ME 274: Basic Mechanics II

Week 12 *– Monday, April 6*

Rigid body kinetics: work/energy

Instructor: Manuel Salmerón

Announcement

New office hours schedule:

- Mondays, 3 – 4 pm
- One of the following:
  1. Thursday morning
  2. Thursday afternoon

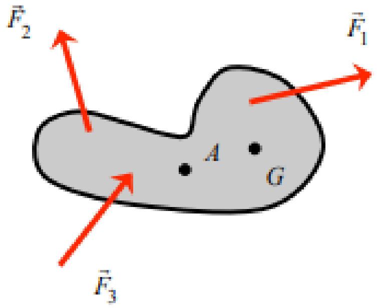
Today’s Agenda

1. Work/Energy for Rigid Bodies
2. Example 5.B.6
3. Activity

Summary: Work/Energy Equation 1

FUNDAMENTAL equation: $T_2 + V_2 = T_1 + V_1 + U_{1\to2}^{(nc)}$
with:

$$T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2 + m\vec{v}_A \cdot \left(\vec{\omega} \times \vec{r}_{G/A}\right)$$



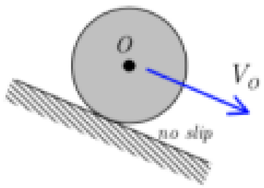
SPECIAL CHOICES FOR POINT "A": If A is <u>*EITHER*</u> the c.m. <u>*OR*</u> a fixed point, then the kinetic energy equation reduces to:

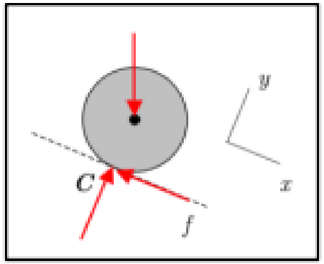
$$T = \underbrace{\boxed{\frac{1}{2} m v_A^2}}_{\text{translation}} + \underbrace{\boxed{\frac{1}{2} I_A \omega^2}}_{\text{rotation}}$$

PARALLEL AXIS THEOREM: As with the Newton/Euler equation, you will need to use the PAT if you choose A to be anything other than the c.m.

SYSTEM CHOICE: Make your system BIG! Include as many components within system to make workless forces INTERNAL (no work on system). Different choice than for Newton/Euler.

ROLLING WITHOUT SLIPPING: The friction force at a no-slip point does not work. Why? Recall that the no-slip point is stationary – no work is done on a stationary point.

