ME 274: Basic Mechanics II

Week 13 *– Monday, April 13*

Rigid body kinetics: summary

Instructor: Manuel Salmerón

Today’s Agenda

1. Kinetics Table
2. Activity

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $where\ \ T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $where\ \ T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \bullet \left(\vec{\omega} \times \vec{r}_{G/A}\right)$

Method	Body model	Fundamental equations
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\,dt = \vec{H}_{O2} - \vec{H}_{O1}$ $where\ \ \vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\,dt = \vec{H}_{A2} - \vec{H}_{A1}$ $where\ \ \vec{H}_A = I_A\vec{\omega}$

Activity: Solve, Explain, Learn

If properly submitted, this activity will be worth **one missed homework** assignment
Submission deadline: **9:20 AM**

You will be assigned one of three dynamics problems at random. Your job is not only to solve your assigned problem, but also to explain it clearly enough that someone else could use your work as a guide.

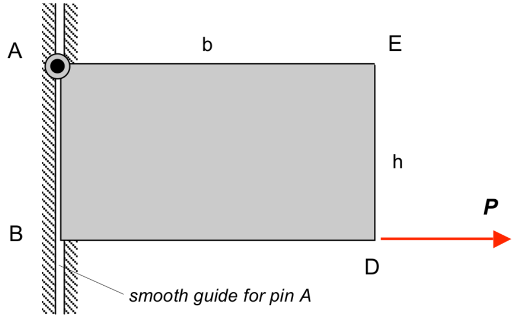
Part A: Solve your assigned problem using the four-step plan. Your work must include:

1. Method selection: Which method did you use, and why?
2. Key ideas, constraints, or tricks that make the solution work
3. Final answer
4. One common mistake to avoid

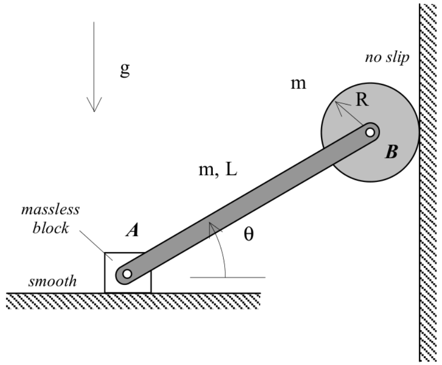
Part B: Listen to the explanations of two classmates who were assigned different problems. Then write down:

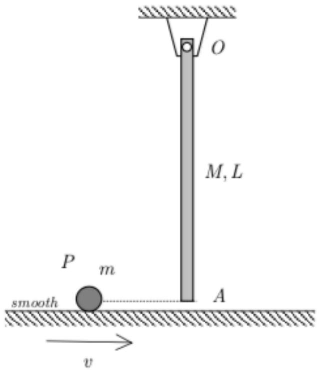
1. Method selection: Which method did they use and why?
2. Key ideas, constraints, or tricks that make the solution work
3. Free-body diagram and summary of the main equations used (no full solution needed)
4. Final answer
5. One common mistake to avoid

Submission: You must submit to Gradescope: (1) the full solution to the problem you solved, (2) answers to the questions in Part A, and (3) answers to the questions in Part B.



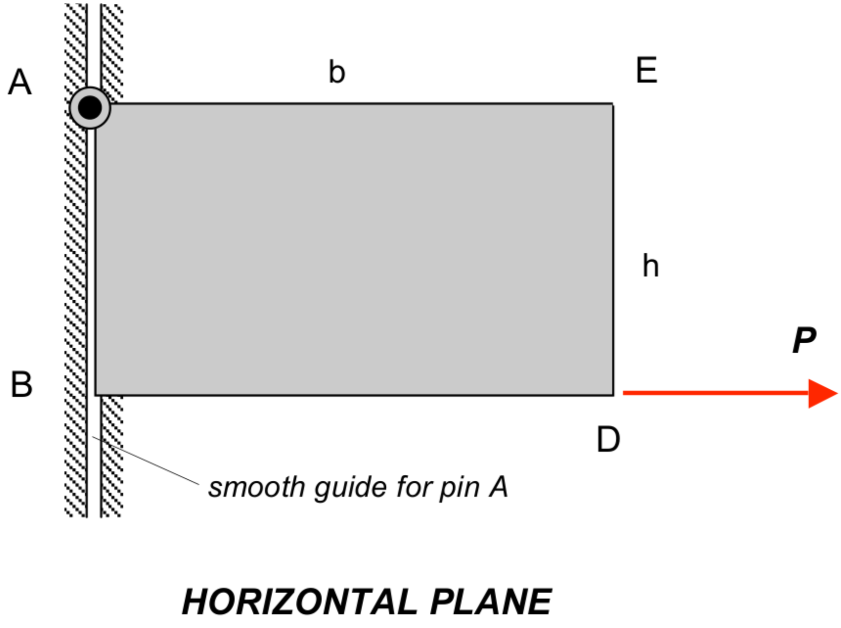
HORIZONTAL PLANE





Method			
Key Ideas, Constraints, or Tricks			
Common Mistakes			
Final Answer			

Given: A uniform plate of mass m is able to move on a smooth horizontal plane with corner A of the plate being constrained to move within a smooth guide in the plane of motion. The plate is initially at rest with edge AB aligned with the guide for A. A force P is applied at corner D, with P acting perpendicular to the guide for A.



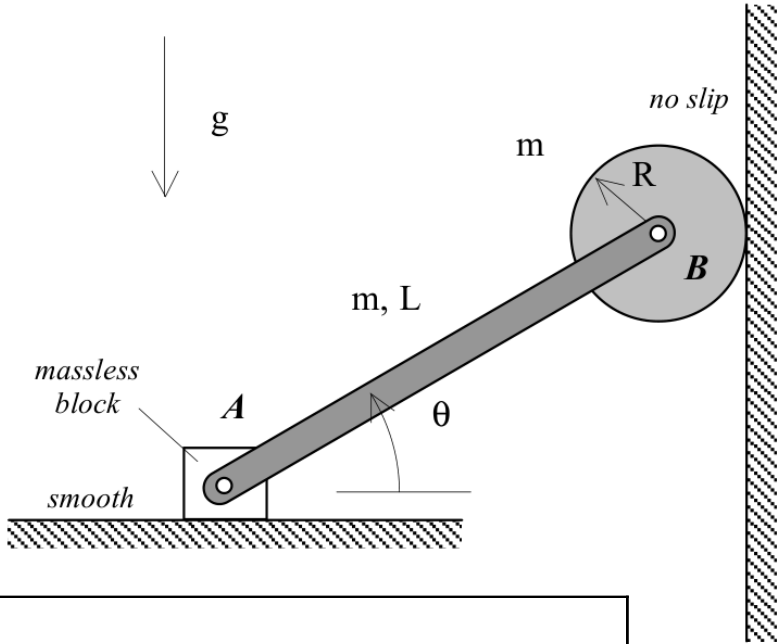
Find: Determine the initial angular acceleration of the plate. Write your answer as a vector.

Use the following parameters in your analysis: $m = 100 kg$, $b = 8\ meters$, $h = 6\ meters$, and $P = 400\ N$.

