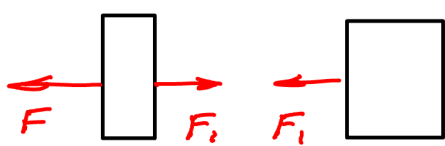
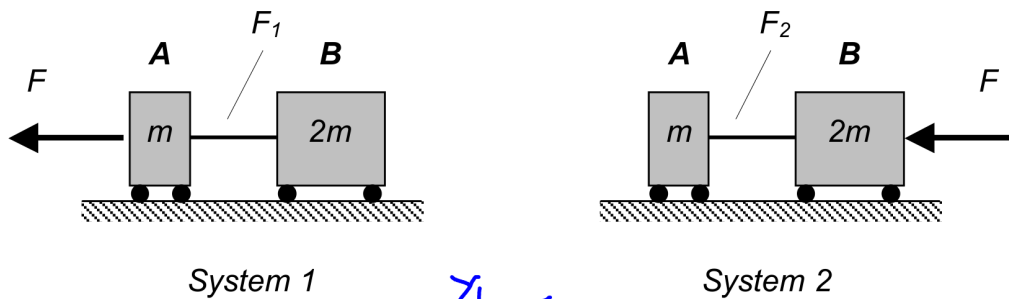


Question C4.1

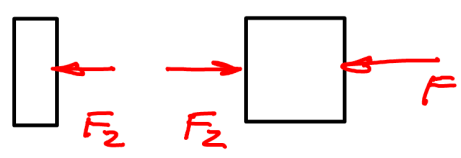
Blocks A and B (having masses of m and $2m$, respectively) are connected by a lightweight, rigid rod. In System 1, a force F acts to the left on block A. In System 2, the same force acts to the left on block B. Let F_1 and F_2 represent the magnitude of the load carried by the rod in Systems 1 and 2, respectively. Circle the answer below that most accurately represents the magnitudes of F_1 and F_2 :

- (a) $F_1 > F_2$
- (b) $F_1 = F_2$
- (c) $F_1 < F_2$
- (d) More information is needed to answer this question

Provide a justification for your answer.



A: $\sum F_x = -F + F_1 = ma$
 B: $\sum F_x = -F_1 = 2ma$



A: $\sum F_x = -F_2 = ma$
 B: $\sum F_x = F_2 - F = 2ma$

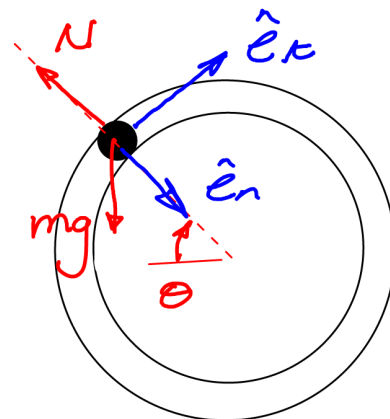
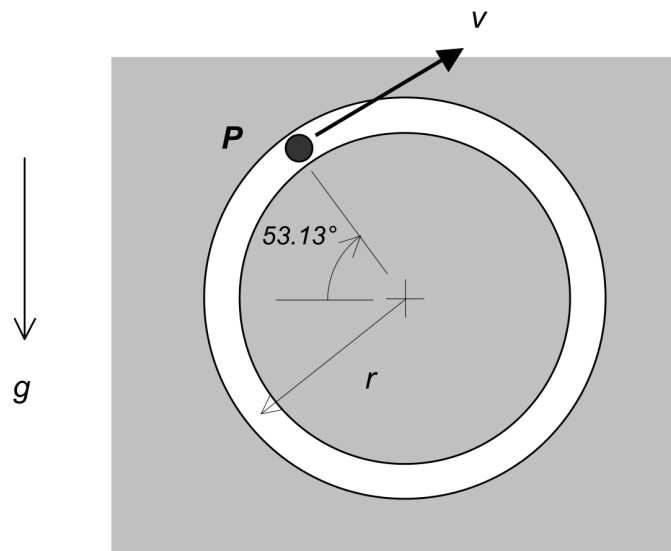
Both 1 & 2 have the same acceleration a .
 Solve for F_1 & F_2

Question C4.2

Particle P travels in a vertical plane within a smooth, circular slot, where the radius of the slot is $r = 0.5$ m. At the position shown below, the speed of P is known to be $v = 3$ m/s. For this position:

- (a) P is in contact with the outer surface of the slot.
- (b) P is in contact with the inner surface of the slot.
- (c) P is in contact with neither surface of the slot.
- (d) More information is needed to answer this question

Provide a justification for your answer.



$$\sum F_n = -N + mg \sin \theta = m \frac{v^2}{r}$$

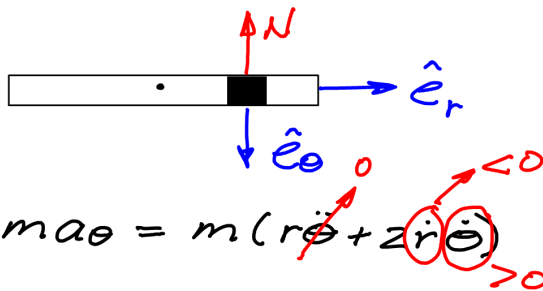
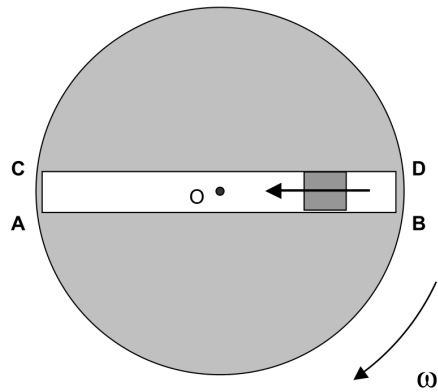
Solve for N & consider sign of N .

Question C4.3

A disk rotates in a clockwise sense with a constant rate of ω about a vertical shaft that passes through the center O of the disk. A particle is moving radially inward toward the shaft O. On which surface of the slot does the particle make contact?

- (a) Surface AB.
- (b) Surface CD.
- (c) Neither surface.
- (d) More information is needed to answer this question.