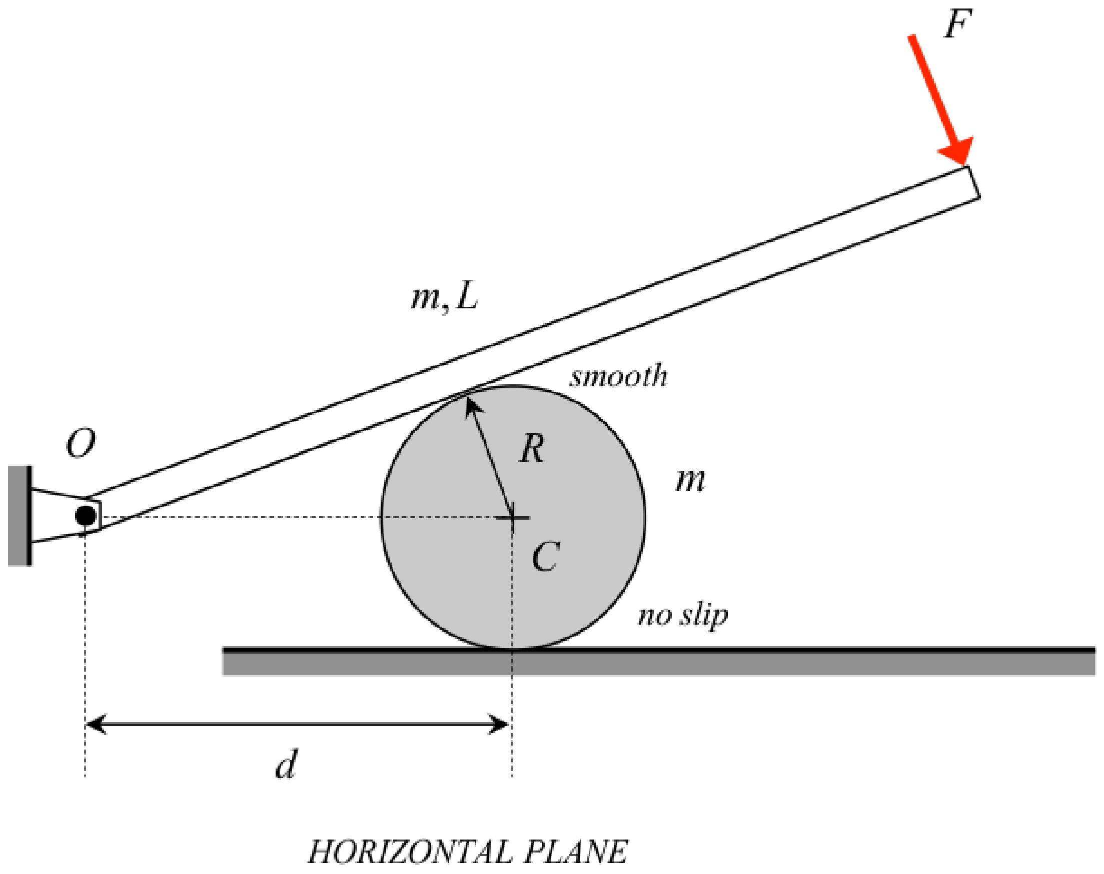




Example 5.B.6

Given: A disk of mass m and radius R is able to roll without slipping on a rough surface. A thin, homogeneous bar of mass m and length L is pinned to ground at O and is brought in contact with the disk at a smooth contacting surface. A constant force F is applied to the free end of the bar with F always perpendicular to the bar. The system is released from rest with the center C of the disk at a disk of d from the pin at O. The system moves in a horizontal plane.

Find: Determine the speed of the disk's center C after C has moved an additional distance d to the right.



Attendance

Solve Example 5.B.6. (again) using the following variations:

1. compute the kinetic energy of the bar using the center of mass of the bar (label it G) as reference instead of the fixed point O, and
2. compute the kinetic energy of the disk using its center of mass instead of the no slip point E.
3. Upload your work to Gradescope and answer the questions asked.

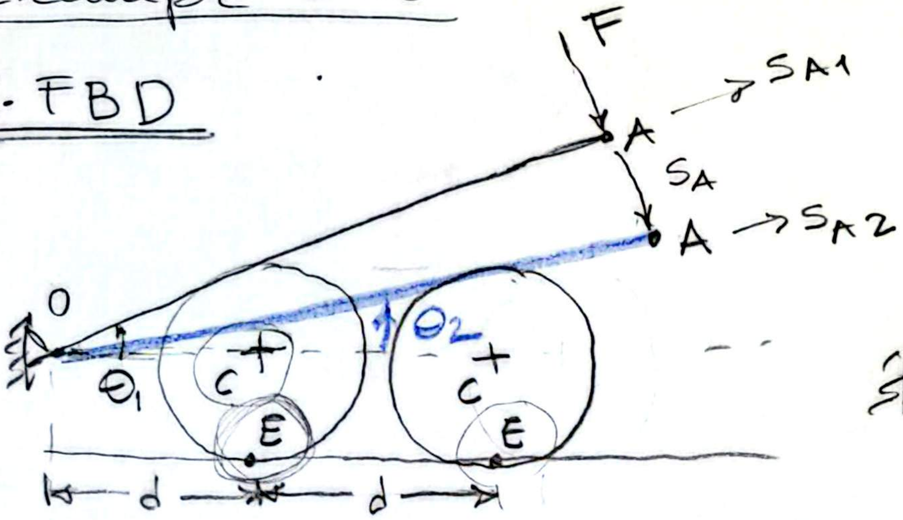
HINT: use rigid-body kinematics, namely:

