

Equation Sheet

Kinematics

$$\begin{aligned}\vec{v} &= \dot{x}\hat{i} + \dot{y}\hat{j} \\ &= v\hat{e}_t \\ &= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\end{aligned}$$

$$\begin{aligned}\vec{a} &= \ddot{x}\hat{i} + \ddot{y}\hat{j} \\ &= \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n \\ &= (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta\end{aligned}$$

$$\begin{aligned}\vec{v}_B &= \vec{v}_A + \vec{v}_{B/A} \\ &= \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A} \\ &= \vec{v}_A + (\vec{v}_{B/A})_{rel} + \vec{\omega} \times \vec{r}_{B/A}\end{aligned}$$

$$\begin{aligned}\vec{a}_B &= \vec{a}_A + \vec{a}_{B/A} \\ &= \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A} \\ &= \vec{a}_A + (\vec{a}_{B/A})_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} \\ &\quad + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})\end{aligned}$$

$$\frac{dv}{dt} = v \frac{dv}{ds}$$

Kinetics

$$\sum \vec{F} = m\vec{a}_G; \quad \sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$$

$$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$$

$$V_{gr} = mgh$$

$$V_{sp} = \frac{1}{2}k\Delta^2$$

$$T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$$

$$U_{1 \rightarrow 2}^{(nc)} = \int_1^2 (\sum \vec{F} \cdot \hat{e}_t) ds = \int_1^2 (F_x dx + F_y dy)$$

Kinetics (continued)

$$e = - \left(\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}} \right)$$

$$\int_1^2 \sum \vec{F}_{ext} dt = \left(\sum_i m_i \vec{v}_i \right)_2 - \left(\sum_i m_i \vec{v}_i \right)_1$$

$$\int_1^2 \sum \vec{M}_A dt = \left(\sum_i (\vec{H}_A)_i \right)_2 - \left(\sum_i (\vec{H}_A)_i \right)_1$$

$$\begin{aligned}(\vec{H}_A)_i &= \vec{r}_{i/A} \times (m_i \vec{v}_i) \\ &= I_A \vec{\omega}\end{aligned}$$

Vibrations

$$M\ddot{x} + C\dot{x} + Kx = F(t) = F_0 \sin \omega t \quad \text{OR} \quad F_0 \cos \omega t$$

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = \frac{1}{M} F(t)$$

$$x_c(t) = e^{-\zeta\omega t} (C \cos \omega_d t + S \sin \omega_d t); \quad 0 \leq \zeta < 1$$

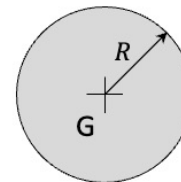
$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$x_p(t) = A \sin \omega t + B \cos \omega t$$

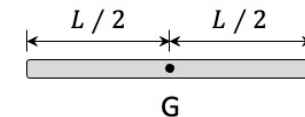
Other

$$I_A = I_G + md^2$$

$$I_A = mk_A^2$$



$$I_G = \frac{1}{2}mR^2$$



$$I_G = \frac{1}{12}mL^2$$