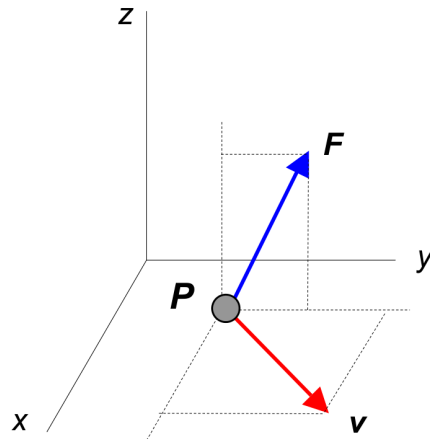
Provide a justification for your answer.



$$\sum F_\theta = -N = ma_\theta = m(\cancel{r\ddot{\theta}} + z\dot{r}\dot{\theta})$$

Question C4.4

Particle P (having a mass of $m = 4$ kg) is traveling with a velocity of $\vec{v} = (15\hat{i} + 20\hat{j})$ m/s. A net force of $\vec{F} = (100\hat{i} + 280\hat{k})$ N acts on P. Determine the rate of change of speed of the particle. Is the speed of P increasing, decreasing or constant at this instant? Justify your answer.

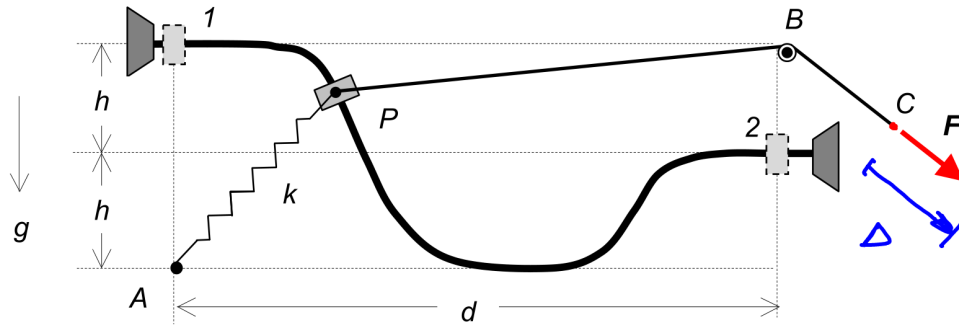


$$\dot{v} = \vec{a} \cdot \hat{e}_t = \left(\frac{\vec{F}}{m}\right) \cdot \frac{\vec{v}}{|\vec{v}|}$$

Look at sign of \dot{v} to see if increasing or decreasing in speed.

Question C4.5

Particle P (of mass m) starts at position 1 with a speed of v_1 and moves to position 2 on a rough guide (coefficient of kinetic friction of μ_k). A spring of stiffness k is attached between P and the fixed point A. The spring is unstretched at position 1. A cable is also attached to P with the cable being pulled over a small pulley at B by a constant force F acting at end C of the cable. At position 2, the particle has a speed of v_2 . Answer the following questions related to the motion of P.



Work done by friction, $U_{1 \rightarrow 2}^{(f)}$: *friction opposes motion $\Rightarrow \vec{f} \cdot \tilde{e}_t < 0$*

- $U_{1 \rightarrow 2}^{(f)} > 0$
- $U_{1 \rightarrow 2}^{(f)} = 0$
- $U_{1 \rightarrow 2}^{(f)} < 0$
- more information is needed about the shape of the guide in order to determine the sign of $U_{1 \rightarrow 2}^{(f)}$.

Work done by the force F , $U_{1 \rightarrow 2}^{(F)}$: *$U_{1 \rightarrow 2} = F\Delta$; use geometry to find Δ (see defn. of Δ above)*

- $U_{1 \rightarrow 2}^{(F)} = Fd$
- $U_{1 \rightarrow 2}^{(F)} = Fh$
- $U_{1 \rightarrow 2}^{(F)} = 2Fh$
- $U_{1 \rightarrow 2}^{(F)} = F(d + 2h)$
- $U_{1 \rightarrow 2}^{(F)} = F(d - h)$
- $U_{1 \rightarrow 2}^{(F)} = F(d - 2h)$
- $U_{1 \rightarrow 2}^{(F)} = F\left(\sqrt{d^2 + h^2}\right)$
- more information is needed about the shape of the guide to determine $U_{1 \rightarrow 2}^{(F)}$.

Question C4.5 (continued)

Spring potential energy at position 2, $(V_2)_{sp}$: $(V_2)_{sp} = \frac{1}{2} k \Delta^2$ (see Δ from earlier question)

- a) $(V_2)_{sp} = \frac{1}{2} k d^2$
- b) $(V_2)_{sp} = \frac{1}{2} k h^2$
- c) $(V_2)_{sp} = \frac{1}{2} k (d - 2h)^2$
- d) $(V_2)_{sp} = \frac{1}{2} k (d^2 - 4h^2)$
- e) $(V_2)_{sp} = \frac{1}{2} k (\sqrt{d^2 + h^2} - 2h)^2$
- f) more information is needed about the shape of the guide in order to determine $(V_2)_{sp}$.

Change in gravitational potential, $\Delta V_{gr} = (V_2)_{gr} - (V_1)_{gr}$: lower potential for 2 (below position at 1)

- a) $\Delta V_{gr} > 0$
- b) $\Delta V_{gr} = 0$
- c) $\Delta V_{gr} < 0$
- d) more information is needed about the shape of the guide in order to determine the sign of ΔV_{gr} .

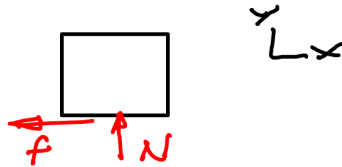
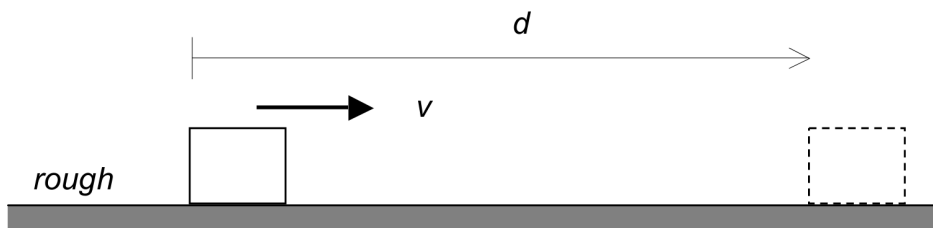
Speed of P at position 2, v_2 , as compared to the speed at position 1, v_1 : work done by friction depends on length and shape of path.

- a) $v_2 > v_1$
- b) $v_2 = v_1$
- c) $v_2 < v_1$
- d) more information is needed about problem in order to compare v_1 and v_2 .