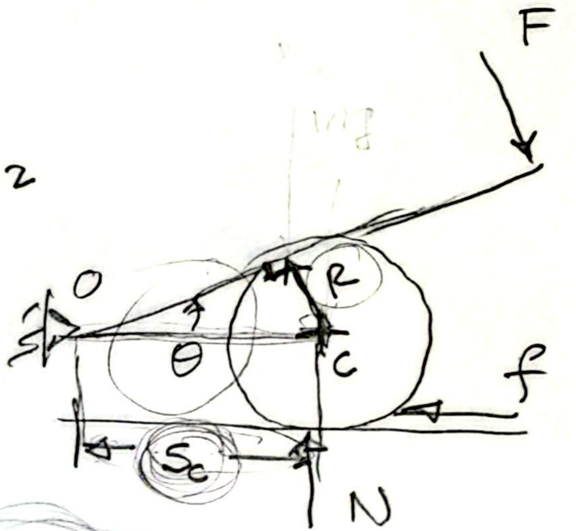
$$\vec{v}_G = \vec{v}_O + \vec{\omega}_{bar} \times \vec{r}_{G/O}$$

$$\vec{v}_C = \vec{v}_E + \vec{\omega}_{disk} \times \vec{r}_{C/E}$$

Example 5B6

1. FBD





2. Kinetics

: $T_1 + V_1 + \mathcal{U}^{(nc)}_{1\to 2} = T_2 + V_2$

$V_1 = V_2 = 0$: horizontal plane

$T_1 = 0$: released from rest

$$\mathcal{U}^{(nc)}_{1\to 2} = \int_{s_{A1}}^{s_{A2}} (\vec{F}\cdot\hat{e}_t)\, ds_A = \int_{s_{A1}}^{s_{A2}} F\, ds_A = F(s_{A2} - s_{A1})$$

$$= FL(\theta_2 - \theta_1)$$

$$\boxed{= FL\,\Delta\theta}$$

Option #1

$$T_{bar,2} = \cancel{\tfrac{1}{2} m v_O^2}^{\,0} + \cancel{m\vec{v}_O}^{\,\vec{0}} \cdot (\vec{\omega}_{bar} \times \vec{r}_{G/O}) + \tfrac{1}{2} I_O\, \omega_{bar}^2$$

$$T_{disk,2} = \cancel{\tfrac{1}{2} m v_E^2}^{\,0} + \cancel{m\vec{v}_E}^{\,\vec{0}} \cdot (\vec{\omega}_{disk} \times \vec{r}_{C/O}) + \tfrac{1}{2} I_E\, \omega_{disk}^2$$

$$I_O = \tfrac{1}{12} mL^2 + m\left(\tfrac{L}{2}\right)^2 = \tfrac{1}{3} mL^2$$

$$I_E = \tfrac{1}{2} mR^2 + mR^2 = \tfrac{3}{2} mR^2$$

} parallel axis theorem

3. Kinematics: we need the ang. speeds!

→ Bar: we need expression for θ:

$$\sin\theta = \frac{R \longrightarrow \text{constant}}{s_c \longrightarrow \text{changes with time}}$$

We are looking for $\omega_{bar} = \dot{\theta}$:

$$\dot{\theta}\cos\theta = -\frac{R\dot{s}_c}{s_c^2} = -\left(\frac{R}{s_c}\right)\frac{\dot{s}_c}{s_c} = -\sin\theta\left(\frac{\dot{s}_c}{s_c}\right)$$

$$\dot{\theta} = -\frac{\sin\theta}{\cos\theta}\left(\frac{\dot{s}_c}{s_c}\right) = -\frac{\dot{s}_c \rightarrow v_c}{s_c}\tan\theta$$

$$\boxed{\dot{\theta} = -\frac{v_c}{s_c}\tan\theta = \omega_{\substack{rod \\ (bar)}}} \quad + 1 \text{ eq.}$$

→ Disk: $v_E = 0 \Rightarrow ICR \equiv E$

Thus: $\boxed{\omega_{disk} = \frac{v_c}{R}}$ + 1 eq.