Method	
Key Ideas/Constraints/Tricks	
Common Mistakes	
Final Answer	

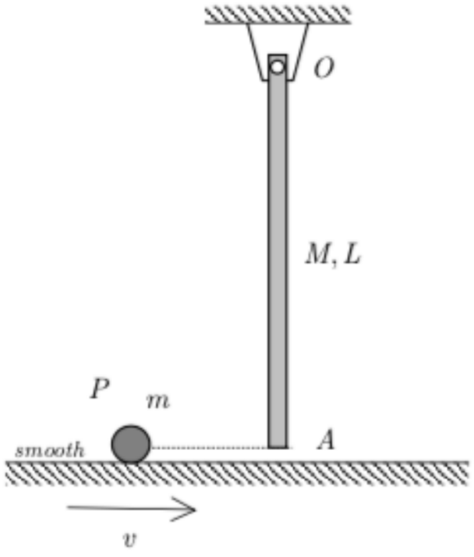
Given: System released from rest with $\theta = 36.87°$. Consider the bar and the disk be homogeneous.

Find: Determine the *angular velocity* of the bar when $\theta = 0$. Use: $m = 100\ kg$, $L = 2\ meters$ and $R = 0.4\ meters$.



Method	
Key Ideas/Constraints/Tricks	
Common Mistakes	
Final Answer	

Given: Particle P (of mass m) slides along a smooth horizontal surface with a speed of v. P then strikes end A of a stationary thin, homogeneous bar (of length L and mass M) that is pinned to ground at O. The coefficient of restitution for the impact of P with end A of the bar is known to be e.



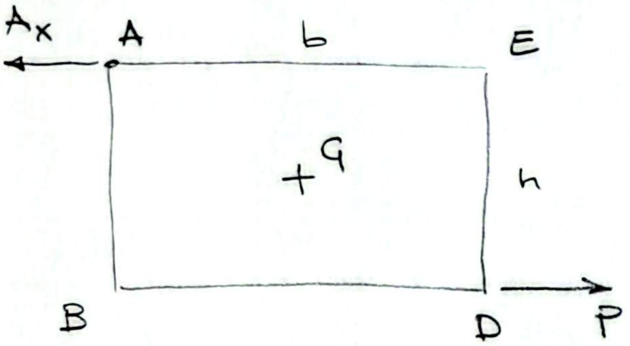
Find: Determine the angular speed of bar OA immediately after the impact. Use the following parameters in your analysis: $m = 10kg$, $M = 30\ kg$, $L = 2\ meters$, $v = 40\,m\,/\,sec$ and $e = 0.5$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.

Method	
Key Ideas/Constraints/Tricks	
Common Mistakes	
Final Answer	

Summer 2009, E2, P2

1. FBD



2. Kinetics

$$\Sigma F_x = P - A_x = m a_{Gx} \quad \cdots (1)$$

$$\Sigma F_y = 0 = m a_{Gy}$$ (slides vertically, so $A_y = 0$!)

$$+\circlearrowleft \; \Sigma M_A = Ph = I_A \alpha + m \vec{r}_{G/A} \times \vec{a}_A$$

$$\vec{r}_{G/A} \times \vec{a}_A = \left(\frac{b}{2}\hat{i} - \frac{h}{2}\hat{j}\right) \times \left(a_{Ax}\hat{i} + a_{Ay}\hat{j}\right)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b/2 & h/2 & 0 \\ a_{Ax} & a_{Ay} & 0 \end{vmatrix} = \left(\frac{b}{2} a_{Ay} - \frac{h}{2} a_{Ax}\right)\hat{k}$$

Also: $I_A = I_G + \frac{m}{4}(b^2+h^2) = \frac{1}{12}m(b^2+h^2) + \frac{m}{4}(b^2+h^2) = \frac{1}{3}m(b^2+h^2)$

So: $I_A \alpha + m\frac{b}{2} a_{Ay} - \cancelto{0}{\frac{h}{2} a_{Ax}} = Ph \quad \cdots (2)$

3. Kinematics

$$\vec{a}_G = \vec{a}_A + \vec{r}_{G/A} \times \vec{\alpha} - \omega^2 \vec{r}_{G/A}$$

$$a_{Gx}\hat{i} + \cancelto{0}{a_{Gy}}\hat{j} = \cancelto{0}{a_{Ax}}\hat{i} + a_{Ay}\hat{j} + \left(\frac{\alpha h}{2}\hat{i} + \frac{\alpha b}{2}\hat{j}\right)$$

In $(\hat{i})$: $a_{Gx} - \frac{\alpha h}{2} = 0 \quad \cdots (3)$

In $(\hat{j})$: $a_{Ay} + \frac{\alpha b}{2} = 0 \quad \cdots (4)$

4. Solve

$$\begin{array}{c} \begin{array}{cccc} a_{Gx} & A_x & a_{Ay} & \alpha \end{array} \\ \begin{bmatrix} m & 1 & 0 & 0 \\ 0 & 0 & mb/2 & I_A \\ 1 & 0 & 0 & -h/2 \\ 0 & 0 & 1 & +b/2 \end{bmatrix} \end{array} \begin{bmatrix} a_{Gx} \\ A_x \\ a_{Ay} \\ \alpha \end{bmatrix} = \begin{bmatrix} P \\ Ph \\ 0 \\ 0 \end{bmatrix}$$

$a_{Gx} = 4.1538$

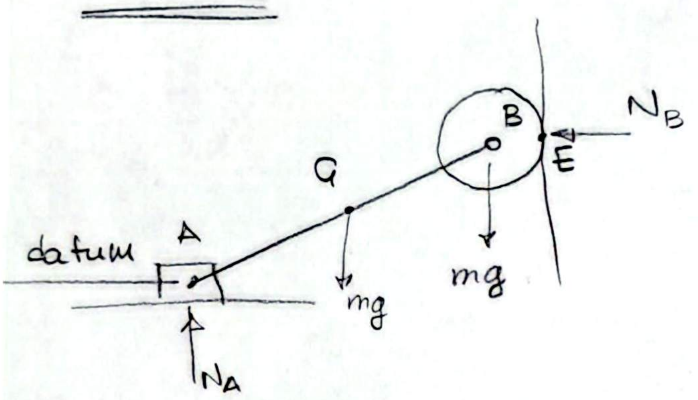
$A_x = -15.38$

$a_{Ay} = -5.5385$

$\boxed{\alpha = 1.3846 \ \frac{rad}{s^2}}$

Summer 2010, E2, P1.