Question C4.6

A block on a rough horizontal surface is given an initial velocity to the right with a speed of $v = 3$ m/s. The block comes to rest after it has slid a distance of $d = 2$ m. If the same block is given an initial speed of $v = 6$ m/s, how far will it slide to the right before stopping?

- (a) 2 m
- (b) 4 m
- (c) 6 m
- (d) 8 m
- (e) 9 m
- (f) The numerical value for the coefficient of kinetic friction is needed to answer this question.



$$\sum F_x = -f = m \frac{dv}{dt} = m v \frac{dv}{ds}$$

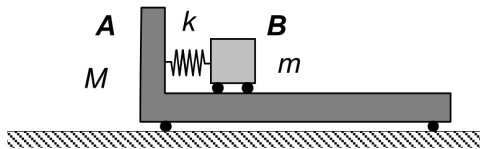
$$\hookrightarrow m \int_{v_0}^0 v dv = -f \int_0^d ds$$

$$-\frac{1}{2} m v_0^2 = -fd$$

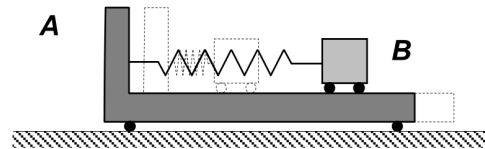
$$\hookrightarrow f = \frac{m v_0^2}{2d}$$

Question C4.7

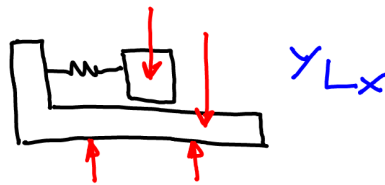
Cart A and block B (having masses of $M = 4 \text{ kg}$ and $m = 2 \text{ kg}$, respectively, are connected by a spring of stiffness $k = 300 \text{ N/m}$. The system is released from rest with the spring being compressed 0.2 m (Position 1). Find the speed of the cart at Position 2 at the instant when the spring is uncompressed/unstretched. Consider all surfaces to be smooth.



Position 1
(at rest)



Position 2
(both A and B moving)



$$\bullet \sum F_x = 0 \Rightarrow M V_{A1} + m V_{B1} = M V_{A2} + m V_{B2}$$

$$\hookrightarrow V_{B2} = -\frac{M}{m} V_{A2} \quad (1)$$

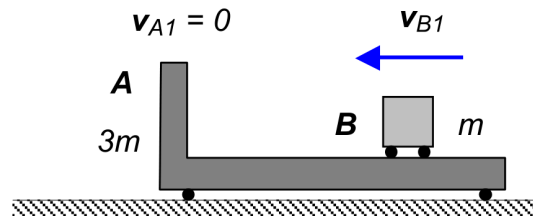
$$\bullet T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} k \Delta_1^2 = \frac{1}{2} m V_{B2}^2 + \frac{1}{2} M V_{A2}^2 \quad (2)$$

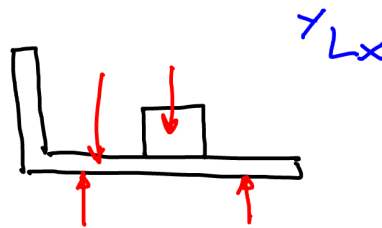
• Solve (1) and (2) for V_{A2}

Question C4.8

Prior to impact, block B is moving to the left with a speed of v_{B1} and cart A is stationary. After impact, block B is stationary, and block A moves to the left with a speed of v_{A2} . What is the numerical value for the coefficient of restitution e for this impact? Consider all surfaces to be smooth.



Position 1
(before impact)



$$\begin{aligned} \bullet \quad \sum F_x = 0 &\Rightarrow 3m v_{A1} + m v_{B1} = 3m v_{A2} + m v_{B2} \\ &\hookrightarrow 3v_{A2} + v_{B2} = v_{B1} \quad (1) \end{aligned}$$

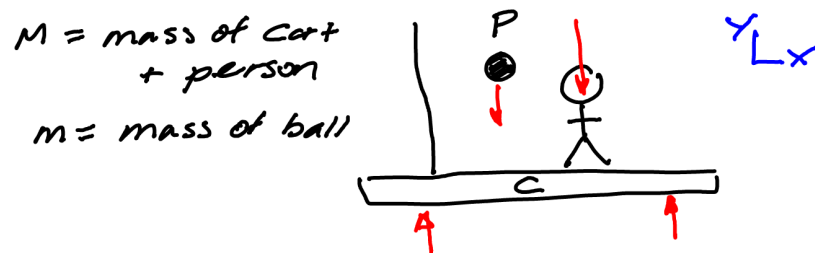
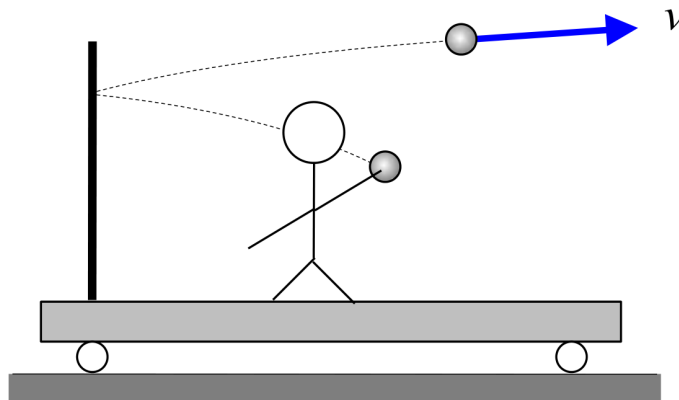
$$\bullet \quad e = \frac{v_{B2} - v_{A2}}{v_{A1} - v_{B1}} = \frac{v_{A2} - v_{B2}}{v_{B1}} \quad (2)$$

Solve (1) & (2) for e .

