

***“Rapid fire” questions from Chapter 5 – Kinetics of Rigid Bodies***  
***ME 274 – cmk***

Attached is a set of problems from past exams in this course that are related to the material on particle kinetics found in Chapter 5. I would like for us to use these problems to sharpen our skills on two issues: 1) deciding which method(s) to use in solving the problem (Newton/Euler equations, the work/energy equation, the linear impulse momentum equations and/or the angular impulse momentum equations), and 2) drawing the appropriate free body diagram(s) for solving the problems. I am calling these “rapid fire” questions in that these are two issues that you with which you need to be extremely comfortable, and that you can handle in a relatively short time at the beginning of the problem solution. Also attached here is page 352 of the course lecture book which can be helpful in guiding you on your choice of method(s) to use in solving.

With this, I challenge you to do these two steps on the following problems. You may continue on with the solution beyond these two steps of course; however, focus on developing skills in answering these two steps first before solving.

# Kinetics Summary

ME 274 - cmk

*Kinetics Table* (page 352 of the lecture book)

Method	Body model	Fundamental equations
<b>Newton-Euler</b> (relating forces to accelerations)	<b>particle</b>	$\sum \vec{F} = m\vec{a}$
	<b>rigid body</b> (G = c.m. and A = any point on body)	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
<b>Work-energy</b> (relating change in speed to change in position)	<b>particle</b>	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	<b>rigid body</b> (G = c.m. and A = any point on body)	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
<b>Linear impulse-momentum</b> (relating change in velocity to change in time)	<b>particle</b>	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_2 - m\vec{v}_1$
	<b>rigid body</b> (G = c.m.)	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
<b>Angular impulse-momentum</b> (relating change in angular velocity to change in time)	<b>particle</b> (O = fixed point)	$\int_{t_1}^{t_2} \sum \vec{M}_O dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	<b>rigid body</b> (A = fixed point or c.m.)	$\int_{t_1}^{t_2} \sum \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A \vec{\omega}$

Notes on the *four-step* method:

### 1. FBD(s)

- Newton/Euler (N/E): typically individual FBDs
- Work/energy (W/E), linear impulse momentum (LIM) and angular impulse/momentum (AIM): typically make it “BIG” (include all moving bodies)

### 2. Kinetics

(see suggestions in the table to the right as to which method(s) to use)

- Choose coordinate system(s) based on the “Given” and “Find”, including consideration of any motion constraints in the system.
- N/E: point G in the Newton equation must be the center of mass for the body
- W/E: mechanical energy is conserved if  $U_{1 \rightarrow 2}^{(nc)} = 0$  (no non-conservative work being done on your system)
- LIM: linear momentum in the x-direction is conserved if  $\sum F_x = 0$  (no net force in the x-direction for your system)
- AIM: angular momentum in the z-direction is conserved if  $\sum M_{Oz} = 0$  (no net moment in the z-direction about point O for your system). For particles, O must be a fixed point. For rigid bodies, O can be either a fixed point or the center of mass.

### 3. Kinematics

- N/E: typically need *acceleration* kinematics
- W/E, LIM and AIM: typically need *velocity* kinematics

### 4. Solve

## Examination No. 2

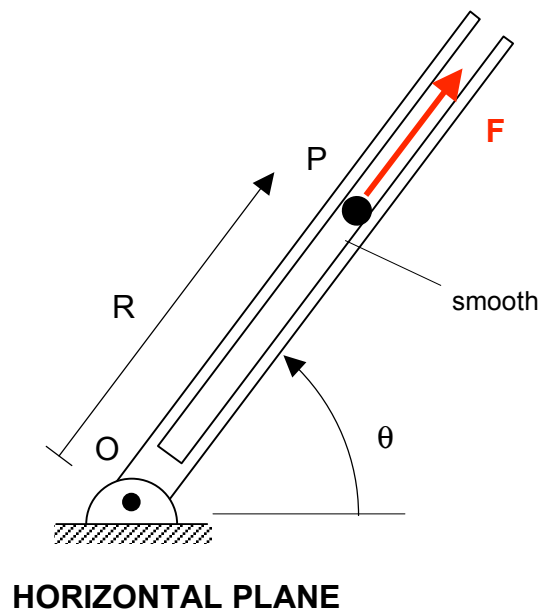
## PROBLEM NO. 1

**Given:** Pellet P having a mass of  $m$  is pulled through a barrel (having negligible mass) by means of radial force  $F = 60R$ , where  $F$  is in Newtons and  $R$  is in meters. The barrel is constrained to move in a HORIZONTAL plane by rotating about shaft passing through point O. The system is released with  $R = R_1$ ,  $\dot{R} = \dot{R}_1$  and  $\dot{\theta} = \dot{\theta}_1$ .