$s_{c_1} = d$

$s_{c_2} = 2d$

4. Solve:

$$FL\Delta\theta = \frac{1}{2}I_0\,\omega_{bar}^2 + \frac{1}{2}I_E\,\omega_{disk}^2$$

$$= \frac{1}{2}I_0\frac{v_{c_2}^2}{s_{c_2}^2}\tan^2\theta_2 + \frac{1}{2}I_E\frac{v_{c_2}^2}{R}$$

$$\boxed{FL\Delta\theta = \left(\frac{mL^2}{24d^2}\tan^2\theta_2 + \frac{3m}{4}\right)v_{c_2}^2}$$

ME 274: Basic Mechanics II

Week 12 *– Wednesday, April 8*

Rigid body kinetics: impulse-momentum

Instructor: Manuel Salmerón

Announcements

- Monday, 3:00 – 4:00 PM
- Thursday, 1:00 – 2:00 PM
- Room TBD

Today’s Agenda

1. Recall Newton-Euler’s Equation
2. Impulse-Momentum Equations for Rigid Bodies
3. Example(s)

Recall Newton-Euler's Equations

$$\sum_{i=1}^{N} \vec{R}_i = \sum_{i=1}^{N} m_i \vec{a}_i = M\vec{a}_G$$

$$\sum_{i=1}^{N} \left(\vec{M}_A\right)_i = \underbrace{\vec{r}_{G/A} \times (M\vec{a}_A)}_{\text{Accounts for the acceleration of } A} + \underbrace{\frac{d}{dt}\left[\sum_{i=1}^{N} \vec{H}_{i/A}\right]}_{\text{How fast are the angular momentums changing}}$$

Recall Newton-Euler's Equations

$$\sum_{i=1}^{N} \vec{R}_i = M\vec{a}_G$$

$$\sum_{i=1}^{N} \left(\vec{M}_A\right)_i = \vec{r}_{G/A} \times (M\vec{a}_A) + I_A\vec{\alpha}$$

Recall Newton-Euler's Equations

$$\sum_{i=1}^{N} \vec{R}_i = M\vec{a}_G$$

$$\sum_{i=1}^{N} \left(\vec{M}_A\right)_i = I_A\vec{\alpha}$$

(assuming A is a fixed point on the rigid body)

Recall Newton-Euler's Equations

$$\int_{t_1}^{t_2} \sum_{i=1}^{N} \vec{R}_i \, dt = \int_{t_1}^{t_2} M\vec{a}_G \, dt$$

$$\int_{t_1}^{t_2} \sum_{i=1}^{N} \left(\vec{M}_A\right)_i \, dt = \int_{t_1}^{t_2} I_A \vec{\alpha} dt$$

(assuming A is a fixed point on the rigid body)

Impulse-Momentum Equations (RB)

$$\int_{t_1}^{t_2} \sum_{i=1}^{N} \vec{R}_i \, dt = M\vec{v}_{G2} - M\vec{v}_{G1}$$

$$\int_{t_1}^{t_2} \sum_{i=1}^{N} \left(\vec{M}_A\right)_i dt = I_A\vec{\omega}_2 - I_A\vec{\omega}_1$$

(A fixed and inside the RB)

$$\int_{t_1}^{t_2} \sum_{i=1}^{N} \left(\vec{M}_A\right)_i dt = \left[I_G\vec{\omega}_2 + \vec{r}_{G/A} \times M\vec{v}_{G2}\right] - \left[I_G\vec{\omega}_1 + \vec{r}_{G/A} \times M\vec{v}_{G1}\right]$$