Question C4.9

You are on a cart that is initially at rest on a smooth track. You throw a ball at a partition that is rigidly mounted on the cart. If the ball bounces off the partition as shown in the figure, then at the instant shown in the figure:

- (a) The cart is moving to the right
- (b) The cart is stationary
- (c) The cart is moving to the left
- (d) More information is needed about the impact of the ball with the partition in order to answer this question

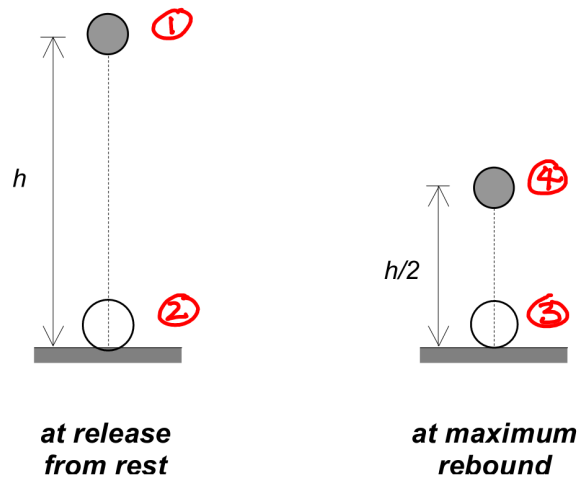


$$\sum F_x = 0 \Rightarrow m v_{px} + M v_{cx} = m v_{p2x} + M v_{c2}$$

$$\hookrightarrow v_{c2} = -\frac{m}{M} v_{p2x}$$

Question C4.10

A sphere of mass m is dropped from rest from a height of h . After impacting a rigid, immovable horizontal surface, the sphere is known to rebound to a height of $h/2$, regardless of the numerical value of h . What is the coefficient of restitution for the impact of the sphere with the surface?



$$\frac{1}{2} m v_2^2 = mgh \Rightarrow v_2 = \sqrt{2gh}$$

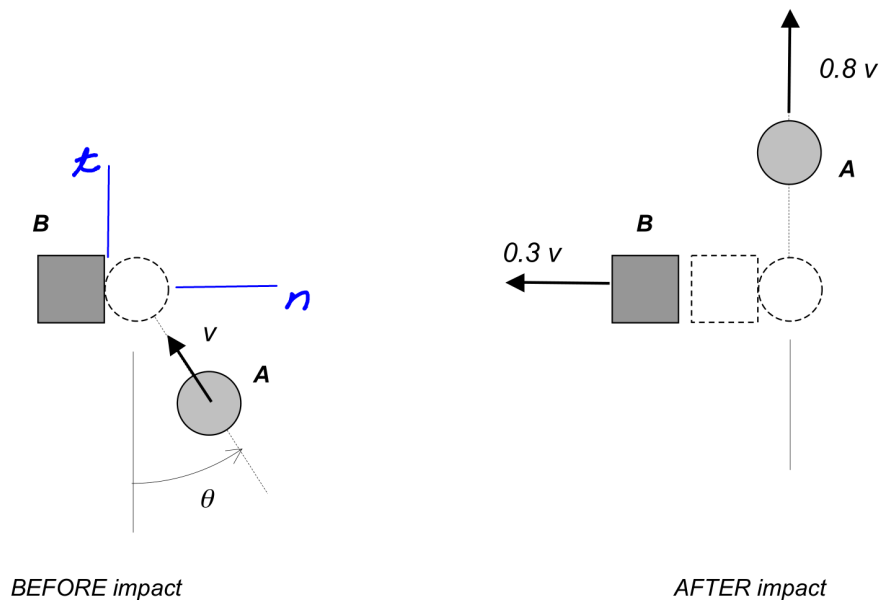
$$e = \frac{v_3}{v_2} \Rightarrow v_3 = ev_2 = e\sqrt{2gh}$$

$$\frac{1}{2} m v_3^2 = mg \frac{h}{2} \Rightarrow 2ghe^2 = gh$$
$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

Question C4.11

Particle A (having a mass of m_A) has a speed of v when it impacts a stationary particle B (having a mass of m_B). After impact, A and B have speeds of $0.8v$ and $0.3v$, respectively, and along the directions shown below. Circle the answer that most accurately represents the relative sizes of the masses of A and B:

- (a) $m_A = 0.3m_B$
- (b) $m_A = 0.5m_B$
- (c) $m_A = m_B$
- (d) $m_A = 2m_B$
- (e) The numerical value for the coefficient of restitution for the impact is needed to answer this question.



$$\underline{A}: \sum F_t = 0 \Rightarrow m_A v \cos \theta = m_A (0.8v)$$

$$\hookrightarrow \cos \theta = 0.8 \Rightarrow \sin \theta = 0.6$$

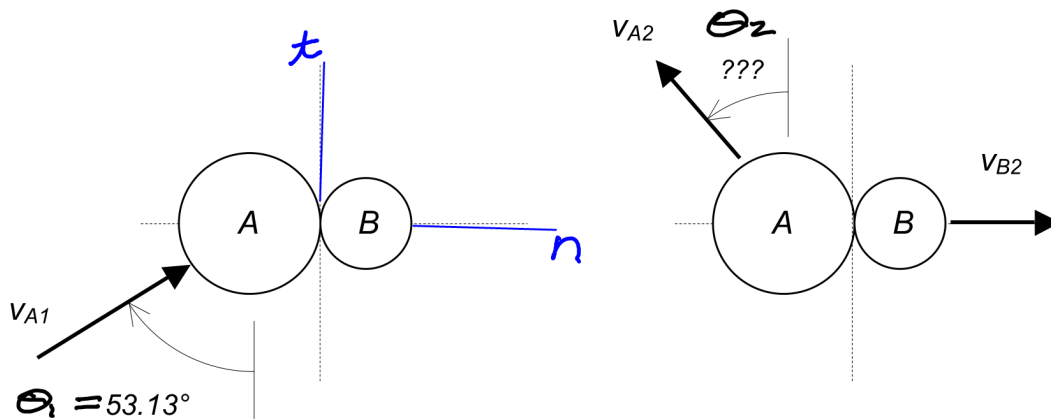
$$\underline{A+B}: \sum F_n = 0 \Rightarrow -m_A v \sin \theta = -m_B (0.3v)$$

$$\hookrightarrow 0.6 m_A = 0.3 m_B$$

$$\hookrightarrow m_A = \frac{1}{2} m_B$$

Question C4.12

Particle A strikes a stationary particle B with a speed of $v_{A1} = 10$ ft/s in the direction shown below on the left. After impact, A has a speed of $v_{A2} = 6$ ft/s, with the direction of travel for A after impact being unknown, and B has a speed of $v_{B2} = 4$ ft/s. All motion of the particles is in a horizontal plane, and the contact surface between A and B is smooth. Determine the numerical value for the coefficient of restitution for the impact of A and B.