1. FBD



2. Kinetics

$$T_1 + V_1 + \mathcal{U}^{(nc)}_{1\to 2} = T_2 + V_2 \quad \cdots (1)$$

$T_1 = \emptyset$; from rest

$$V_1 = \underbrace{mg\left(\tfrac{L}{2}\sin\theta_1\right)}_{\text{rod}} + \underbrace{mg\left(L\sin\theta_1\right)}_{\text{disk}}$$

$$\mathcal{U}^{(nc)}_{1\to 2} = \emptyset$$

$$V_2 = \emptyset \quad : \quad \underline{\theta_2 = \emptyset}$$

$$T_2 = \underbrace{\tfrac{1}{2} m v_{G_2}^2 + \tfrac{1}{2} I_G \omega_{r_2}^2}_{\text{rod}} + \underbrace{\tfrac{1}{2} m v_{B_2}^2 + \tfrac{1}{2} I_B \omega_{B_2}^2}_{\text{disk}}$$

3. Kinematics

$$\vec{v}_B = \cancel{\vec{v}_E}^{\,\vec{\emptyset}} + \vec{\omega}_B \times \vec{r}_{B/E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega_{B2} \\ -R & 0 & 0 \end{vmatrix} = -\omega_{B2} R \hat{j} \quad \cdots (2)$$

$$\vec{v}_G = \vec{v}_B + \vec{\omega}_r \times \vec{r}_{G/B} = -\omega_{B_2} R \hat{j} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega_{r_2} \\ -\tfrac{L}{2}c\theta & -\tfrac{L}{2}s\theta & 0 \end{vmatrix} = -\omega_{r_2} \frac{L}{2} \hat{j}$$

$$-L/2 \qquad 0$$

$$\vec{v}_G = -\left(\omega_{B_2} R + \omega_{r_2} \frac{L}{2}\right)\hat{j} \quad \cdots (3)$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_r \times \vec{r}_{B/A} = v_A \hat{i} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega_r \\ Lc\theta & Ls\theta & 0 \end{vmatrix} = v_A \hat{i} - \omega_r L s\theta \hat{i} + \omega_r L c\theta \hat{j}$$

In $(\hat{j})$: $v_B = \omega_r L\cos\theta \quad \cdots (4)$

4. Solve

$$\tfrac{1}{2} m\left(\omega_{B_2} R + \omega_{r_2}\tfrac{L}{2}\right)^2 + \tfrac{1}{2} I_G \omega_r^2 + \tfrac{1}{2}\omega_{r_2}^2 L^2 \cos^2\theta + \tfrac{1}{2} I_B \omega_{B_2}^2$$
$$= mg\tfrac{L}{2}\sin\theta_1 + mg\, L\sin\theta_1$$

MATLAB:

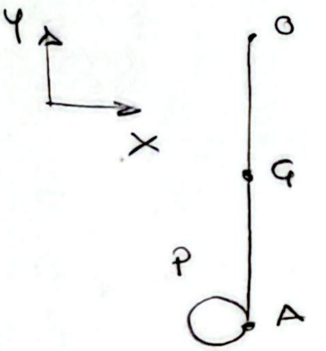
$$v_{B_{y_2}} = -\omega_{B2} R = \omega_{r_2} L\cos\theta$$

$$\boxed{\omega_{r_2} = 2.19\ \frac{\text{rad}}{\text{s}}}$$

$$v_{G_2} = -\omega_{B_2} R + \omega_{r_2} \frac{L}{2}$$

Summer 2010, EF, P2 (impulse-momentum)

1. FBD



2. Kinetics

$$\Sigma F_x = 0: \quad m v_{P_1} + M \cancelto{0}{v_{G x_1}} = m v_{P_2} + M v_{G x_2}$$

$$\Sigma M_O = 0: \quad (\vec{H}_O)_1 = (\vec{H}_O)_2$$

$$I_O = \frac{1}{12} M L^2 + M\left(\frac{L}{2}\right)^2$$

$$(\vec{H}_O)_1 = \vec{r}_{P/O} \times m\vec{v}_P = (-L\hat{j}) \times m(v_P\,\hat{i}) = m v_{P_1} L\,\hat{k}$$

$$(\vec{H}_O)_2 = m v_{P_2} L + I_O\,\omega_2$$

$$e = -\frac{v_{P_2} - v_{A_2}}{v_{P_1} - \cancelto{0}{v_{A_1}}} \Longrightarrow v_{P_1} e = v_{A_2} - v_{P_2}$$

So:

$$m v_{P_2} + M v_{G x_2} = m v_{P_1} \quad \cdots (1)$$

$$m L v_{P_2} + I_O \omega_2 = m L v_{P_1} \quad \cdots (2)$$

$$-v_{P_2} + v_{A_2} = e v_{P_1} \quad \cdots (3)$$

4 unknowns, 3 eqs.

3. Kinematics

$$\vec{v}_A = \vec{v}_O + \vec{\omega} \times \vec{r}_{A/O} = (\omega\hat{k}) \times (-L\hat{j}) = \omega L\,\hat{i}$$

In $(\hat{i})$: $v_{A_2} = \omega_2 L \quad \cdots (4)$

$$\begin{bmatrix} m & M & 0 \\ mL & 0 & I_O \\ -1 & 0 & L \end{bmatrix} \begin{bmatrix} v_{P_2} \\ v_{G x_2} \\ \omega_2 \end{bmatrix} = \begin{bmatrix} m v_{P_1} \\ m L v_{P_1} \\ e v_{P_1} \end{bmatrix} \rightsquigarrow$$

$v_{P_2} = 10$ m/s

$v_{G x_2} = 10$ m/s.

$$\boxed{\omega_2 = 15 \ \frac{rad}{s}}$$

ME 274: Basic Mechanics II

Week 13 – *Wednesday, April 15*

Vibrations: Equations of Motion

Instructor: Manuel Salmerón

Today’s Agenda

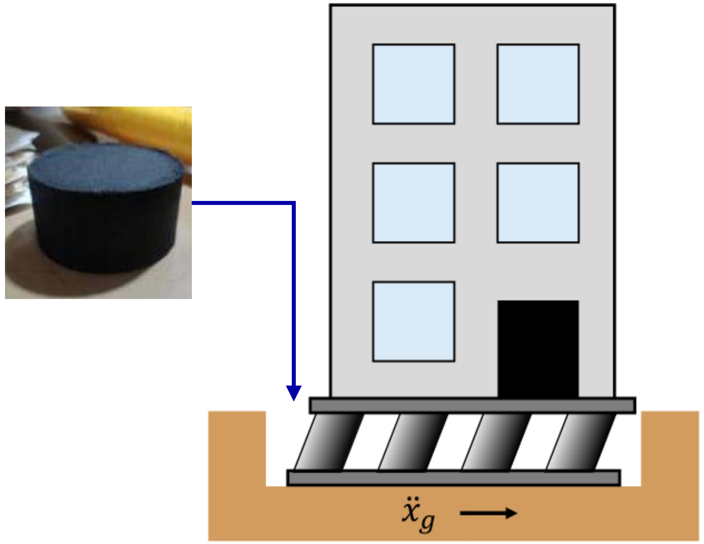
1. Motivating examples
2. Definition of vibration
3. Idealized elements of vibration
4. Equations of motion
5. Example

