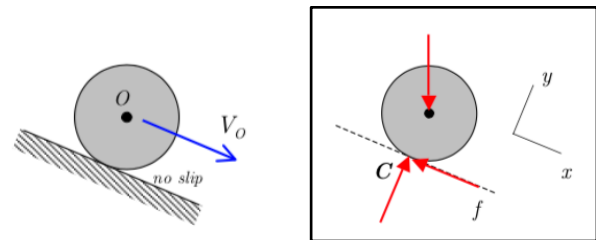
$$T = \boxed{\frac{1}{2} m v_A^2} + \boxed{\frac{1}{2} I_A \omega^2}$$

translation rotation

PARALLEL AXIS THEOREM: As with the Newton/Euler equation, you will need to use the PAT if you choose A to be anything other than the c.m.

SYSTEM CHOICE: Make your system BIG! Include as many components within system to make workless forces INTERNAL (no work on system). Different choice than for Newton/Euler.

ROLLING WITHOUT SLIPPING: The friction force at a no-slip point does not work. Why? Recall that the no-slip point is stationary – no work is done on a stationary point.



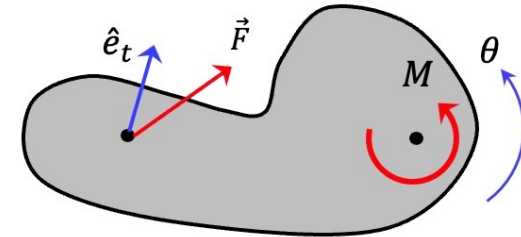
Summary: Work/Energy Equation 2

FUNDAMENTAL equations: $T_2 + V_2 = T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)}$ with:

$$T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2$$

where A = EITHER c.m. OR fixed point, and:

$$U_{1 \rightarrow 2}^{(nc)} = \int_1^2 (\vec{F} \cdot \vec{e}_t) ds + M\Delta\theta$$



SOLUTION METHOD: the four-step plan

Kinetics: Four-Step Problem Solving Method

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2. **Kinetics equations.** At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1.
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Draw free-body diagram of entire system for work/energy.

Be sure to use the correct mass moment of inertia for your choice of point "A". Use PAT if necessary.

Typically the most difficult step. Recall the rigid body kinematics from Chapter 2.

If you are short equations, go back to Step 3 – Kinematics.

Summary: Impulse/Momentum Equations 1

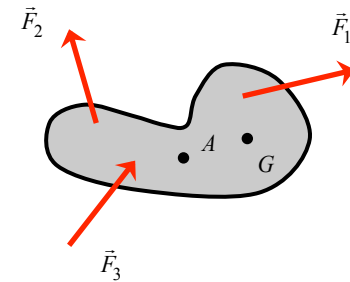
FUNDAMENTAL equations (for your chosen “system”):

$$m\vec{v}_{G2} = m\vec{v}_{G1} + \int_1^2 \vec{F} dt$$

$$\vec{H}_{A2} = \vec{H}_{A1} + \int_1^2 \vec{M}_A dt$$

for A = c.m. OR fixed point, with:

$\vec{H}_A = I_A \vec{\omega} \quad ; \text{ for a RIGID BODY}$ $= m\vec{r}_{P/A} \times \vec{v}_P \quad ; \text{ for a particle } P$

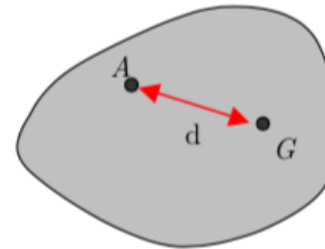


SYSTEM CHOICE: Make your system big (make as many forces internal as possible to take advantage of conservation).

PARALLEL AXIS THEOREM: You will need to use the PAT if you choose A to be anything other than the c.m.:

$$I_A = \boxed{I_G} + md^2$$

cm



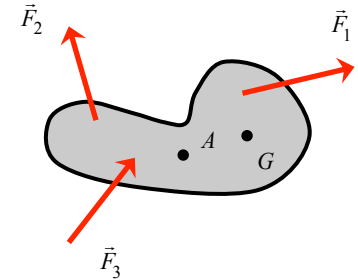
Summary: Impulse/Momentum Equations 2

FUNDAMENTAL equations (for A = cm OR fixed point):

$$m\vec{v}_{G2} = m\vec{v}_{G1} + \int_1^2 \vec{F} dt$$

$$\vec{H}_{A2} = \vec{H}_{A1} + \int_1^2 \vec{M}_A dt$$

$\vec{H}_A = I_A \vec{\omega} \quad ; \text{ for a RIGID BODY}$ $= m\vec{r}_{P/A} \times \vec{v}_P \quad ; \text{ for a particle P}$
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SOLUTION METHOD: the four-step plan

Kinetics: Four-Step Problem Solving Method

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Draw free-body diagram of entire system for impulse momentum.

Be sure to use the correct mass moment of inertia for your choice of point "A". Use PAT if necessary.

Typically the most difficult step. Recall the rigid body kinematics from Chapter 2.

If you are short equations, go back to Step 3 – Kinematics.

Summary: Particle and Planar Rigid Body Kinetics

WHICH TOOL(s) TO USE?

Put effort up front deciding on which method(s) to use: *Newton-Newton/Euler, work/energy, linear impulse momentum or angular impulse momentum*. Use the Kinetics Table in Section 5.D of the lecture book as a guide.