(A fixed and outside the RB)

Example 5.C.1

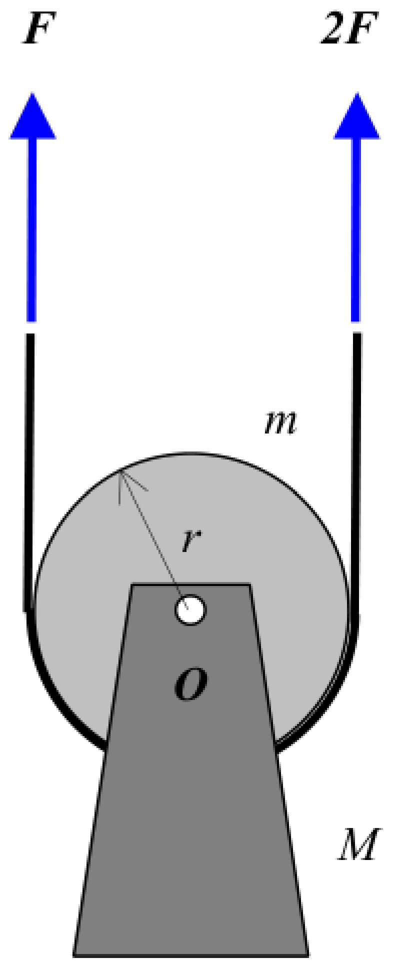
Given: Constant forces of F and $2F$ are applied at the free ends of the hoisting cable as shown. The velocity of O is 4 m/s down and the angular velocity ω_1 of the drum (having a centroidal radius of gyration of $k_O = 0.4$ m) is counterclockwise at time $t = 0$.

Find: Determine, after 10 s:

(a) The velocity of O; and

(b) The angular velocity of the drum.

Use the following parameters in your analysis: $\omega_1 = 2$ rad/s, $r = 0.4$ m, $m = 20$ kg, $M = 30$ kg and $F = 100$ N.

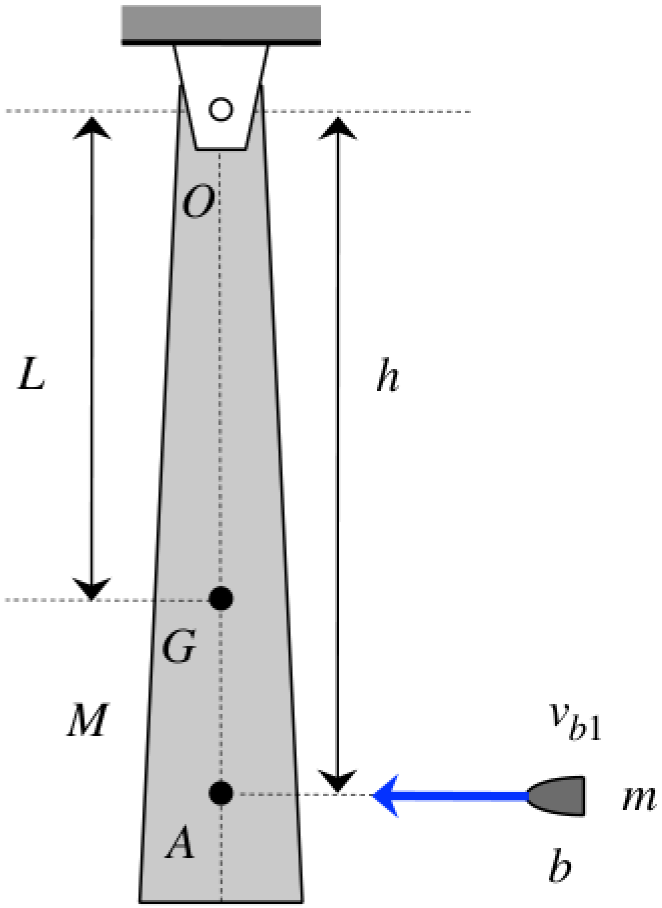


Example 5.C.5

Given: A bullet having a mass of m strikes a stationary pendulum at a distance of h from the pivot O with a speed of v_{b1}. The pendulum has a mass of M and has a radius of gyration of k_O about the pivot O. The bullet imbeds itself immediately within the pendulum at A on impact.