BEFORE impact

AFTER impact

$$\underline{A}: \sum F_t = 0 \Rightarrow m_A v_{A1} \cos \theta_1 = m_A v_{A2} \cos \theta_2 \quad (1)$$

$$\underline{A+B}: \sum F_n = 0 \Rightarrow m_A v_{A1} \sin \theta_1 = -m_A v_{A2} \sin \theta_2 + m_B v_{B2} \quad (2)$$

$$e = \frac{v_{B2} - (-v_{A2} \sin \theta_2)}{v_{A1} \sin \theta_1} = \frac{v_{B2} + v_{A2} \sin \theta_2}{v_{A1} \sin \theta_1} \quad (3)$$

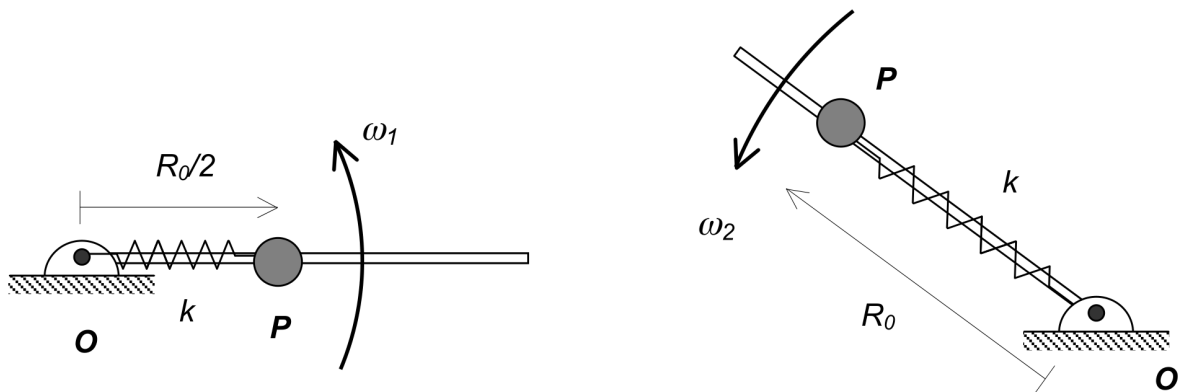
Solve (1) & (3) for e .

Question C4.13

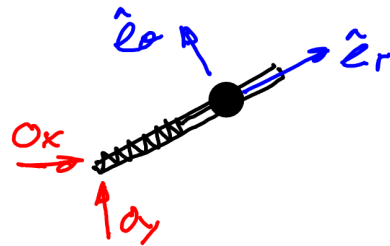
A particle P is free to slide on a smooth, lightweight bar. The bar is free to rotate in a horizontal plane about a vertical axis passing through end O of the bar. A spring of stiffness k and unstretched length R_0 is connected between P and O. The spring is compressed to half of its unstretched length and released when the bar has a rotational speed of ω_1 . After release, P reaches a position when the spring is unstretched. At this position, the rotational speed of the bar is ω_2 .

Suppose now the experiment is repeated except the stiffness of the spring is doubled to a value of $2k$. As a result of this change, the value of ω_2 is now:

- (a) Decreased
- (b) The same
- (c) Increased
- (d) More information is needed in order to answer this question.



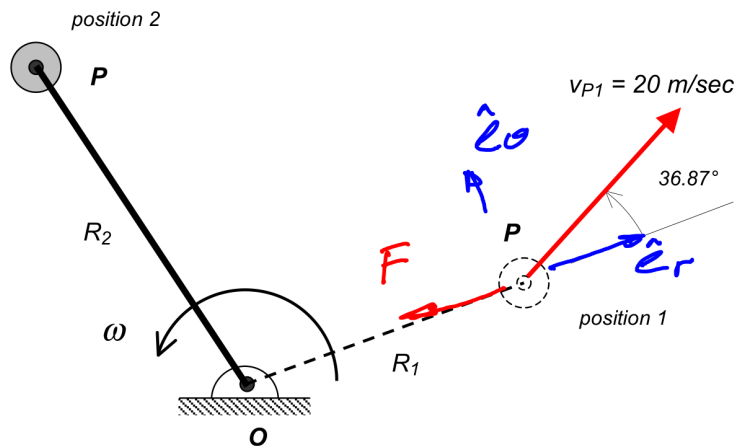
HORIZONTAL PLANE



$$\begin{aligned} \sum M_o = 0 &\Rightarrow \vec{H}_o = \text{const} \\ \omega / \vec{H}_o &= m \vec{r}_{p/o} \times \vec{v}_p \\ &= m(R \hat{e}_r) \times (\dot{R} \hat{e}_r + R\omega \hat{e}_\theta) \\ &= mR^2\omega \hat{k} \leftarrow \text{ind. of } k \end{aligned}$$

Question C4.14

Particle P (having a mass of m) is able to slide on a smooth horizontal surface. An extensible cord (having a stiffness of $k = 50 \text{ N/m}$ and unstretched length of $R_0 = 2 \text{ m}$) is attached between P and a fixed point O in the plane of motion for P. At position 1, P is released with a velocity as shown below with $R_1 = 2 \text{ m}$. Assuming that the cord remains taut for all time, find the angular speed ω of the cord about point O when P is at position 2 where $R_2 = 4 \text{ m}$.



$$\sum \vec{M}_O = \vec{0} \Rightarrow \vec{H}_O = \text{const.}$$

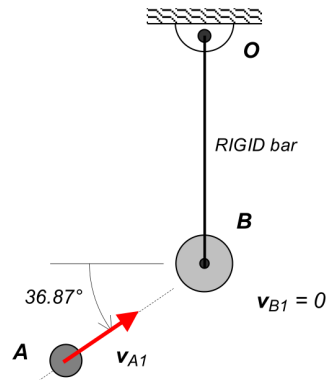
$$\begin{aligned} \omega \quad \vec{H}_O &= m \vec{r}_{P/O} \times \vec{v}_P = m(R \vec{e}_r) \times (\dot{R} \vec{e}_r + R\omega \vec{e}_\theta) \\ &= mR^2 \omega \vec{k} \end{aligned}$$

$$\Rightarrow \vec{H}_{O1} = \vec{H}_{O2} \Rightarrow mR_1^2 \omega_1 = mR_2^2 \omega_2$$

$$\hookrightarrow \omega_2 = \left(\frac{R_1}{R_2}\right)^2 \omega_1$$

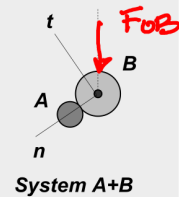
Question C4.15

Particle B is attached to a rigid bar BO with BO pinned to ground at O. Particle A strikes the stationary particle B with a speed of v_{A1} in the direction shown. The coefficient of restitution for this impact is $e < 1$. Consider all surfaces to be smooth and all motion to be in a horizontal plane. Respond to the following true/false questions below.