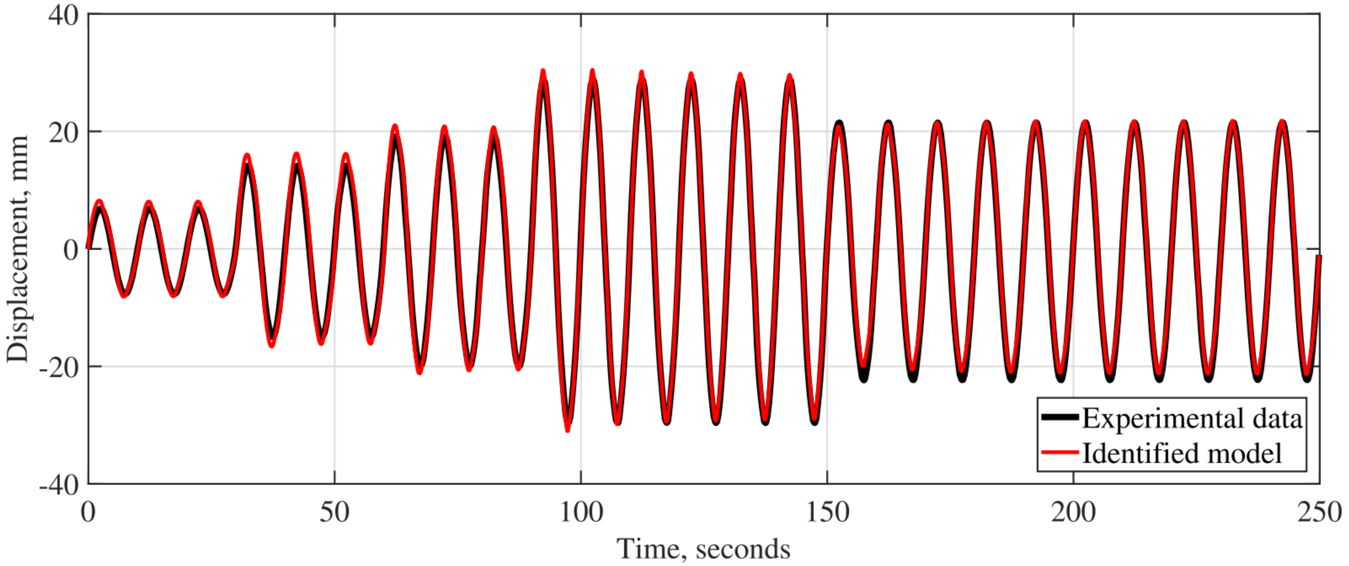
Motivating Examples

Definitions

Oscillation: repetitive or periodic variation, typically in time, of some measure about a central value (often a point of equilibrium) or between two or more different states.

Vibrations: mechanical oscillations about an equilibrium point.

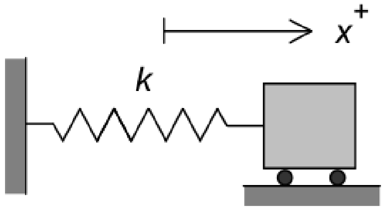




Idealized elements of vibration

Why do these systems "come back and forth"?

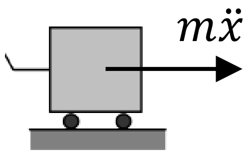
- Spring-like elements (columns, coils)
- Restoring force, resistance to displacement



$$F_{sp} = -kx$$

Why don't they return instantly?

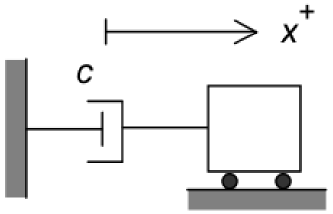
- Mass-like elements (car's body, slabs)
- Inertial effect, resistance to acceleration



$$\sum F = m\ddot{x}$$

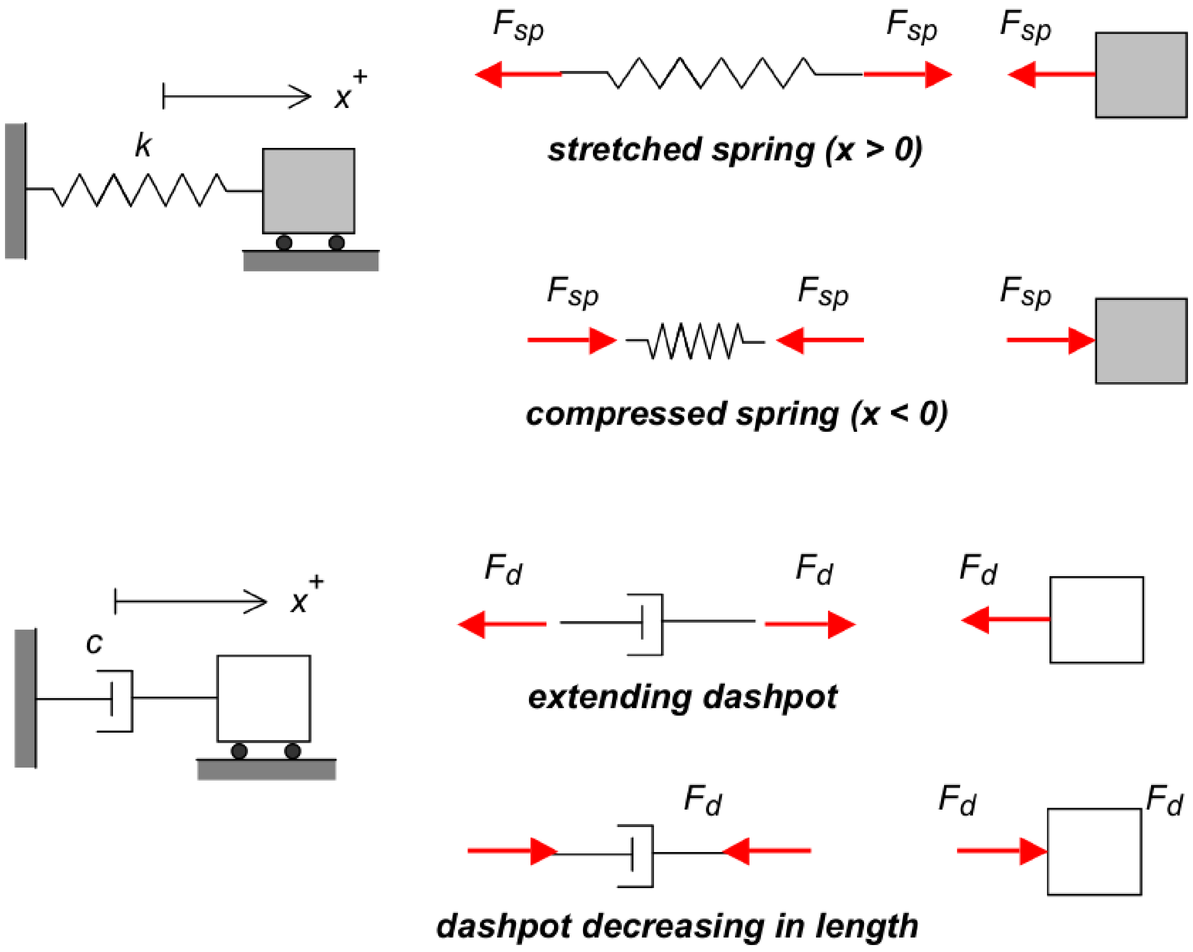
Why don't they keep moving forever?