Find: Determine the angular speed of the pendulum immediately after impact occurs.

Use the following parameters in your analysis: $v_{b1} = 1500$ ft/s, $m = 0.1$ slugs, $M = 2$ slugs, $h = 4$ ft, $k_O = 3$ ft and $L = 3$ ft.



Moment of inertia

68 languages

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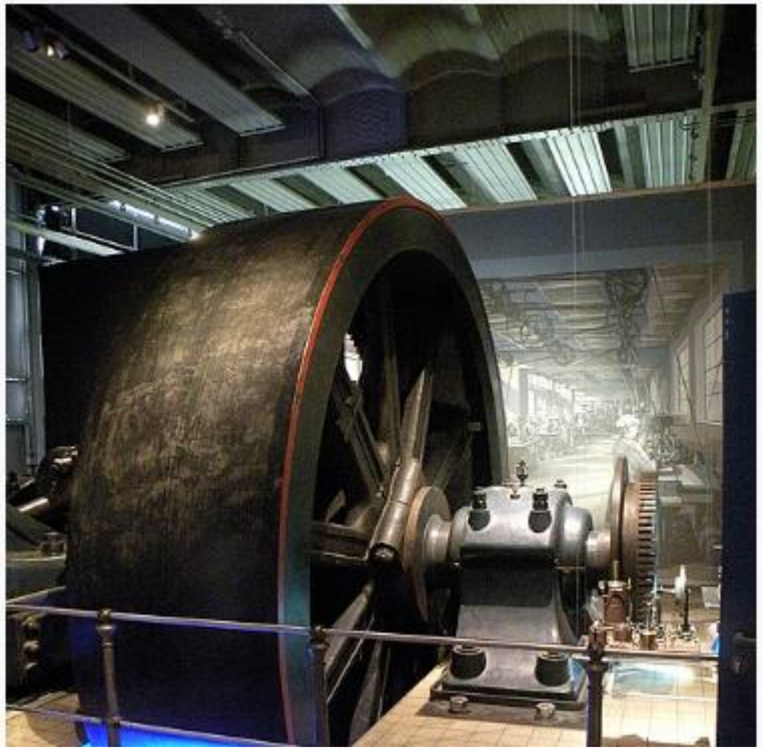
From Wikipedia, the free encyclopedia

For the quantity also known as the "area moment of inertia", see Second moment of area.

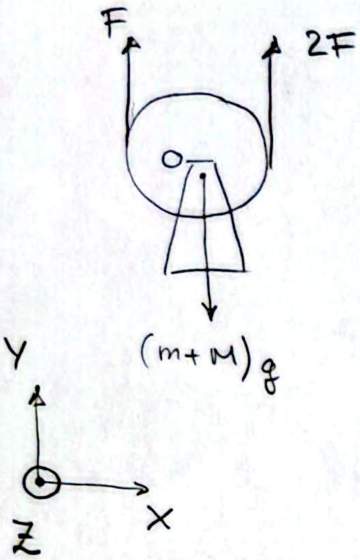
The **moment of inertia** (also known as **mass moment of inertia**, **angular/rotational mass**, **second moment of mass**, or **rotational inertia**) is a measure of how difficult it is to change the rotation rate of a rigid body about a given axis. It is the ratio between the torque applied and the resulting angular acceleration about that axis.[1]:279[2]:261 It plays the same role in rotational motion as mass does in linear motion. A body's moment of inertia about a particular axis depends both on the mass and its distribution relative to the axis, increasing with mass and distance from the axis.