For System A+B

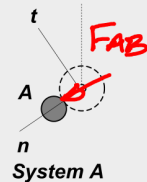
linear momentum in the n -direction is conserved: TRUE or FALSE
 linear momentum in the t -direction is conserved: TRUE or FALSE
 angular momentum about point O is conserved: TRUE or FALSE
 mechanical energy is conserved: TRUE or FALSE



$$\begin{aligned} \sum F_n &\neq 0 \\ \sum F_t &\neq 0 \\ \sum M_O &= 0 \\ e &\neq 1 \end{aligned}$$

For System A

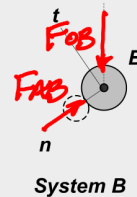
linear momentum in the n -direction is conserved: TRUE or FALSE
 linear momentum in the t -direction is conserved: TRUE or FALSE
 angular momentum about point O is conserved: TRUE or FALSE
 mechanical energy is conserved: TRUE or FALSE



$$\begin{aligned} \sum F_n &\neq 0 \\ \sum F_t &= 0 \\ \sum M_O &\neq 0 \\ F_{AB} &\neq 0 \end{aligned}$$

For System B

linear momentum in the n -direction is conserved: TRUE or FALSE
 linear momentum in the t -direction is conserved: TRUE or FALSE
 angular momentum about point O is conserved: TRUE or FALSE
 mechanical energy is conserved: TRUE or FALSE

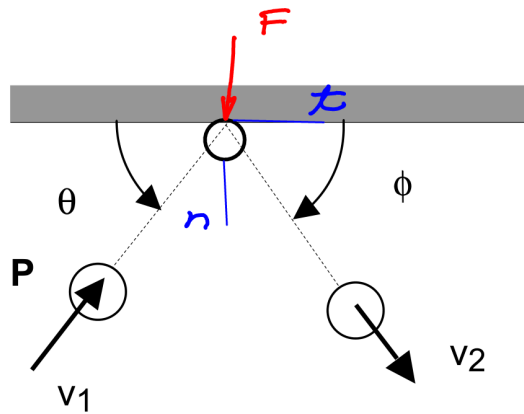


$$\begin{aligned} \sum F_n &\neq 0 \\ \sum F_t &\neq 0 \\ \sum M_O &\neq 0 \\ F_{AB} &\neq 0 \end{aligned}$$

Question C4.16

Particle P (of mass m) is traveling on a smooth horizontal surface with a speed of v_1 and angle θ when it strikes a smooth wall. The coefficient of restitution between the wall and the particle is $0 < e < 1$. Circle the answer below that most accurately describes the angle ϕ at which the particle rebounds from the wall.

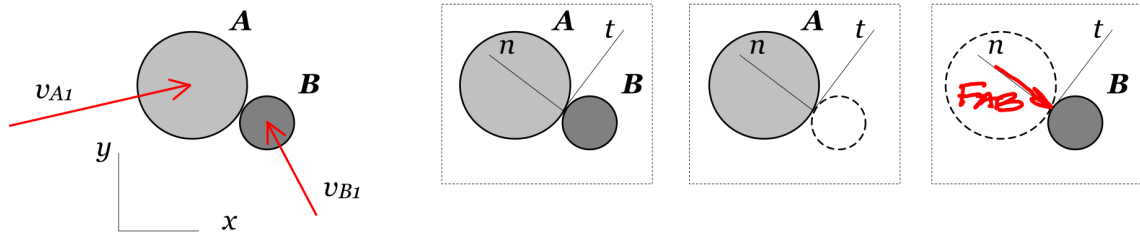
- (a) $\phi < \theta$
- (b) $\phi = \theta$
- (c) $\phi > \theta$
- (d) $\phi = 0$
- (e) $\phi = 90^\circ$



$$\sum F_x = 0 \Rightarrow m v_1 \cos \theta = m v_2 \cos \phi \Rightarrow \frac{v_2}{v_1} = \frac{\cos \theta}{\cos \phi}$$
$$e = \frac{v_2 \sin \phi}{v_1 \sin \theta} = \frac{\tan \phi}{\tan \theta}$$

Question C4.17

Particles A and B impact each other as shown below.



Circle all correct responses below (there may be more than one correct response).

For the system made up of B alone during impact:

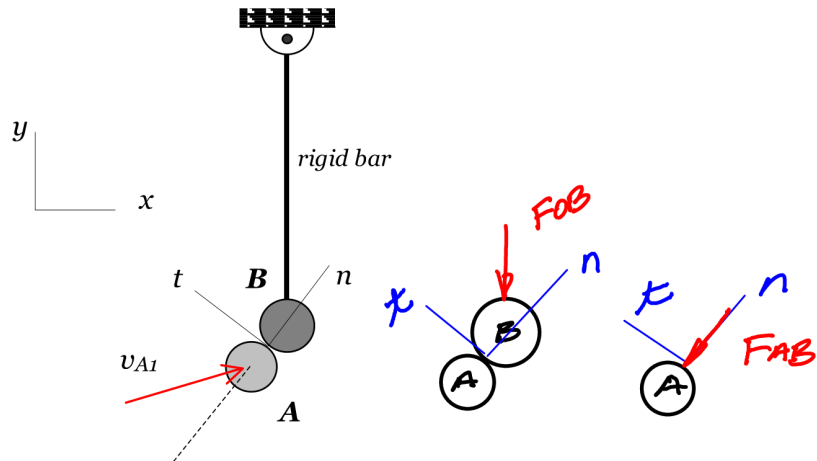
- (a) linear momentum in the x -direction is conserved $\Sigma F_x \neq 0$
- (b) linear momentum in the y -direction is conserved $\Sigma F_y \neq 0$
- (c) linear momentum in the n -direction is conserved $\Sigma F_n \neq 0$
- (d) linear momentum in the t -direction is conserved $\Sigma F_t = 0$
- (e) none of the above

For the system made up of A+B during impact:

- (a) linear momentum in the x -direction is conserved $\Sigma F_x = 0$
- (b) linear momentum in the y -direction is conserved $\Sigma F_y = 0$
- (c) linear momentum in the n -direction is conserved $\Sigma F_n = 0$
- (d) linear momentum in the t -direction is conserved $\Sigma F_t = 0$
- (e) none of the above

Question C4.18

Particle A strikes a stationary particle B with a speed of v_{A1} . Particle B is attached to a rigid bar that is pinned to ground at the top. Circle all of the correct responses below. Some problems might have more than one correct response.