**Find:** For the instant when  $R = R_2$ :

- determine the rotation rate of the barrel,  $\dot{\theta}_2$ .
- determine the value of  $\dot{R}_2$ .

Use the following parameters in your analysis:  $m = 20\text{ kg}$ ,  $R_1 = 1.5\text{ meters}$ ,  $\dot{R}_1 = 4\text{ m/sec}$ ,  $\dot{\theta}_1 = 8\text{ rad/sec (CCW)}$  and  $R_2 = 3\text{ meters}$ .



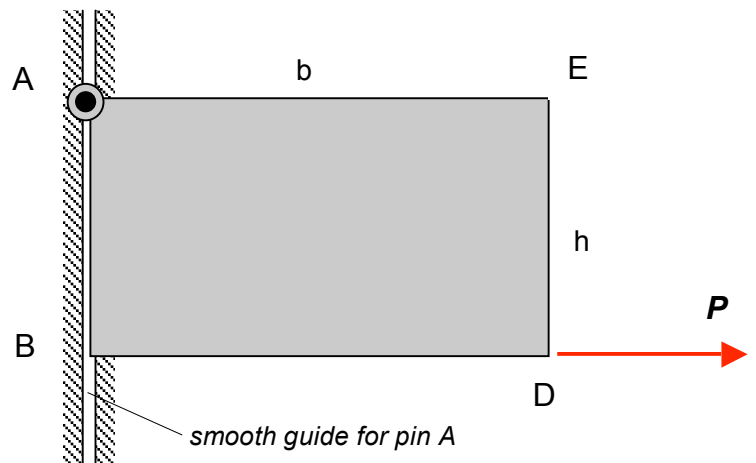
## Examination No. 2

## PROBLEM NO. 2

**Given:** A uniform plate of mass  $m$  is able to move on a smooth horizontal plane with corner A of the plate being constrained to move within a smooth guide in the plane of motion. The plate is initially at rest with edge AB aligned with the guide for A. A force  $P$  is applied at corner D, with  $P$  acting perpendicular to the guide for A.

**Find:** Determine the initial angular acceleration of the plate. Write your answer as a vector.

Use the following parameters in your analysis:  $m = 100\text{kg}$  ,  $b = 8\text{ meters}$  ,  $h = 6\text{ meters}$  , and  $P = 400\text{ N}$  .



**HORIZONTAL PLANE**

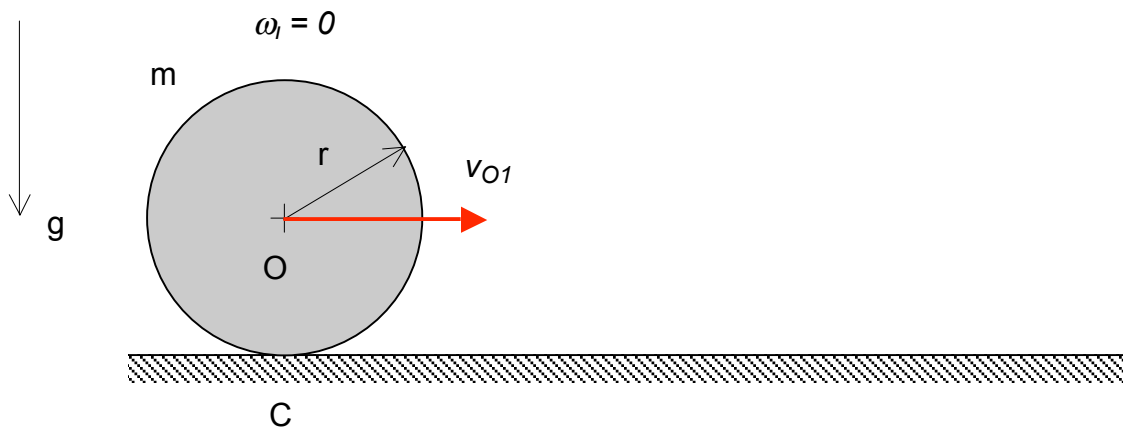
## Final Examination

## PROBLEM NO. 4

**Given:** The homogeneous disk shown below has a mass of  $m$  and outer radius of  $r$ . The disk is placed on a rough horizontal surface (coefficients of static and kinetic friction of  $\mu_s$  and  $\mu_k$ , respectively) with a uniform speed (i.e., zero angular velocity) of  $v_{O1}$  to the right.