Moment of inertia



Example S.C.1.

1. FBD



2. Kinetics

Linear Impulse Momentum:

$$\int_{t_1}^{t_2} \Sigma \vec{F}\, dt = (m+M)\vec{v}_{G2} - (m+M)\vec{v}_{G1}$$

$$(Y): \int_0^{10} \left[3F - (m+M)g\right] dt = (m+M)v_{G2} - (m+M)v_{G1}$$

$$\left[3F - (m+M)g\right] 10 = (m+M)v_{G2} - (m+M)v_{G1}$$

$$\vec{v}_{G2} = \frac{30F - 10(m+M)g}{m+M}\,\hat{j}$$

Angular Impulse Momentum: "O" fixed & inside RB

$$\int_{t_1}^{t_2} \Sigma \vec{M}_O\, dt = I_O\vec{\omega}_2 - I_O\vec{\omega}_1$$

$$I_O = mk_O^2$$

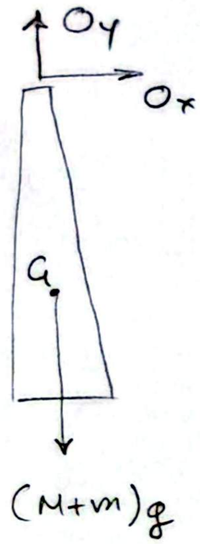
$$(Z): \int_0^{10} \left[2Fr - Fr\right] dt + I_O\omega_1 = I_O\omega_2$$

$$10Fr + mk_O^2\omega_1 = mk_O^2\omega_2$$ (assume only the disk spins)

$$\vec{\omega}_2 = \frac{10Fr + mk_O^2\omega_1}{mk_O^2}\,\hat{k}$$

Example S.C.S.

1. FBD



2. Kinetics

$$\sum \vec{M}_o = 0$$

$$(\vec{H}_o)_1 = (\vec{H}_o)_2$$

→ Instant 1: only bullet moving,

$$(\vec{H}_o)_1 = \vec{r}_{1,bull} \times m\vec{v}_1 \longrightarrow \text{particle!}$$

$$= -h\hat{j} \times m(-v_{1,bull}\,\hat{i})$$

$$= -m\,h\,v_{bull,1}\,\hat{k}$$

→ Instant 2: bullet + pendulum

$(\vec{H}_o)_2 = I_o\,\omega_2\,\hat{k}$ → the entire system spins @ ω_2 $\frac{rad}{s}$ ... but what is I_o?

(see Wikipedia)

Thus: $I_o = (I_{o,\,pend}) + (I_{o,\,bull})$ } both of them, together, "resist" rotation

$$I_o = (M k_o^2) + (\underbrace{0}_{\text{particle, no moment of inertia}} + \underbrace{mh^2}_{\text{little contribution to the system: parallel axis theorem.}})$$

ME 274: Basic Mechanics II

Week 12 *– Friday, April 10*

Rigid body kinetics: impulse-momentum

Instructor: Manuel Salmerón

Today’s Agenda

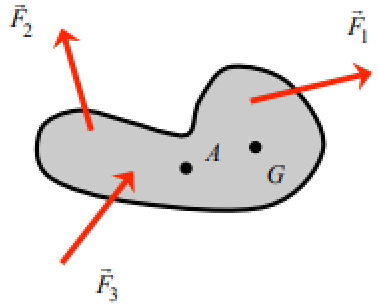
1. Impulse-Momentum Equations for Rigid Bodies
2. Examples