- Dashpot-like elements (shock-absorbers, dampers)
- Damping force, resistance to velocity



$$F_d = -c\dot{x}$$

Idealized elements of vibration

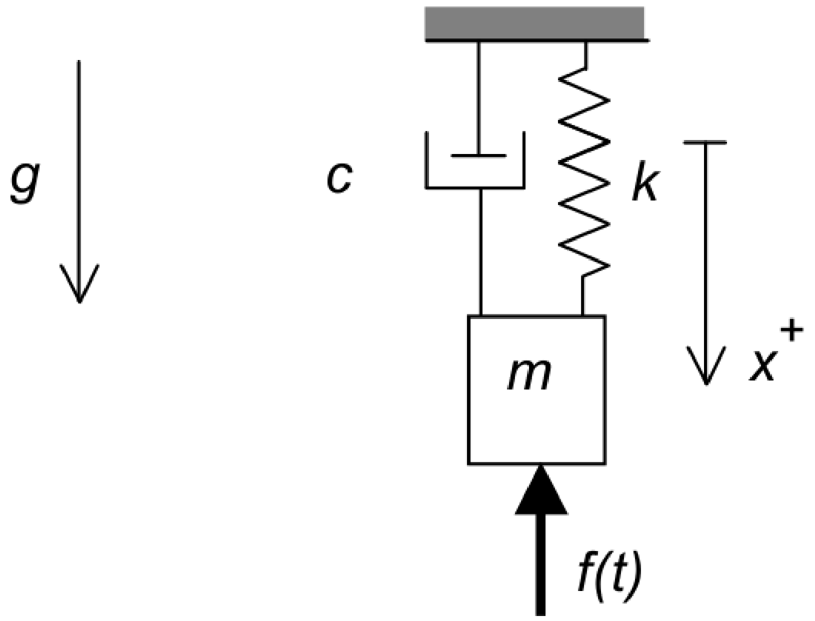


Equations of motion

Example 6.A.1

Given: A particle of mass m is supported by a spring of stiffness k and a dashpot with damping constant c. A vertical force $f(t)$ acts on the particle as shown. Let x describe the position of the particle, where x is measured from the position of the particle when the spring is unstretched.

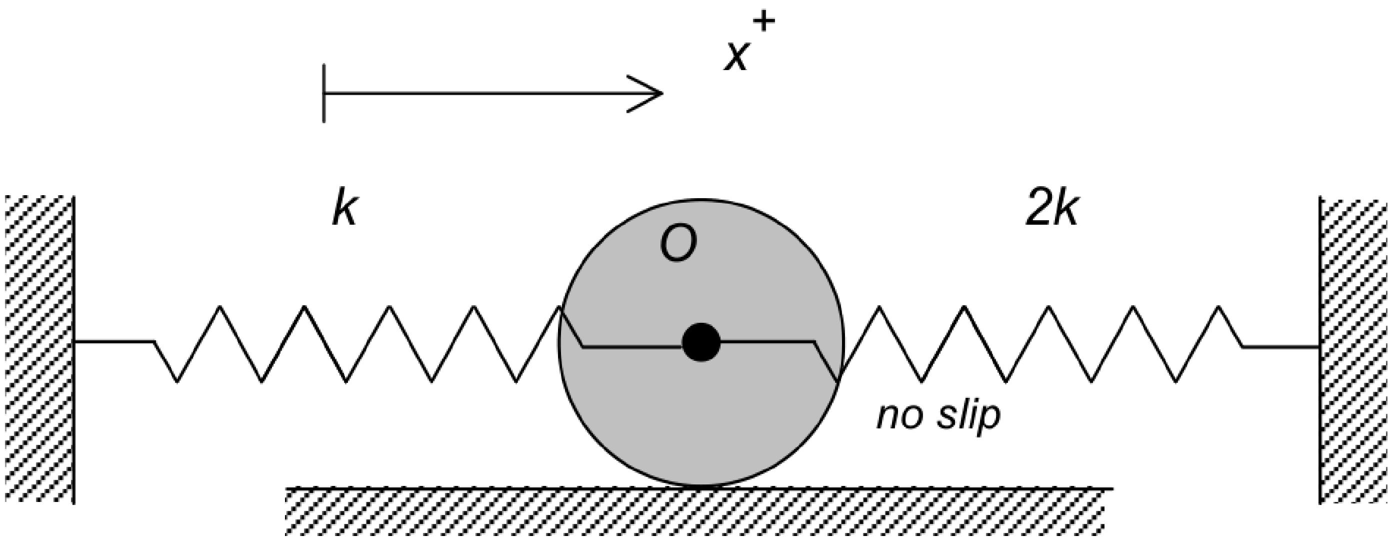
Find: Determine the EOM of this system in terms of the coordinate x.



Example 6.A.2

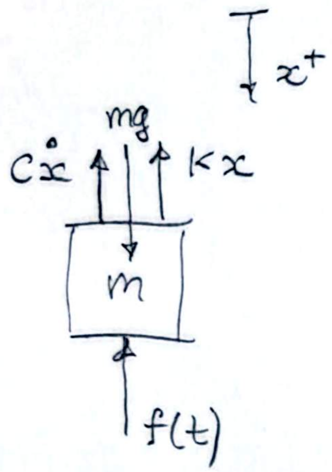
Given: A homogeneous disk of mass m and radius r rolls without slipping on a rough horizontal surface. Two springs, having stiffnesses of k and $2k$, are attached between the disk center O and ground, as shown below. Let x describe the position of O, where the springs are unstretched when $x = 0$.

Find: Determine the EOM for the disk in terms of the coordinate x.



Example 6.A.1

1. FBD



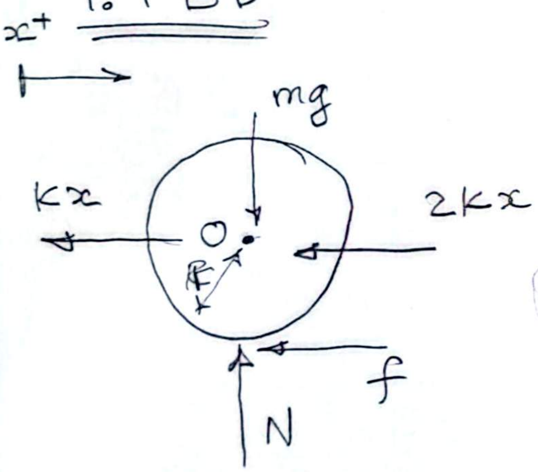
2. Kinetics

$$\Sigma F_x = -kx - c\dot{x} + mg - f(t) = m\ddot{x}$$

$$\boxed{m\ddot{x} + c\dot{x} + kx = mg - f(t)}$$

Example 6.A.2

1. FBD



2. Kinetics

$$\Sigma F_x = -kx - 2kx - f = m\ddot{x} \quad \cdots (1)$$

$$\Sigma F_y = N - mg = 0$$

$$\circlearrowleft + \; \Sigma M_o = -fr = I_o \alpha \quad \cdots (2)$$

3. Kinematics

No slipping: $x = -r\theta \longrightarrow \ddot{x} = -r\ddot{\theta} \quad \cdots (3)$