PARTICLE or RIGID BODY?

How is a particle distinguished from a rigid body? For a *particle*, we have:

$$\sum \bar{M}_G = I_G \bar{\alpha} = \bar{0} \quad (\text{EITHER } I_G = 0 \text{ OR } \bar{\alpha} = \bar{0})$$

THE FOUR-STEP PLAN: Follow it...it is your friend!

Method	Body model	Fundamental equations
Newton-Euler <i>(relating forces to accelerations)</i>	particle	$\sum \bar{F} = m\bar{a}$
	rigid body <i>(G = c.m. and A = any point on body)</i>	$\sum \bar{F} = m\bar{a}_G$ $\sum \bar{M}_A = I_A \bar{\alpha} + m\bar{r}_{G/A} \times \bar{a}_A$
Work-energy <i>(relating change in speed to change in position)</i>	particle	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body <i>(G = c.m. and A = any point on body)</i>	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\bar{v}_A \cdot (\bar{\omega} \times \bar{r}_{G/A})$
Linear impulse-momentum <i>(relating change in velocity to change in time)</i>	particle	$\int_{t_1}^{t_2} \sum \bar{F} dt = m\bar{v}_2 - m\bar{v}_1$
	rigid body <i>(G = c.m.)</i>	$\int_{t_1}^{t_2} \sum \bar{F} dt = m\bar{v}_{G2} - m\bar{v}_{G1}$
Angular impulse-momentum <i>(relating change in angular velocity to change in time)</i>	particle <i>(O = fixed point)</i>	$\int_{t_1}^{t_2} \sum \bar{M}_O dt = \bar{H}_{O2} - \bar{H}_{O1}$ where $\bar{H}_O = m\bar{r}_{P/O} \times \bar{v}_P$
	rigid body <i>(A = fixed point or c.m.)</i>	$\int_{t_1}^{t_2} \sum \bar{M}_A dt = \bar{H}_{A2} - \bar{H}_{A1}$ where $\bar{H}_A = I_A \bar{\omega}$

Summary: Vibrations 1 (EOMs)

1. *GOAL?* To derive the differential equation of motion (EOM):

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

2. *HOW?*

Good news: You already know how to do this! The four-step plan:

1) FBD; 2) Newton/Euler; 3) Kinematics; 4) Solve (here “solve” means combining together into a single EOM).

3. *THE CATCH:* You EOM must describe motion for all time, not just a single instant in time. Define your coordinate(s) at the start, and stick with them throughout. If you do, you are all set.
4. *WHAT'S NEXT?* We will spend the rest of the semester solving our differential EOMs. You already know how to do that also! **Woo Hoo!**

Summary: Vibrations - Free Undamped Response

EOM: For free response of undamped system:

$$m\ddot{x} + kx = 0$$

STANDARD FORM OF EOM: Divide EOM by “m” to get:

$$\ddot{x} + \omega_n^2 x = 0$$

where $\omega_n = \sqrt{\frac{k}{m}} = \text{natural frequency}$

SOLUTION OF EOM:

$$x(t) = C \cos \omega_n t + S \sin \omega_n t$$

HOW TO FIND THE RESPONSE COEFFICIENTS, C AND S?

Enforce the initial conditions on the solution:

$$x(0) = x_0 = C \cos 0 + S \sin 0 \Rightarrow C = x_0$$

$$\dot{x}(0) = \dot{x}_0 = -C\omega_n \sin 0 + S\omega_n \cos 0 \Rightarrow S = \frac{\dot{x}_0}{\omega_n}$$

Summary: Vibrations - damped free response

EOM: For free response of damped system:

$$m\ddot{x} + c\dot{x} + kx = 0$$

STANDARD FORM OF EOM: Divide EOM by “m” to get:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = 0$$

where

$$2\zeta\omega_n = c / m \Rightarrow \zeta = c / 2\sqrt{km} = \text{damping ratio}$$

$$\omega_n = \sqrt{k / m} = \text{natural frequency}$$

SOLUTION OF EOM: For $0 \leq \zeta < 1$ (UNDERdamped)

$$x(t) = e^{-\zeta\omega_n t} (C \cos \omega_d t + S \sin \omega_d t) \quad ; \quad \omega_d = \omega_n \sqrt{1 - \zeta^2}$$

HOW TO FIND THE RESPONSE COEFFICIENTS, C AND S?

Enforce the initial conditions on the solution:

$$x(0) = x_0 = e^0 (C \cos 0 + S \sin 0) \Rightarrow C = x_0$$

$$\dot{x}(0) = \dot{x}_0 = -\zeta\omega_n e^0 (C \cos 0 + S \sin 0) + \omega_d e^0 (-C \sin 0 + S \cos 0) \Rightarrow S = \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d}$$

Summary: Vibrations - Forced Response 1

EOM: For forced response:

$$M\ddot{x} + C\dot{x} + Kx = F_0 \sin \Omega t \Rightarrow \ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = \frac{F_0}{M} \sin \Omega t$$

TOTAL RESPONSE: Since this is a linear EOM, we can write:

$$x(t) = x_C(t) + x_P(t)$$

where $x_C(t) = e^{-\zeta\omega_n t} (C \cos \omega_d t + S \sin \omega_d t)$

HOW TO FIND PARTICULAR SOLUTION?

$$x_P(t) = A \sin \Omega t + B \cos \Omega t$$

Substitute into EOM and solve for A and B . If the system is undamped, then you will find that $B = 0$.