**Find:** Determine the elapsed time,  $\Delta t$ , during which the disk travels to the right before slipping ceases between the disk and the horizontal surface. Note that slipping ceases when the contact point C has zero velocity.

Use the following parameters in your analysis:  $m = 100\text{kg}$ ,  $r = 0.5\text{ meters}$ ,  $\mu_s = 0.8$ ,  $\mu_k = 0.5$  and  $v_{O1} = 10\text{ m / sec}$ .



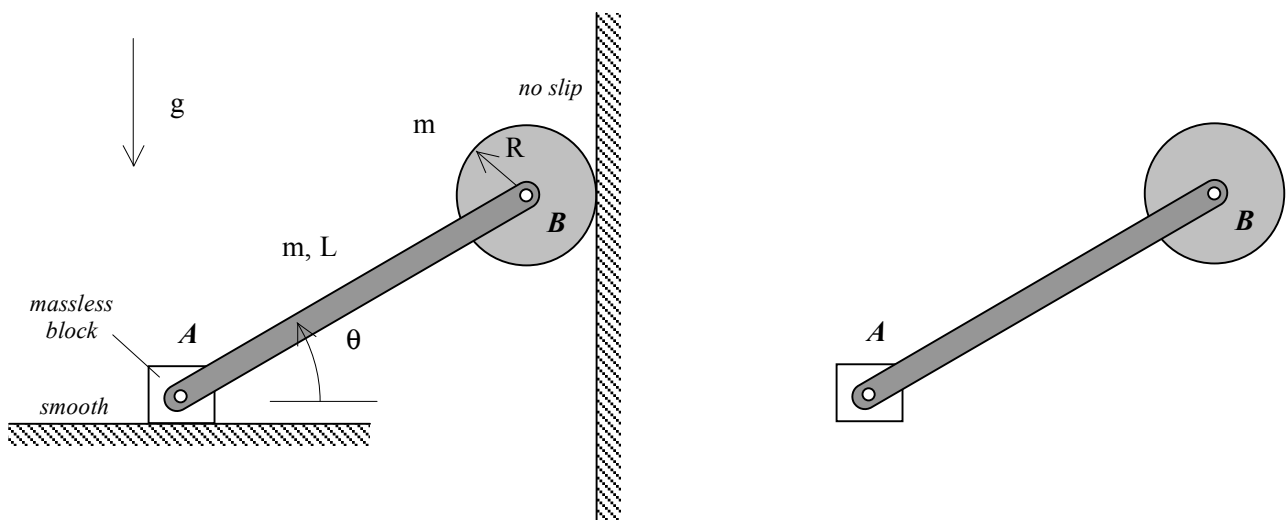
**ME 274 – Summer 2010**  
**Examination No. 2**  
**PROBLEM NO. 1**

**Name** \_\_\_\_\_

**Given:** System released from rest with  $\theta = 36.87^\circ$ . Consider the bar and the disk to be homogeneous.

**Find:** Determine the *angular velocity* of the bar when  $\theta = 0$ . Use:  $m = 100 \text{ kg}$ ,  $L = 2 \text{ meters}$  and  $R = 0.4 \text{ meters}$ .

Please clearly indicate the four steps in a neat and orderly presentation of your work.