For the system made up of A alone during impact:

- (a) linear momentum in the x-direction is conserved $\Sigma F_x \neq 0$
- (b) linear momentum in the y-direction is conserved $\Sigma F_y \neq 0$
- (c) linear momentum in the n-direction is conserved $\Sigma F_n \neq 0$
- (d) linear momentum in the t-direction is conserved $\Sigma F_t = 0$
- (e) none of the above

For the system made up of A+B during impact:

- (a) linear momentum in the x-direction is conserved $\Sigma F_x = 0$
- (b) linear momentum in the y-direction is conserved $\Sigma F_y \neq 0$
- (c) linear momentum in the n-direction is conserved $\Sigma F_n \neq 0$
- (d) linear momentum in the t-direction is conserved $\Sigma F_t \neq 0$
- (e) none of the above

Question C4.20

Consider the system shown below where A, B and E masses of m , $2m$ and $2m$, respectively. The system starts from rest with $\theta = 0$. Consider the motion of the system through a second position when $\theta = 90^\circ$. Answer the following.

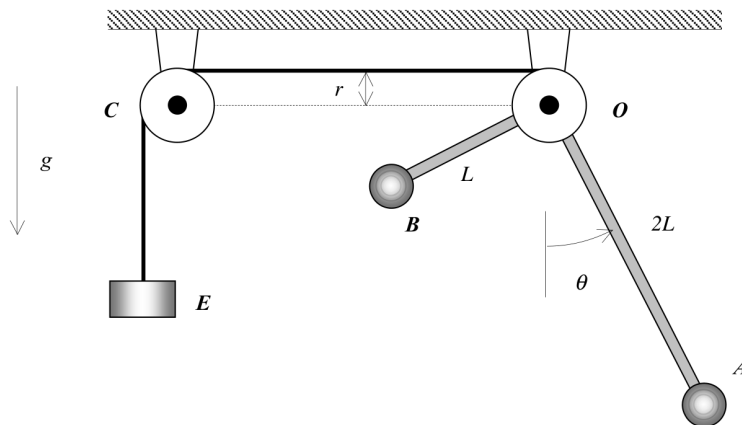
TRUE or FALSE: Energy is conserved for each particle individually. *Look at FBD of each particle.*

TRUE or FALSE: Energy is conserved for the total system of A+B+E. *Look at FBD of A+B+E*

TRUE or FALSE: Linear momentum in the horizontal direction is conserved for the total system of A+B+E. *Look at FBD of A+B+E*

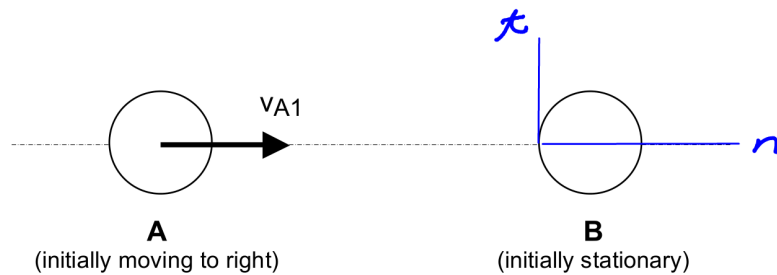
TRUE or FALSE: Linear momentum in the vertical direction is conserved for the total system of A+B+E. *Look at FBD of A+B+E*

TRUE or FALSE: Angular momentum about point O is conserved for the system of A+B. *Look at FBD of A+B*



Question C4.21

Sphere A, having a mass of M , initially moves to the right with a speed of v_{A1} . Sphere A then strikes sphere B, having a mass of $2M$, which is initially at rest. Sphere A has zero velocity after impacting B. What is the coefficient of restitution between A and B?



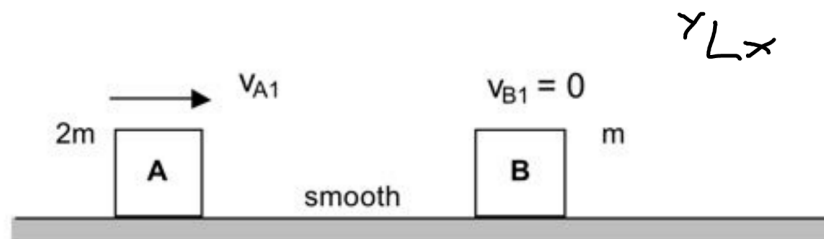
Spheres move in a horizontal plane

$$\begin{aligned} \underline{A+B}: \quad & \cancel{M} v_{A1} + 2M \cancel{v_{B1}} = \cancel{M} v_{A2} + 2M v_{B2} \\ & \rightarrow v_{A1} = 2v_{B2} \Rightarrow \frac{v_{B2}}{v_{A1}} = \frac{1}{2} \\ e = & \frac{v_{B2} - \cancel{v_{A2}}}{\cancel{v_{A1}} - \cancel{v_{B1}}} = \frac{v_{B2}}{v_{A1}} \end{aligned}$$

Question C4.22

Particle A strikes stationary particle B with a coefficient of restitution of $e = 1$. Circle the correct expression below for the velocity of A after impact.

- (a) $v_{A2} = 0$
- (b) v_{A2} is to the left
- (c) v_{A2} is to the right
- (d) Additional information is needed to answer this question



$$\underline{A+B}: \sum F_x = 0 \Rightarrow 2m v_{A1} + m \cancel{v_{B1}} = 2m v_{A2} + m v_{B2}$$
$$\hookrightarrow 2v_{A2} + v_{B2} = 2v_{A1} \quad (1)$$

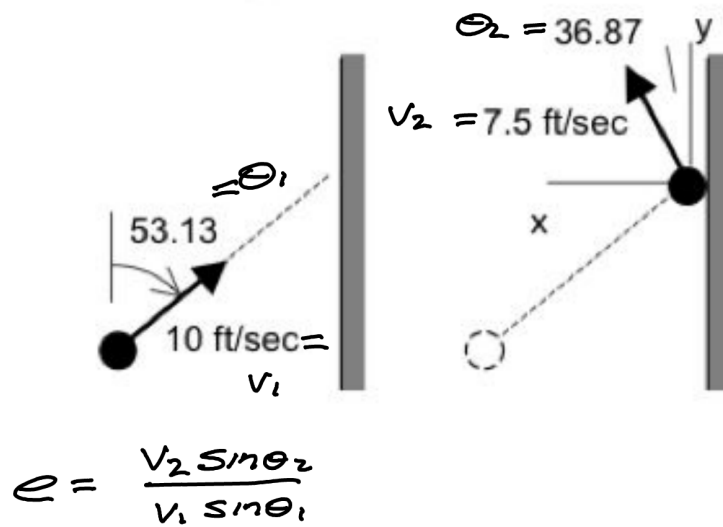
$$e = \frac{v_{B2} - v_{A2}}{\cancel{v_{A1}} - v_{B1}} \Rightarrow -v_{A2} + v_{B2} = e v_{A1} = v_{A1} \quad (2)$$

Solve for v_{A1} . Look at sign.