ENFORCING IC's: The initial conditions are enforced on the TOTAL solution $x(t)$, NOT on the complementary solution $x_C(t)$!

Summary: Vibrations - Forced Response 2

EOM: For forced response:

$$M\ddot{x} + C\dot{x} + Kx = F_0 \sin \Omega t \Rightarrow \ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = \frac{F_0}{M} \sin \Omega t$$

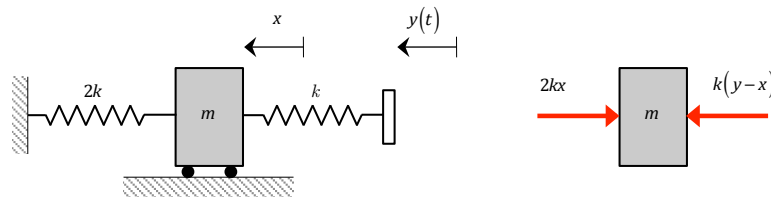
PARTICULAR SOLUTION

$$x_p(t) = A \sin \Omega t + B \cos \Omega t$$

Substitute into EOM and solve for A and B . If the system is undamped, then you will find that $B = 0$.

BASE EXCITATION: For many problems, the energy input is through a moving support (prescribed motion, $y(t)$), rather than an applied force.

Example:



$$\sum F_x = -2kx + k(y-x) = m\ddot{x} \Rightarrow m\ddot{x} + 3kx = \boxed{ky(t)} \text{ base excitation}$$

Sample exam problems

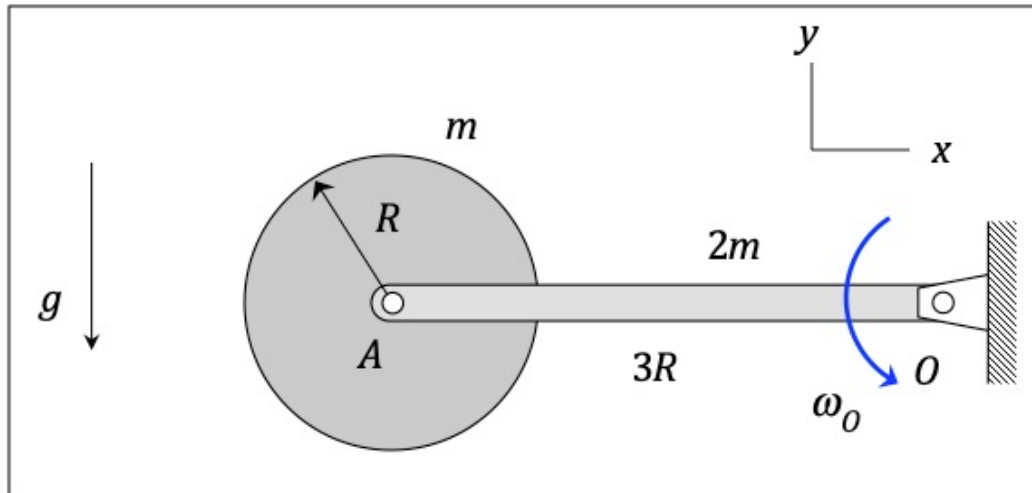
Attached here is a set of sample exam questions from past terms in the course. These problems were chosen from past terms without regard to the specific questions that will appear on your exam. Please do not use these questions as an indication of which specific topics will appear on this term's midterm.

Please use these questions to help you prepare for the midterm this term. We will not be providing solutions for these questions, as we want you to use the questions to prepare under the exam-like situation of not knowing the answer, and you working through ways to check your work on your own, rather than looking at the answer to check your work. If you have questions regarding the solution of these problems, please check with your instructor and/or the TAs.

Please note that Weekly Joys has a good number of sample exam questions from past terms for which solutions are provided.

Examination No. 2 (REGULAR)

PROBLEM NO. 1 – 20 points

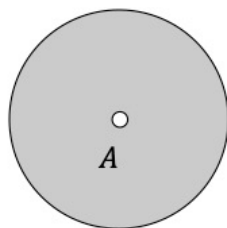


Given: Thin homogeneous bar OA (having a mass of $2m$ and length of $3R$) is pinned to ground at end O. A homogeneous, circular disk (of radius R and of mass m) is pinned to bar OA at end A. The system is released with bar OA being horizontal and rotating in the counterclockwise sense with an angular speed of ω_0 , and with the disk having zero angular speed.

Find: It is desired to know the angular acceleration of the disk and the acceleration of point A, on release. Please follow the four steps provided below, and present your work within the appropriate steps.

Solution:

STEP 1: Draw individual free body diagrams of the disk and bar OA below.



STEP 2: Kinetics

ME 274 – Summer 2022
Examination No. 2 (REGULAR)
PROBLEM NO. 1 – continued

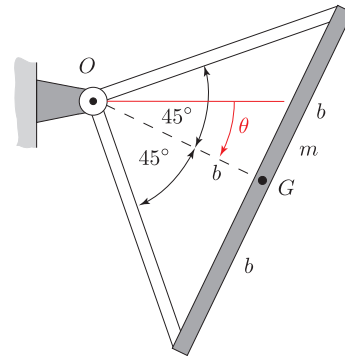
Name _____

STEP 3: Kinematics

STEP 4: Solve for the angular acceleration of the disk and the acceleration of point A.
Write your answers as vectors. Leave your answers in terms of, at most: m , g , R and ω_0 .