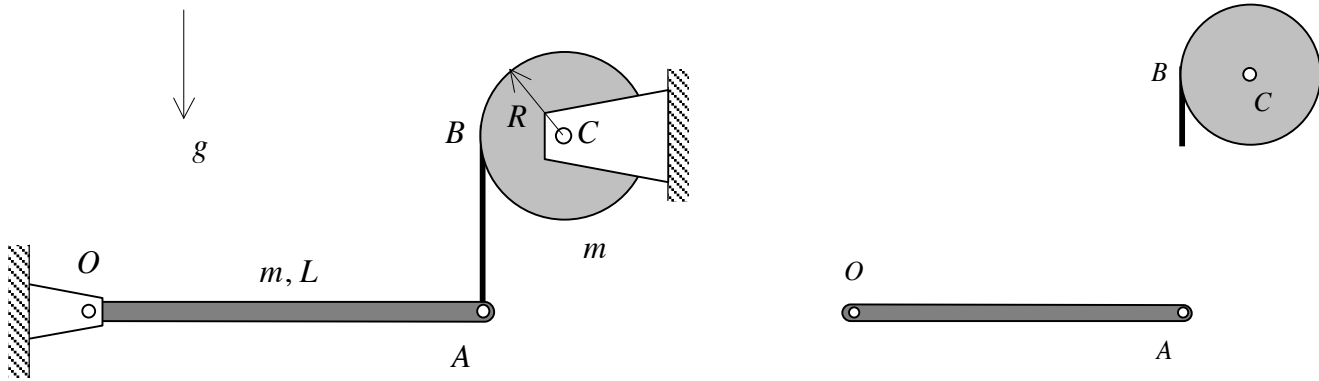
**ME 274 – Summer 2010**  
**Examination No. 2**  
**PROBLEM NO. 3**

**Name** \_\_\_\_\_

**Given:** Homogeneous bar OA (of length  $L$  and mass  $m$ ) is pinned to ground at end O. End A of the bar is connected to a cable that is wrapped around a homogeneous disk (of mass  $m$  and outer radius  $R$ ), with the disk being pinned to ground at its center C. Assume that the cable does not slip on the disk. The system is released from rest with OA being horizontal and the cable being vertical.

**Find:** Determine the *angular acceleration of the disk* on release. Use the following:  $m = 10\text{kg}$ ,  $L = 4\text{ meters}$  and  $R = 2\text{ meters}$ .

Please clearly indicate the four steps in a neat and orderly presentation of your work.



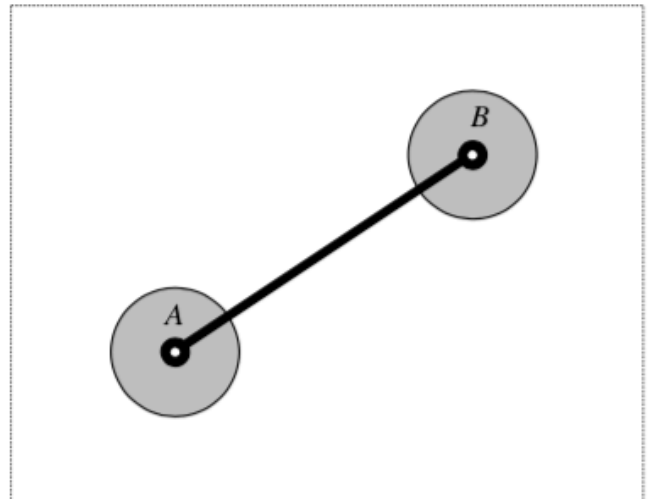
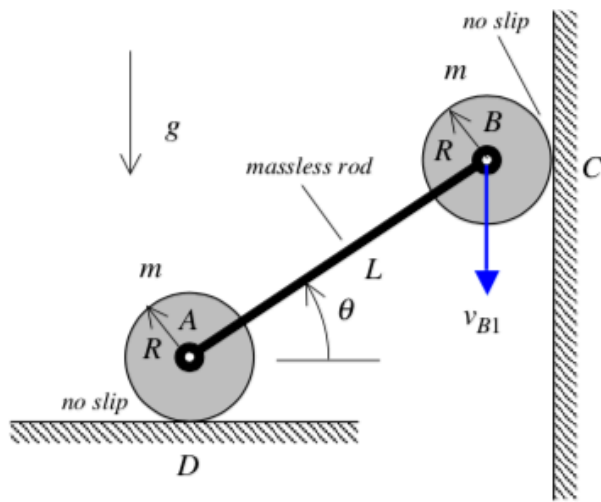
Final Exam

PROBLEM NO. 1

**Given:** System released at  $\theta = 53.13^\circ$  with the center of the disk moving downward with a speed of  $v_{B1}$ . Consider the bar and the disk to be homogeneous.