$\ddot{\theta} \equiv \alpha$

4. Solve

From (1): $f = -3kx - m\ddot{x} \quad \cdots (4)$

(4) in (2): $-(-3kx - m\ddot{x})r = I_o \alpha = I_o \left(-\dfrac{\ddot{x}}{r}\right)$

$$(3kx + m\ddot{x})r + \frac{I_o}{r}\ddot{x} = 0$$

$$\left(3kx + m\ddot{x}\right)\cancel{r} + \left(\frac{1}{2}m r^{\cancel{2}}\right)\left(\frac{\ddot{x}}{\cancel{r}}\right) = 0$$

$$3kx + \underline{\underline{m\ddot{x}}} + \frac{1}{2}\underline{\underline{m\ddot{x}}} = 0$$

$$\boxed{\frac{3}{2}m\ddot{x} + 3kx = 0}$$

ME 274: Basic Mechanics II

Week 13 *– Friday, April 17*

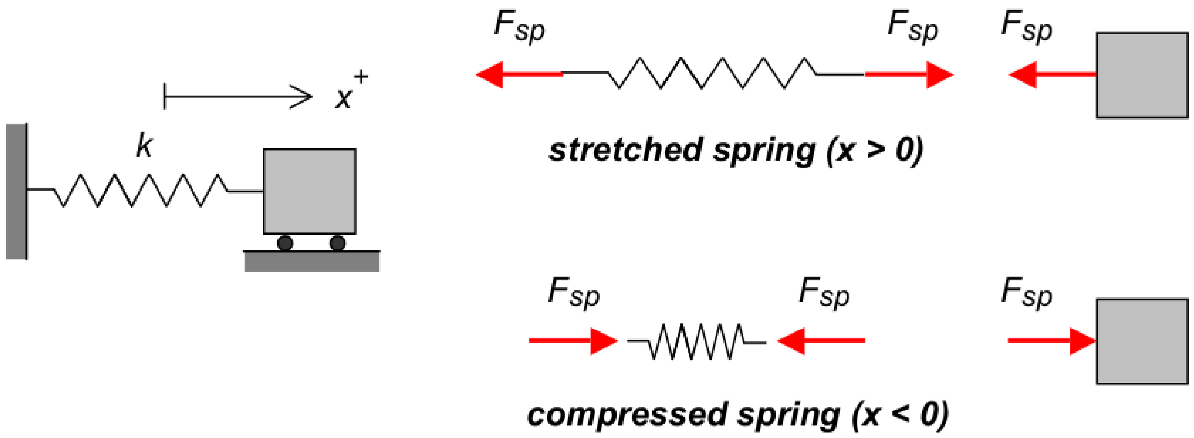
Vibrations: Natural Frequency and Free Response

Instructor: Manuel Salmerón

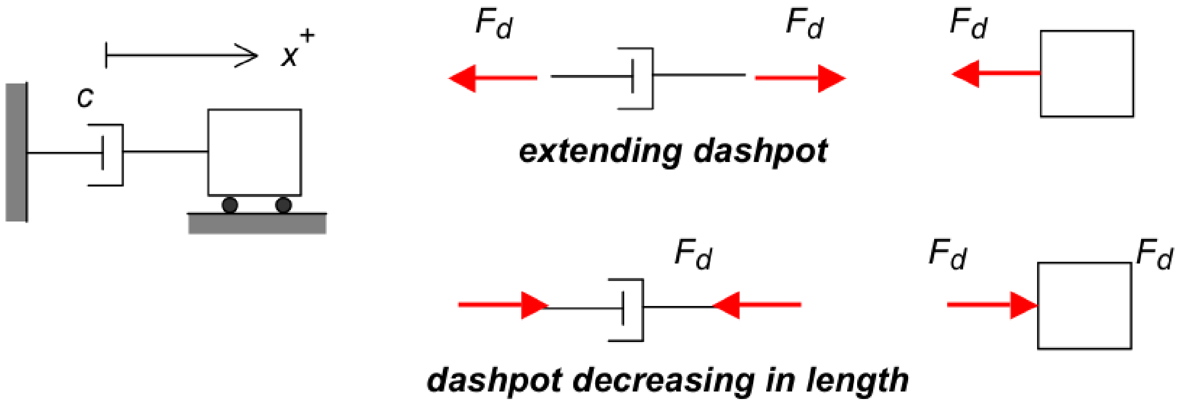
Today’s Agenda

1. Spring and dashpot forces
2. Example
3. Static deformation
4. Natural frequency
5. Free vibration response

Spring and dashpot forces



$$F_{sp} = -kx$$



$$F_d = -c\dot{x}$$

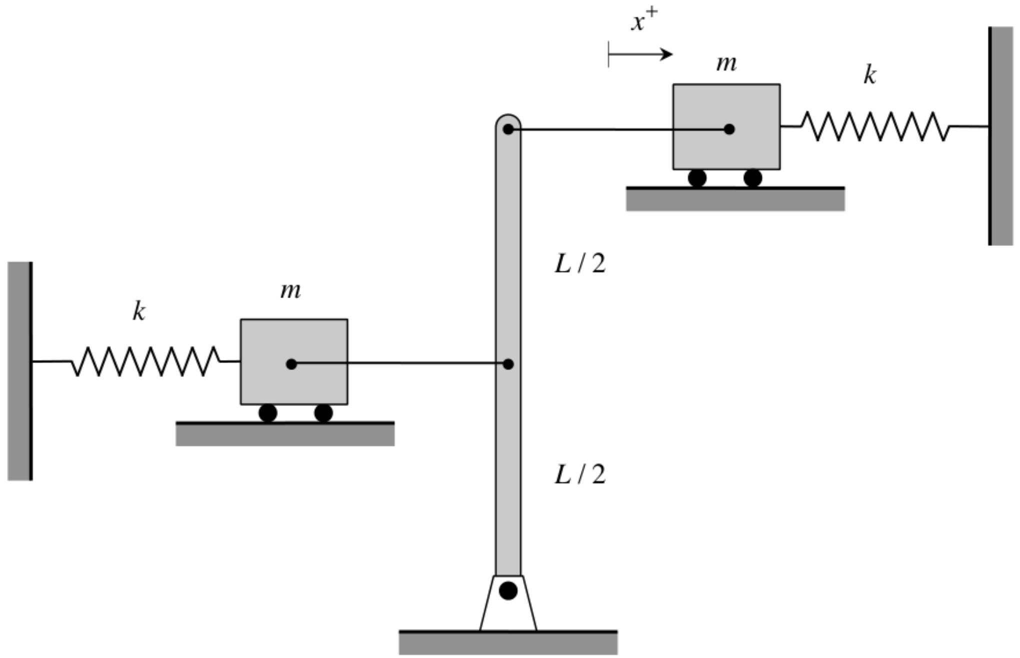
Example 6.B.4

Given: The system shown below consists of two masses, each of mass m, which are connected by a massless bar with the bar able to rotate about a pin joint at its lower end. Each mass is connected to ground by a spring of stiffness k, and moves along a smooth surface. The coordinate x is measured from the position of the particle when the bar is vertical. The springs are unstretched when $x = 0$. Assume small motions of the system; i.e., restrict the motion of the bar to small angles of rotation.