Problem 2, Part 2 (18 points): _____

Given: A uniform slender bar of mass m and length $2b$ is mounted in a right-angle "L-bracket" of negligible mass. It is released from rest at $\theta = 0$, i.e., when points G and O are on the same horizontal line. Use g as the gravitational constant.



Find: Determine the following quantities:

- Draw a free-body diagram of the bar in its initial position. Clearly indicate any datum that you use on the free-body diagram.
- Use the *principles of work energy* to derive an expression for the angular velocity ω in terms of b , g , and θ .

Express your answers as vectors.

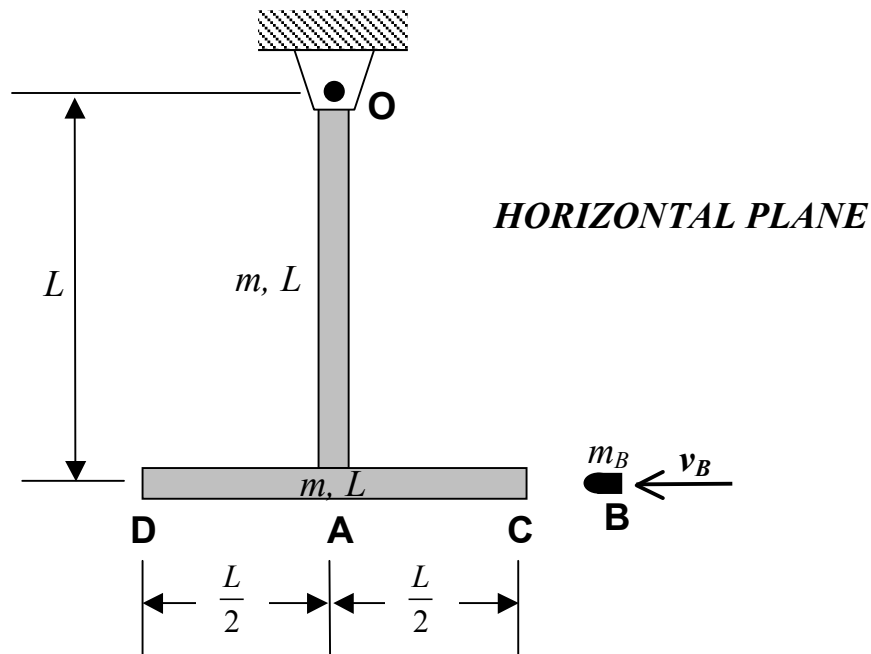
Solution:

ME 274 – Spring 2010
Examination No. 3
PROBLEM NO. 2

Name _____

Given: A T-shaped bar consisting of a bar (of mass m , length L) welded to the end of another bar (m, L) that is pinned at O . A bullet (mass m_B) strikes a T-shaped bar at end C with a speed v_B . The bullet becomes embedded in the bar immediately after the impact. All motion of the system is in a **HORIZONTAL PLANE**.

Find: Determine the angular velocity of the T-shaped bar immediately after impact.



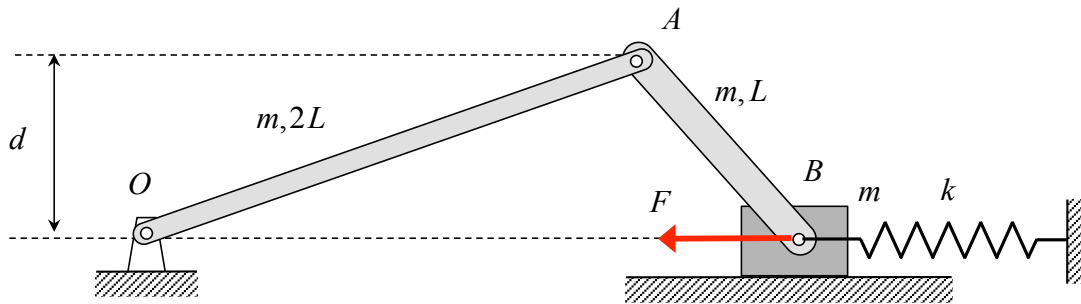
Use the following parameters in your analysis: $L = 2$, $m = 3$ kg, $m_B = 0.02$ kg, $v_B = 150$ m/s.

ME 274 – Spring 2017
Examination No. 3
PROBLEM No. 2 (20 pts.)

Name (Print) _____
 (Last) (First)
Instructor _____

Given: A mechanism is made up of three bodies: a homogeneous rigid link OA, a homogeneous rigid link AB and a rigid block B. Each body has a mass of m . Link OA is joined to link AB at pin joint A, with end O of OA pinned to ground at O. End B of AB is pinned to block B, with block B being constrained to move along a smooth surface. A spring of stiffness k is attached between B and ground. A constant force F acts on block B. The system is released from rest with $d = d_1$, a position at which the spring is unstretched/uncompressed. Consider all surfaces to be smooth.

Find: Determine the angular velocity of link AB when $d = d_2 = L$. Use the following parameters in your analysis: $m = 10 \text{ kg}$, $L = 0.5 \text{ m}$, $k = 1000 \text{ N/m}$, $F = 500 \text{ N}$ and $d_1 = 0.3 \text{ m}$.