**Find:** Determine the *angular velocity* of disk B when  $\theta = 0$ . Use:  $m = 100 \text{ kg}$ ,  $L = 3 \text{ meters}$ ,  $v_{B1} = 4 \text{ m / sec}$  and  $R = 0.5 \text{ meters}$ .

Please clearly indicate the four steps in a neat and orderly presentation of your work.



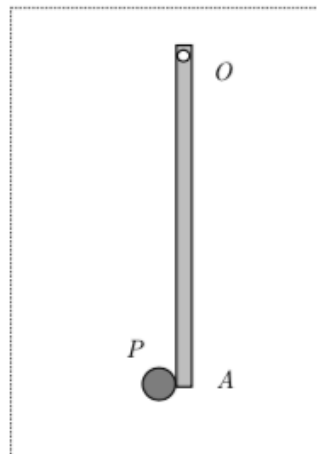
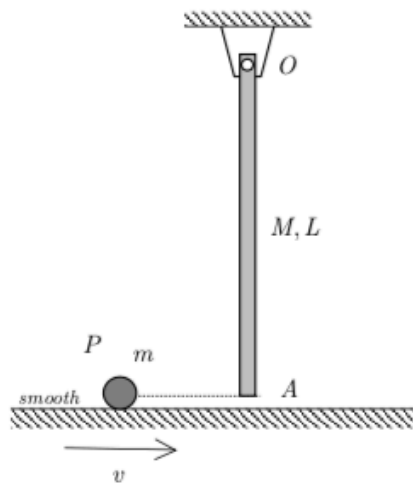
## Final Exam

## PROBLEM NO. 2

**Given:** Particle P (of mass  $m$ ) slides along a smooth horizontal surface with a speed of  $v$ . P then strikes end A of a stationary thin, homogeneous bar (of length  $L$  and mass  $M$ ) that is pinned to ground at O. The coefficient of restitution for the impact of P with end A of the bar is known to be  $e$ .

**Find:** Determine the angular speed of bar OA immediately after the impact. Use the following parameters in your analysis:  $m = 10\text{ kg}$ ,  $M = 30\text{ kg}$ ,  $L = 2\text{ meters}$ ,  $v = 40\text{ m / sec}$  and  $e = 0.5$ .

Please clearly indicate the four steps in a neat and orderly presentation of your work.



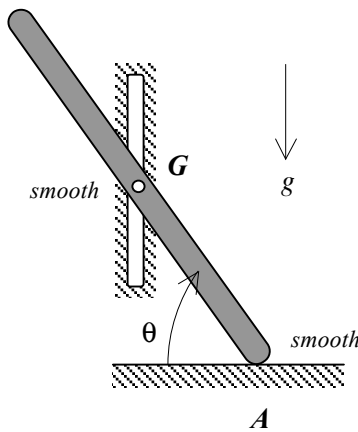
## Examination No. 2

## PROBLEM NO. 1

**Given:** The system shown below is released from rest. The bar is homogeneous with a mass of  $m$  and a length of  $L$ . The center of the mass of the bar,  $G$ , is constrained to move within a smooth vertical guide. End A of the bar is constrained to move on a smooth horizontal surface.

**Find:** Determine the *angular acceleration* of the bar on release. Use the following parameter values in your work:  $m = 100 \text{ kg}$ ,  $L = 2 \text{ meters}$  and  $\theta = 53.13^\circ$ .

Please clearly indicate the four steps in a neat and orderly presentation of your work.