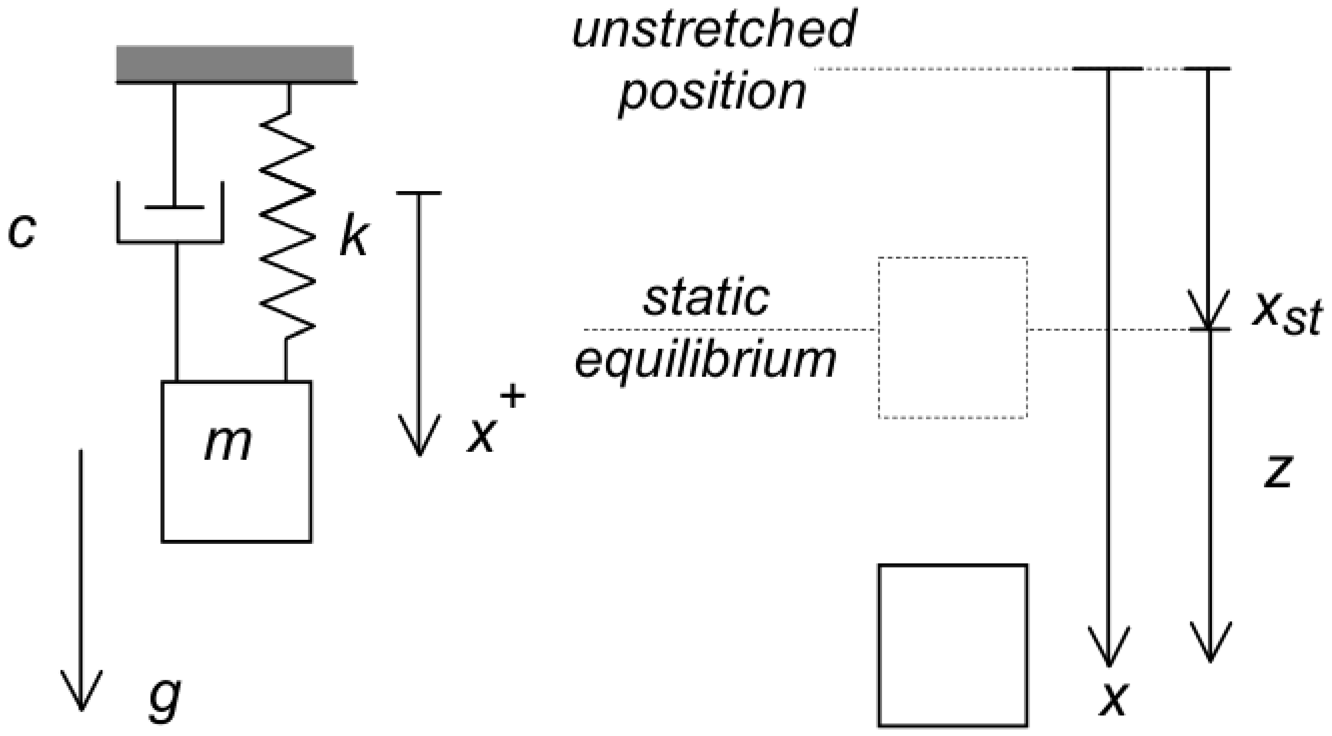
Find:

(a) Determine the equation of motion of this system in terms of the coordinate x; and

(b) Determine the natural frequency of the system.



Static deformation



$$m\ddot{x} + c\dot{x} + kx = mg$$

At static equilibrium: $\dot{x} = \ddot{x} = 0$

$$kx = mg \rightarrow x_{st} = \frac{mg}{k}$$

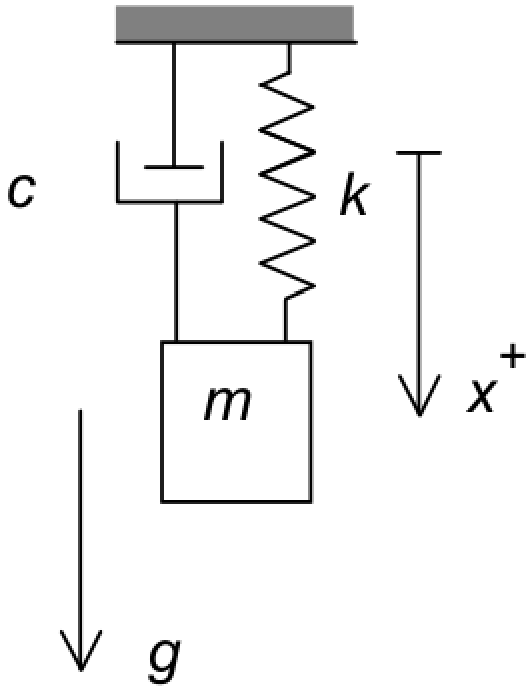
New coordinate: $z = x - x_{st}$

Derivatives: $\dot{z} = \dot{x}, \ddot{z} = \ddot{x}$

$$m\ddot{z} + c\dot{z} + kz = 0$$

EOM in z-coordinates

Natural frequency



$$m\ddot{z} + c\dot{z} + kz = 0$$

Assume damping is very, very small: $c \approx 0$

$$m\ddot{z} + kz = 0$$

Acceleration is given by:

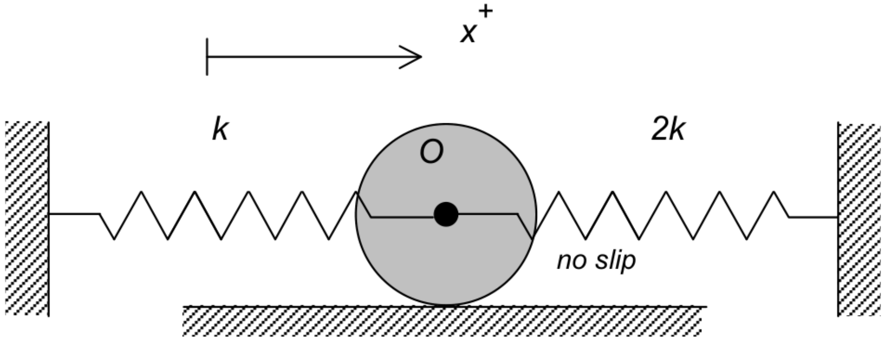
$$\ddot{z} = -\boxed{\frac{k}{m}}z$$

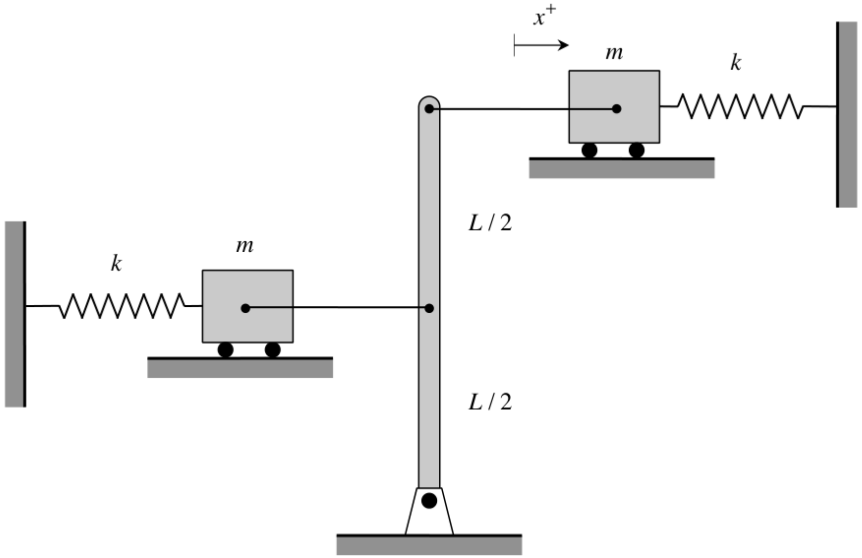
$$\omega_n = \sqrt{k/m}$$

Natural frequency: rate/pace at which a system oscillates in the absence of disturbance.