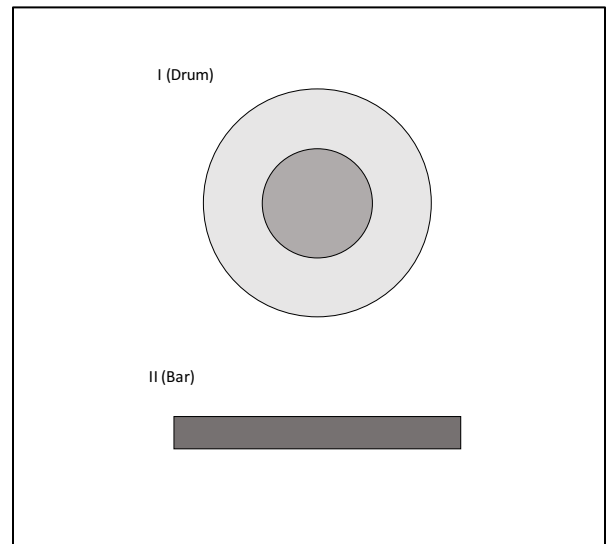
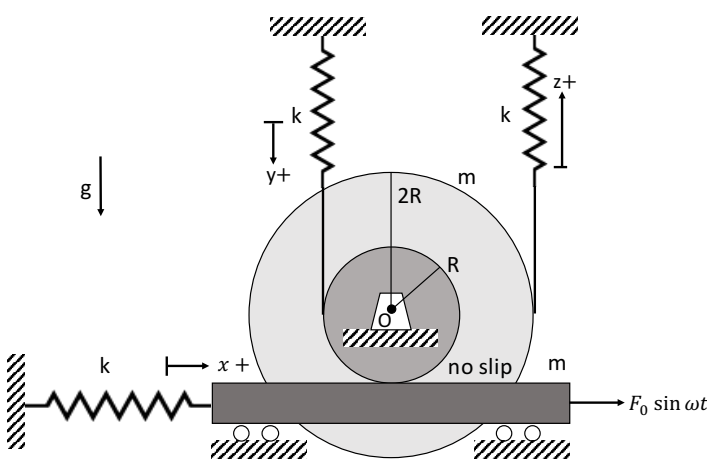
HORIZONTAL PLANE

Given: The system shown below consists of a drum and a bar. The drum is pinned to the ground at its center of mass, O, with an inner radius of R , an outer radius $2R$, a radius of gyration about point O is $k_O = R$, and is connected to two linear springs, each with spring constant k . The bar has a mass m and a length L , is connected to a linear spring with spring constant k , and is in no slip contact with the bottom of the inner surface of the drum. An external force equal to $F(t) = F_0 \sin \omega t$ is applied to the right end of the bar. The variable x measures the horizontal displacement of the bar. The variables y and z measure the change in length of the two vertical springs from their equilibrium position. Note that when $x = y = z = 0$ all springs are not stretched. The gravitational force acts downwards.

Please use the following numerical values: $R = 0.1\text{m}$, $L = 0.3\text{m}$, $k = 100\text{N/m}$, $m = 3\text{kg}$, $F_0 = 1\text{N}$, and $\omega = 12 \text{ rad/s}$. Please use the provided figure for your FBDs.

Find: For this problem, determine:

- the equation of motion (EOM) of the system expressed in terms of x ;
- the natural frequency ω_n ; and
- the particular solution $x_p(t)$ of the EOM. You must derive the form of the solution for the response coefficients – do not write down these from memory.
- Is the particular solution $x_p(t)$ *in phase* or *out of phase* with the forcing $F(t)$?

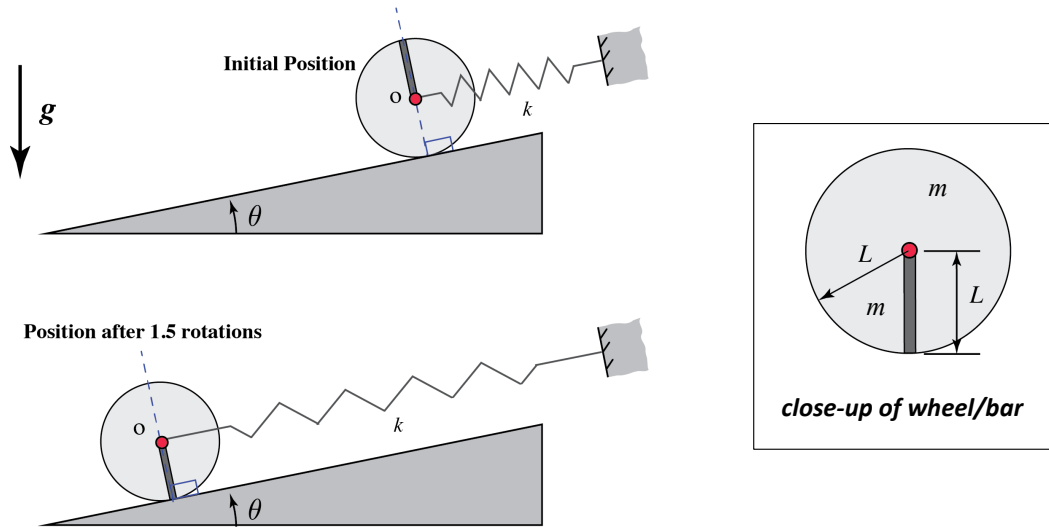


PLEASE USE THIS FIGURE FOR YOUR FBD

ME 274 – Spring 2017
Final Examination
PROBLEM No. 2 (20 pts.)