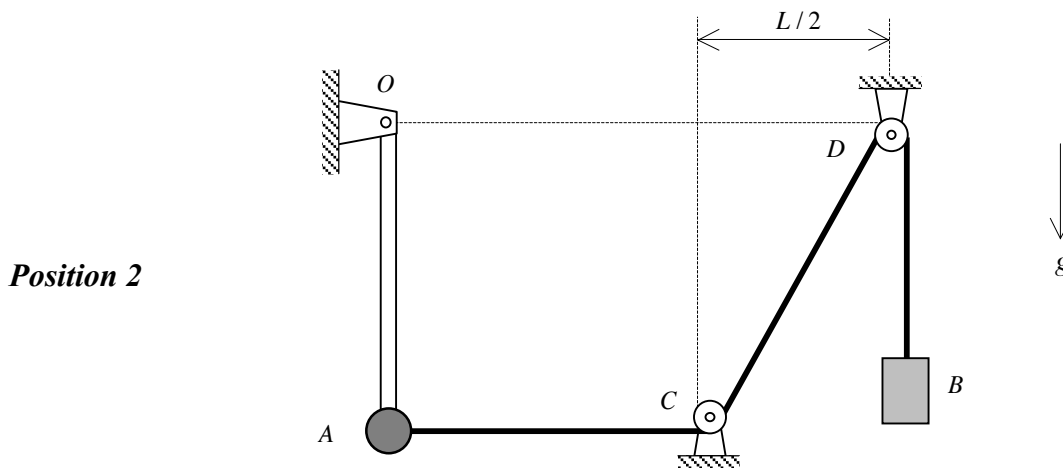
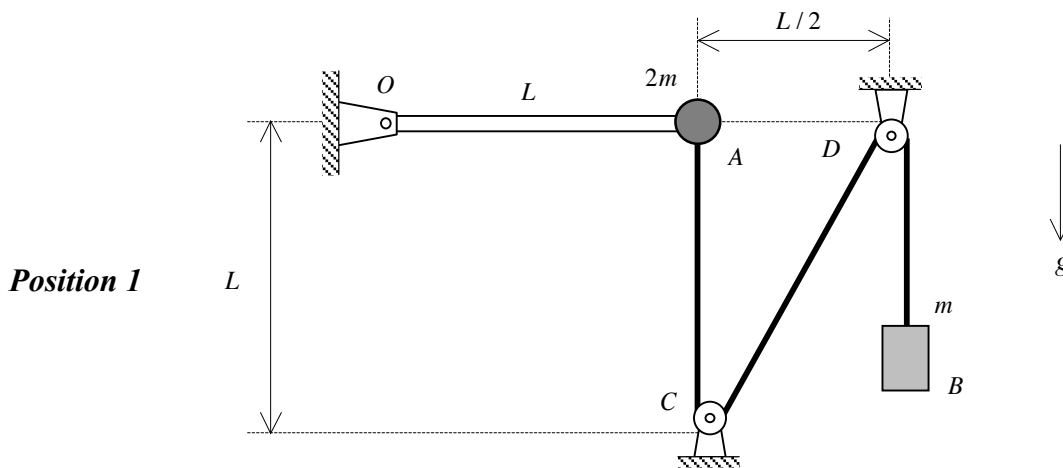
Examination No. 2

PROBLEM NO. 3

**Given:** Particle A (of mass  $2m$ ) is attached to a rigid bar of negligible mass. Particle A is also connected to a cable that is wrapped around two pulleys and connected to particle B on its other end. The system is released from rest with OA being horizontal and with section AC of the cable being vertical. Assume that the radii of the pulleys to negligible.

**Find:** Determine the *angular velocity* of the bar at Position 2 where it has rotated  $90^\circ$  CW to a vertical orientation. (At Position 2, section AC of the cable is horizontal.) Use the following parameter values in your work:  $m = 10\text{kg}$  and  $L = 4\text{ meters}$ .

Please clearly indicate the four steps in a neat and orderly presentation of your work.