ω_n for previous examples

$$\omega_n = \sqrt{k/m}$$
Natural frequency: rate/pace at which a system oscillates in the absence of disturbance.





$$\frac{3}{2}m\ddot{x} + 3kx = 0$$

$$\omega_n^2 = \frac{3k}{(3/2)m} = \frac{2k}{m}$$

$$\frac{11}{6}m\ddot{x} + \frac{3}{2}kx = 0$$

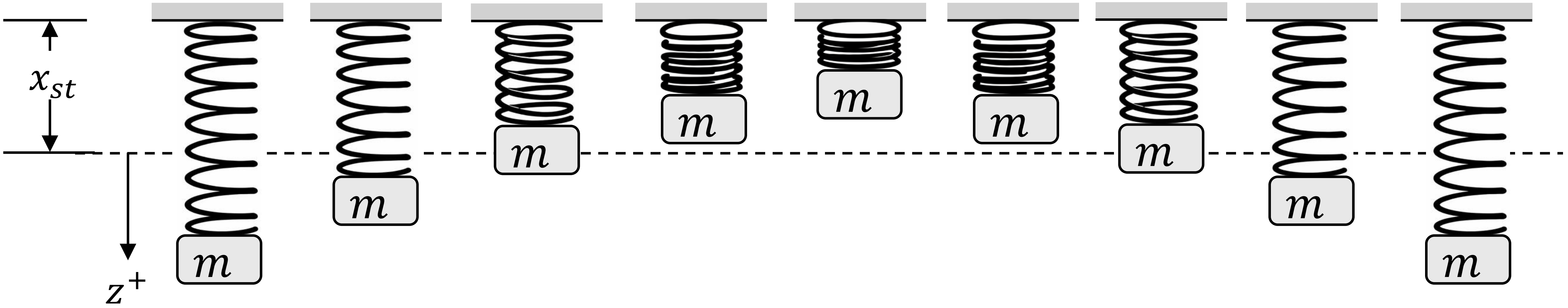
$$\omega_n^2 = \frac{(3/2)k}{(11/6)m} = \frac{9k}{11m}$$

Free vibration response

The response is cyclical, so it can be expressed as:

$$z(t) = A\cos\omega_n t + B\sin\omega_n t$$
$$\dot{z}(t) = -A\omega_n\sin\omega_n t + B\omega_n\cos\omega_n t$$

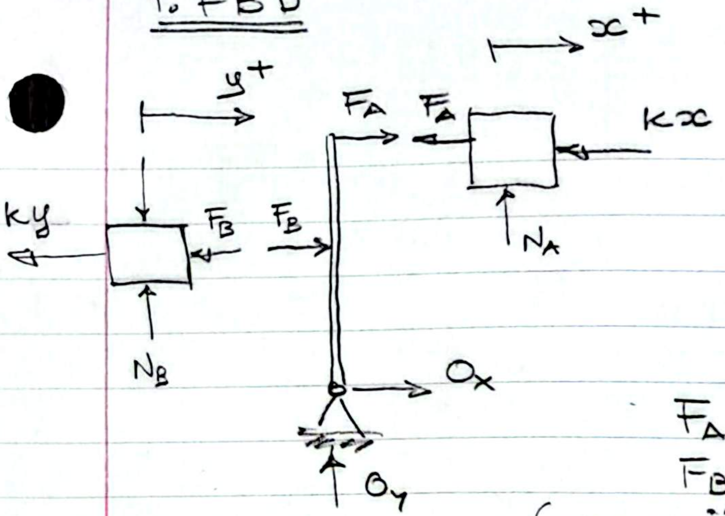
$$\ddot{z} = -\frac{k}{m}z = -\omega_n^2 z$$



$z > 0$	$z > 0$	$z = 0$	$z < 0$	$z < 0$	$z < 0$	$z = 0$	$z > 0$	$z > 0$
$\ddot{z} < 0$	$\ddot{z} < 0$	$\ddot{z} = 0$	$\ddot{z} > 0$	$\ddot{z} > 0$	$\ddot{z} > 0$	$\ddot{z} = 0$	$\ddot{z} < 0$	$\ddot{z} < 0$
$\dot{z} = 0$	$\dot{z} < 0$	$\dot{z} < 0$	$\dot{z} < 0$	$\dot{z} = 0$	$\dot{z} > 0$	$\dot{z} > 0$	$\dot{z} > 0$	$\dot{z} = 0$

Example 6.B.4

1. FBD



2. Kinetics

(A): $\Sigma F_x = -F_A - kx = m\ddot{x}$ (1)

(B): $\Sigma F_y = -ky - F_B = m\ddot{y}$ (2)

Bar: $\circlearrowright \Sigma M_o = F_A L + F_B \frac{L}{2} = I_o \ddot{\theta}$ (3)

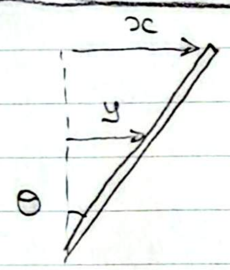
$I_o = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{3}mL^2$

$F_A = -kx - m\ddot{x}$ (from (1))

$F_B = -ky - m\ddot{y}$ (from (2))

$(-kx - m\ddot{x})L + (-ky - m\ddot{y})\frac{L}{2} = I_o\ddot{\theta}$

3. Kinematics: we want everything in $\ddot{x}$



$x = L\sin\theta \xrightarrow[\text{small angles}]{} x \approx L\theta$

$y = \frac{L}{2}\sin\theta \xrightarrow[\text{small angles}]{} y \approx \frac{L}{2}\theta$

Thus: $\ddot{x} = L\ddot{\theta}$, $\ddot{y} = \frac{L}{2}\ddot{\theta} = \frac{L}{2}\underbrace{\left(\frac{\ddot{x}}{L}\right)}_{\ddot{\theta}} = \frac{1}{2}\ddot{x}$

4. Solve (get EOM)

$(-kx - m\ddot{x})L + \left(-k\frac{x}{2} - m\frac{\ddot{x}}{2}\right)\frac{L}{2} = I_o \frac{\ddot{x}}{L} = \frac{1}{3}mL\ddot{x}$

$-\frac{3}{2}k\cancel{L}x - \frac{3}{2}m\cancel{L}\ddot{x} - \frac{1}{3}m\cancel{L}\ddot{x} = 0$

$\left(\frac{1}{3} + \frac{3}{2}\right)m\ddot{x} + \frac{3}{2}kx = 0$

$\frac{2+9}{6} = \frac{11}{6}$ $\qquad \frac{11}{6}m\ddot{x} + \frac{3}{2}kx = 0$

$\frac{18}{22} = \frac{9}{11}$

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