Name (Print) _____
 (Last) (First)
Instructor _____

Given: A homogeneous wheel of mass m and radius L has a uniform bar welded to it as shown in the figure below. The bar has mass m and length L . The center O of the wheel is attached to a spring of stiffness k . The wheel is allowed to roll without slip down an inclined surface of angle θ , with the wheel not losing contact with the incline as it rolls. At the initial position shown, the system is at rest, and the spring is initially unstretched. At the final position shown, the wheel has made 1.5 rotations from the initial position. *Note at both the initial and final positions the bar is aligned with a direction that is normal to the incline.*

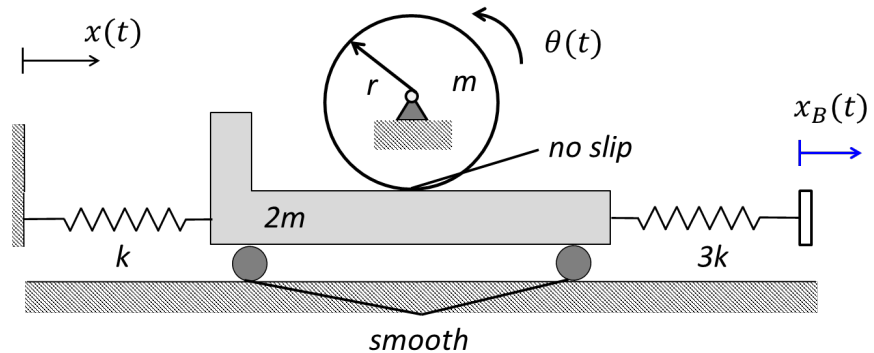


Find: For this problem:
 a) Fill out the table on the next page showing the kinetic and potential energies at the initial and final positions. State your answers in terms of given parameters above. Be sure to indicate the location of your gravitation datum line on the figure above. Use the space below for your work.

PROBLEM NO. 4 (20 points)

Given:

A homogeneous disk of radius R and mass m is connected to the ground at point O . The disk is in no slip contact with a block of mass $2m$ that is connected to a grounded wall through a spring of stiffness k . The block is further connected to a second spring of stiffness $3k$ and a massless base subjected to a prescribed base displacement $x_B(t) = b \sin \Omega t$. Let $x(t)$ represent the horizontal motion of the block, where the left spring is unstretched when $x = 0$. Let $\theta(t)$ represent the angular motion of the disk.



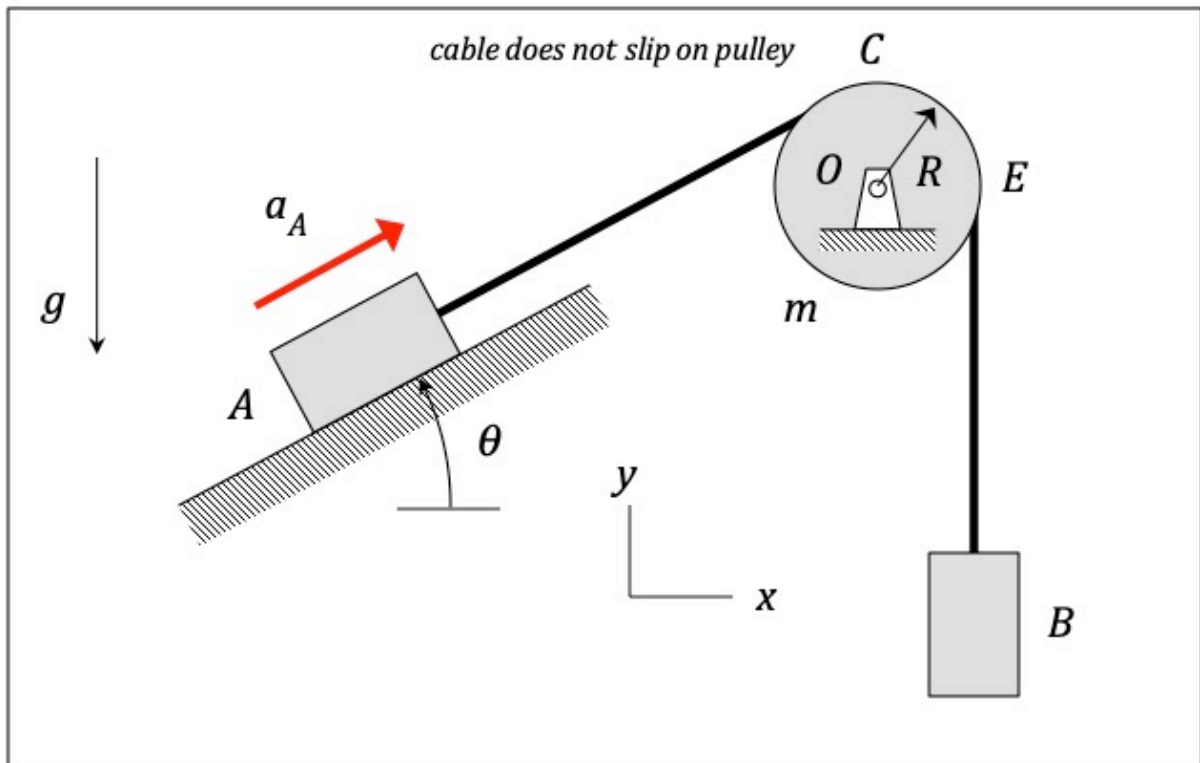
Find:

- Derive the dynamical equation of motion (EOM) of the system in terms of $\theta(t)$.
- Determine the undamped natural frequency of the system.
- Derive the forced response of the system $\theta_{\square}(t)$. Show the complete derivation process.
- If the forcing frequency is $\Omega = \sqrt{\frac{3k}{2m}}$. Under these conditions, would the system be in-phase, or out-of-phase? Justify your answer.