**PLEASE START YOUR ANALYSIS ON THE NEXT PAGE.**

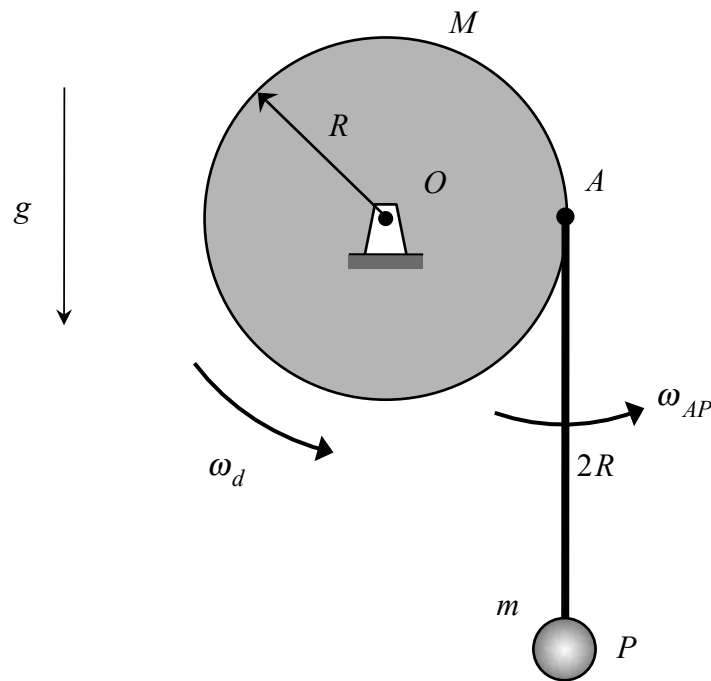
## Examination No. 2

## PROBLEM NO. 1

**Given:** A homogeneous disk with an outer radius of  $R$  and of mass  $M$  is pinned to ground at its center  $O$ . Rigid link  $AP$ , of negligible mass and length  $2R$ , is pinned to the outer perimeter of the disk at  $A$ . Particle  $P$  (of mass  $m$ ) is attached to the free end of link  $AP$ . At the instant shown below:  $A$  is directly to the right of  $O$ ;  $P$  is directly below  $A$ ; link  $AP$  is rotating in the counterclockwise sense with an angular speed of  $\omega_{AP}$ ; and, the disk is rotating in the counterclockwise sense with an angular speed of  $\omega_d$ .

**Find:** For the position shown, determine the angular acceleration of the disk and the angular acceleration of link  $AP$ . Write your answers as vectors. Leave your answers in terms of:  $m$ ,  $M$ ,  $R$ ,  $g$ ,  $\omega_d$  and  $\omega_{AP}$ .

Please clearly indicate the four steps in a neat and orderly presentation of your work.




---

Work appearing above this line will NOT be graded.

## Final Examination

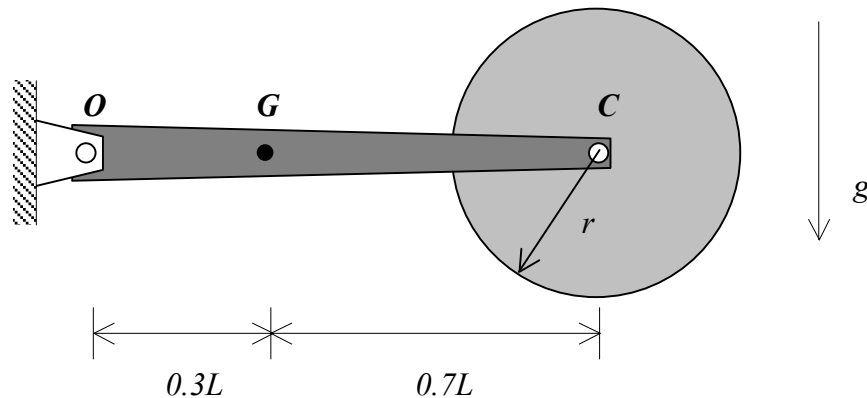
## PROBLEM NO. 2

**Given:** Link OC and the homogeneous circular disk are released from rest when OC is horizontal. The disk is pinned at its center to end C of link OC. Link OC has a mass of  $m$  and a radius of gyration about its center of mass G of  $k_G$ . The disk has a mass of  $M$  and an outer radius of  $r$ . Assume all joints to be smooth.

**Find:** Determine the angular speed of link OC when OC has rotated to a vertical orientation.

Use the following:  $m = 10\text{kg}$ ,  $L = 1.2\text{ meters}$ ,  $k_G = 0.3$ ,  $M = 8\text{kg}$  and  $r = 0.4\text{ meters}$ .

Please clearly indicate the four steps in a neat and orderly presentation of your work.



---

Work appearing above this line will NOT be graded.

## Final Examination

## PROBLEM NO. 1 – 20 points

**Given:** A homogeneous disk of mass  $m$  and outer radius  $R$  is attached to a horizontal shaft at the disk's center  $O$ . An inextensible cable is wrapped around the outer radius of the disk, as shown. Block A (having a mass of  $2m$ ) is attached to one end of the cable, and block B (having a mass of  $m$ ) is attached to the other end of the cable. A second cable is attached to block B, with a force  $F$  applied to the other end of this cable. The system is released from rest. Assume that the cable does not slip on the disk, and that the pulley around which the second cable is wrapped has negligible mass.

**Find:** For this problem:

- Draw individual free body diagrams of the disk, block A and block B.
- Determine the angular acceleration of the disk on release. Write your answer as a vector.