Question C4.23

A particle moving in a smooth horizontal plane approaches a wall with a speed of 10 ft/s. After impact, the particle leaves the wall with a speed of 7.5 m/s. The coefficient of restitution is:

- (a) $e = 0.375$
- (b) $e = 0.75$
- (c) $e = 0.5625$
- (d) $e = 1.0$



Question C4.24

Particles A and B are attached to a rigid bar with the bar being pinned to ground at point O. A bullet b strikes particle A and sticks. Consider a system made up of b, A, B and the rod. Circle all answers below that correctly describe this system during impact.

(a) linear momentum is conserved

$$\Sigma \vec{F} \neq \vec{0}$$

(b) angular momentum about A is conserved

$$\Sigma \vec{M}_A \neq \vec{0}$$

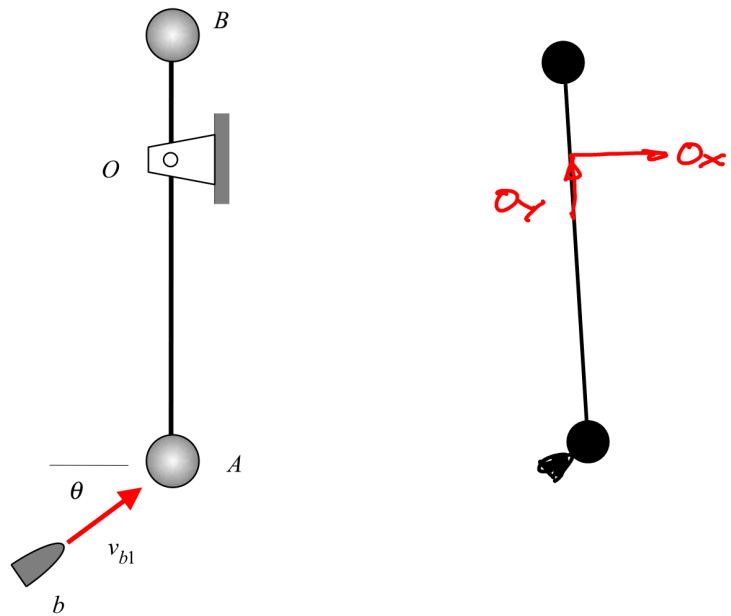
(c) angular momentum about O is conserved

$$\Sigma \vec{M}_O = \vec{0}$$

(d) energy is conserved

$$e = 0$$

(e) none of the above



Question C4.25

Particle A (of mass m) is released from rest at elevation 1. Particle B (of mass m and connected to lightweight bar OB) is also released from rest at elevation 1.

Circle the answer below that correctly describes the speeds of A and B (v_{A2} and v_{B2} , respectively) at elevation 2.

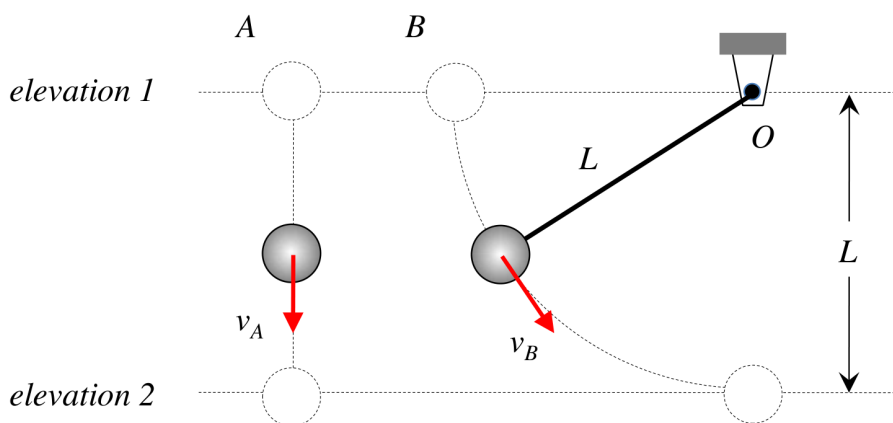
- (a) $v_{A2} > v_{B2}$
- (b) $v_{A2} = v_{B2}$
- (c) $v_{A2} < v_{B2}$

Justify your response with equations and/or words.

Circle the answer below that correctly describes the times required for A and B to reach elevation 2 (t_{A2} and t_{B2} , respectively).

- (a) $t_{A2} > t_{B2}$
- (b) $t_{A2} = t_{B2}$
- (c) $t_{A2} < t_{B2}$

Justify your response with equations and/or words.



$y \downarrow$

A: $\Sigma F_y = mg = may$
 $\hookrightarrow a_y = g$

B: $\Sigma F_y = mg - F \cos \theta = may$
 $\hookrightarrow a_y = g - \frac{F}{m} \cos \theta < g$

Question C4.26

In System A shown below on the left, particle P is connected to a pin joint at O with a lightweight, rigid bar of length L. Bullet b impacts the stationary particle P with a speed of v_1 , and after impact the bullet sticks to P. System B is identical to System A except the rigid bar is replaced by an inextensible string of length L. Let $(v_{1,min})_A$ represent the minimum value of v_1 that is required for particle P in System A to reach position 2, a position where P is at a distance of L immediately above O. Let $(v_{1,min})_B$ represent the minimum value of v_1 that is required for P in System B to reach position 2. Circle the response below that most accurately describes the relative magnitudes of $(v_{1,min})_A$ and $(v_{1,min})_B$:

- (a) $(v_{1,min})_A > (v_{1,min})_B$
- (b) $(v_{1,min})_A = (v_{1,min})_B$
- (c) $(v_{1,min})_A < (v_{1,min})_B$

Justify your response with equations and/or words.