Examination No. 2 (REGULAR)

PROBLEM NO. 3 – 20 points TOTAL

NOTE: You are not required to show your work on Problem 3. There is no partial credit awarded for the different parts of the problem.



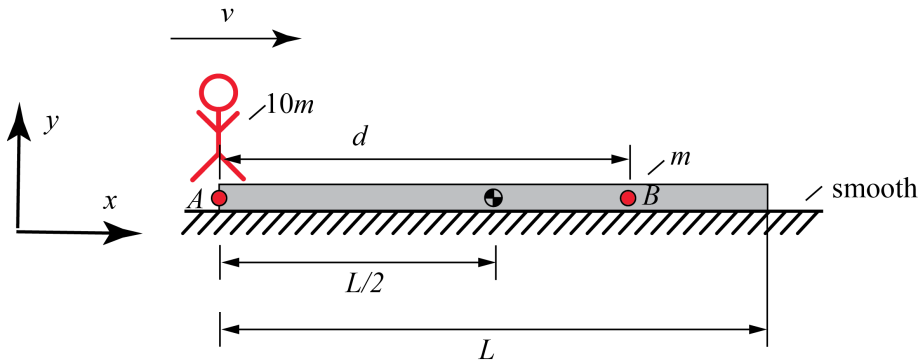
Blocks A and B are connected by an inextensible cable. The cable is supported by a homogeneous pulley of mass m and radius R , with the cable not slipping on the pulley. It is known that A has an acceleration of a_A up the incline. Let T_{AC} and T_{BE} represent the tensions in sections AC and BE of the cable, respectively.

PART A (2 points) – choose the correct response

- $T_{AC} > T_{BE}$
- $T_{AC} = T_{BE}$
- $T_{AC} < T_{BE}$
- More information is needed to answer this question.

PART D - 2 points

A boy of mass $10m$ stands on a thin board of mass m . The board sits on ice, assume the ice is a frictionless surface. The boy and the board are initially at rest. The boy walks from A to B at a constant speed v .



Fill in the bubble next to the answer that correctly describes the board's motion while the boy walks.

The board moves in the **positive x** direction with the magnitude of its absolute velocity is **greater** than the magnitude of the boy's velocity.

The board moves in the **negative x** direction with the magnitude of its absolute velocity is **greater** than the magnitude of the boy's velocity.

The board moves in the **positive x** direction with the magnitude of its absolute velocity is **greater** than the magnitude of the boy's velocity.

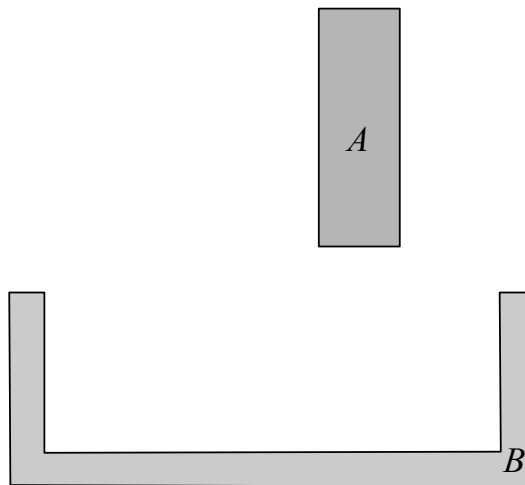
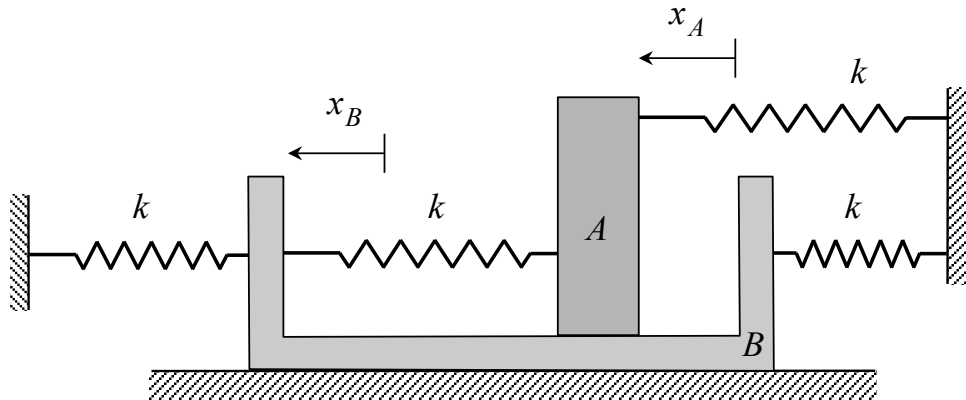
The board moves in the **negative x** direction with the magnitude of its absolute velocity is **greater** than the magnitude of the boy's velocity.

The board does not move.

PART C – 5 points

The system shown below moves within a horizontal plane. The displacements of blocks A and B in the system shown below are defined as x_A and x_B , respectively, with the positive senses for these displacements shown in the figure. The springs are unstretched when $x_A = x_B = 0$. Blocks A and B each have a mass of m . Complete the free body diagrams of blocks A and B below. Assume all surfaces to be smooth. Be sure to label the spring forces with in terms of the variables x_A , x_B and k , and clearly indicate the *direction* of these forces.

HORIZONTAL PLANE

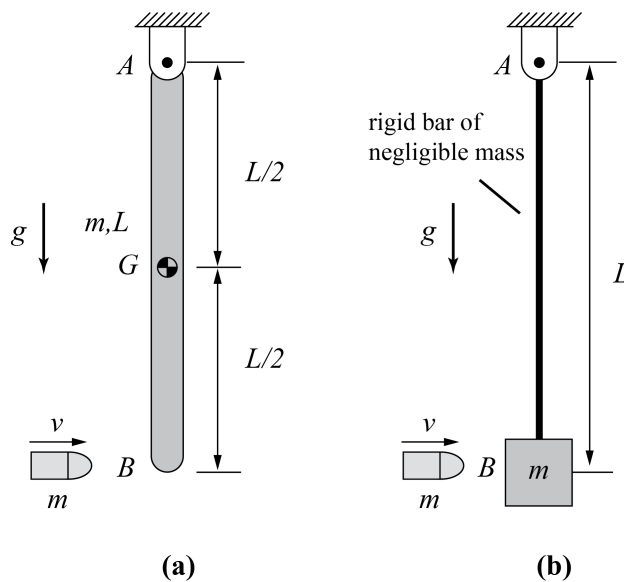


PART E- 2 points

Consider the two systems to the left labeled (a) and (b) respectively in the vertical plane. Each system contains a bullet with **absolute velocity, v** .

In system (a) the bullet strikes a bar that is initially at rest. The bar has mass, m , and length, L . The bullet imbeds it itself in the bar at B , and the bullet and bar rotate about A .

In system (b) the bullet strikes a mass, m , which is initially at rest. The mass is attached to a rigid bar of negligible mass. The bullet imbeds it itself in the mass at B . After impact the rod, mass and bullet rotate about A .



Fill in the bubble next to the answer that correctly describes the bullet's motion after impact.

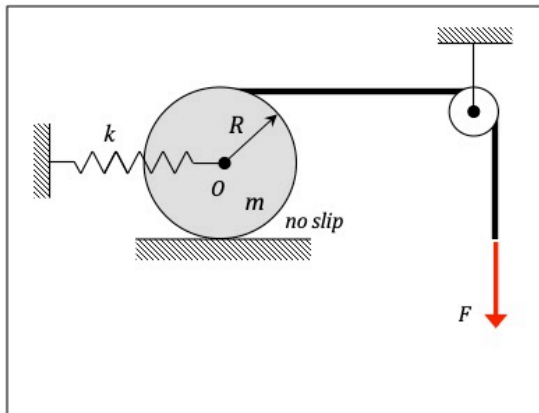
After impact the velocity of the bullet in system (a) **is greater than** the velocity of the bullet in system (b) after impact.

After impact the velocity of the bullet in system (a) **is less than** the velocity of the bullet in system (b) after impact.

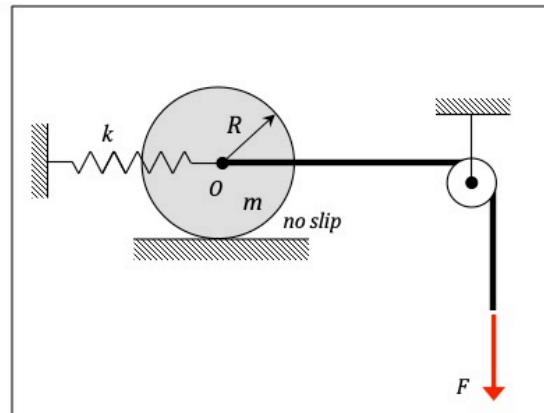
After impact the velocity of the bullet in system (a) **equals** the velocity of the bullet in system (b) after impact.

Final Examination (REGULAR)

PROBLEM NO. 4 (continued)



System I



System II

PART B – 1 pt.

The force F for both Systems I and II pulls the center O of the disk to the right through a distance of d . Let $U_{1 \rightarrow 2}^{(I)}$ and $U_{1 \rightarrow 2}^{(II)}$ represent the work done for F for Systems I and II, respectively.

- $U_{1 \rightarrow 2}^{(I)} > U_{1 \rightarrow 2}^{(II)}$
- $U_{1 \rightarrow 2}^{(I)} = U_{1 \rightarrow 2}^{(II)}$
- $U_{1 \rightarrow 2}^{(I)} < U_{1 \rightarrow 2}^{(II)}$
- More information is needed in order to answer this question.

PART F - 5 points

Particle P is launched with a speed of v_0 and moves along a projectile path under the action of only its weight (no air resistance). The particle reaches its highest point at A, with A being at a height of h above the launch point. Determine the radius of curvature of the path of P at A. Your answer should be in terms of, at most, v_0 , h and g .