Summary: Impulse/Momentum Equations 1

FUNDAMENTAL equations (for your chosen "system"):

$$m\vec{v}_{G2} = m\vec{v}_{G1} + \int_1^2 \vec{F}\,dt$$

$$\vec{H}_{A2} = \vec{H}_{A1} + \int_1^2 \vec{M}_A\,dt$$

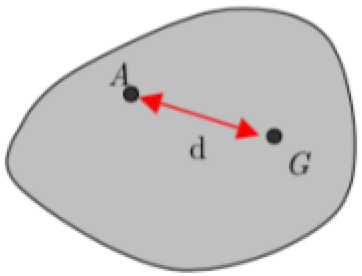


for A = c.m. ***OR*** fixed point, with:

$$\boxed{\begin{aligned} \vec{H}_A &= I_A\vec{\omega} && ;\quad \textit{for a RIGID BODY} \\ &= m\vec{r}_{P/A} \times \vec{v}_P && ;\quad \textit{for a particle P} \end{aligned}}$$

SYSTEM CHOICE: Make your system big (make as many forces *internal* as possible to take advantage of conservation).

PARALLEL AXIS THEOREM: You will need to use the PAT if you choose A to be anything other than the c.m.: $I_A = \underbrace{\boxed{I_G}}_{cm} + md^2$



Example 5.C.9

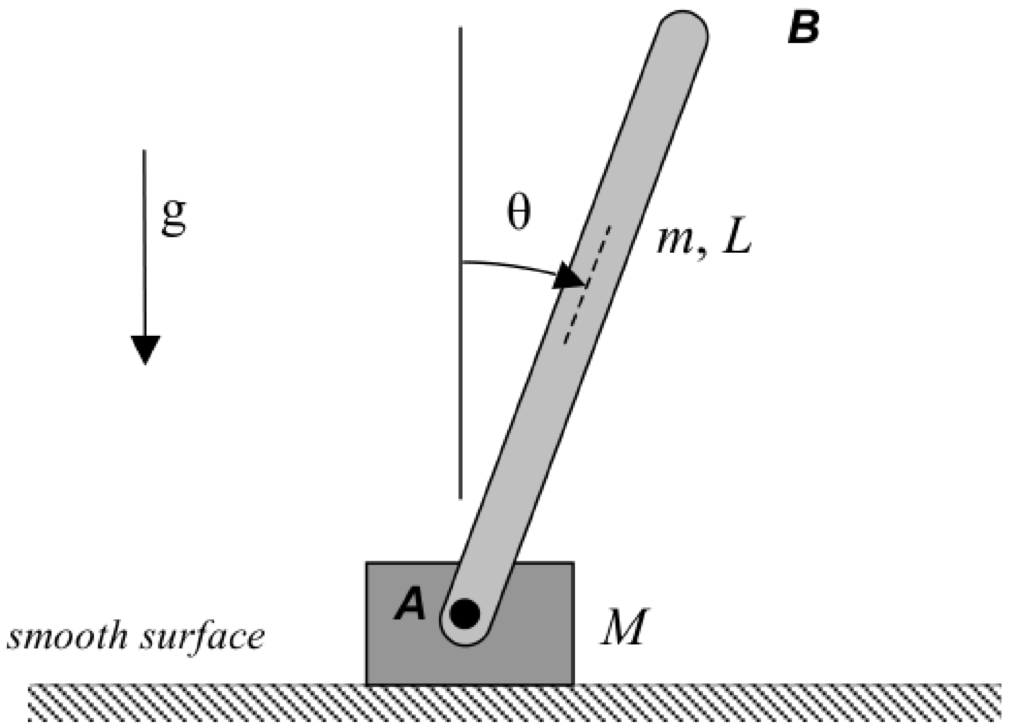
Given: A rod of mass m and length L is pinned to a block of mass M. The block can slide on a frictionless horizontal surface. The rod is released from rest when $\theta = 30°$.

Find: Determine:

(a) The angular velocity of the rod when $\theta = 90°$; and

(b) The velocity of the block when $\theta = 90°$.

Consider using both the work-energy and linear impulse-momentum equations in your solution. Use the following parameters in your analysis: $m = 10$ kg, $M = 30$ kg and $L = 2$ m.



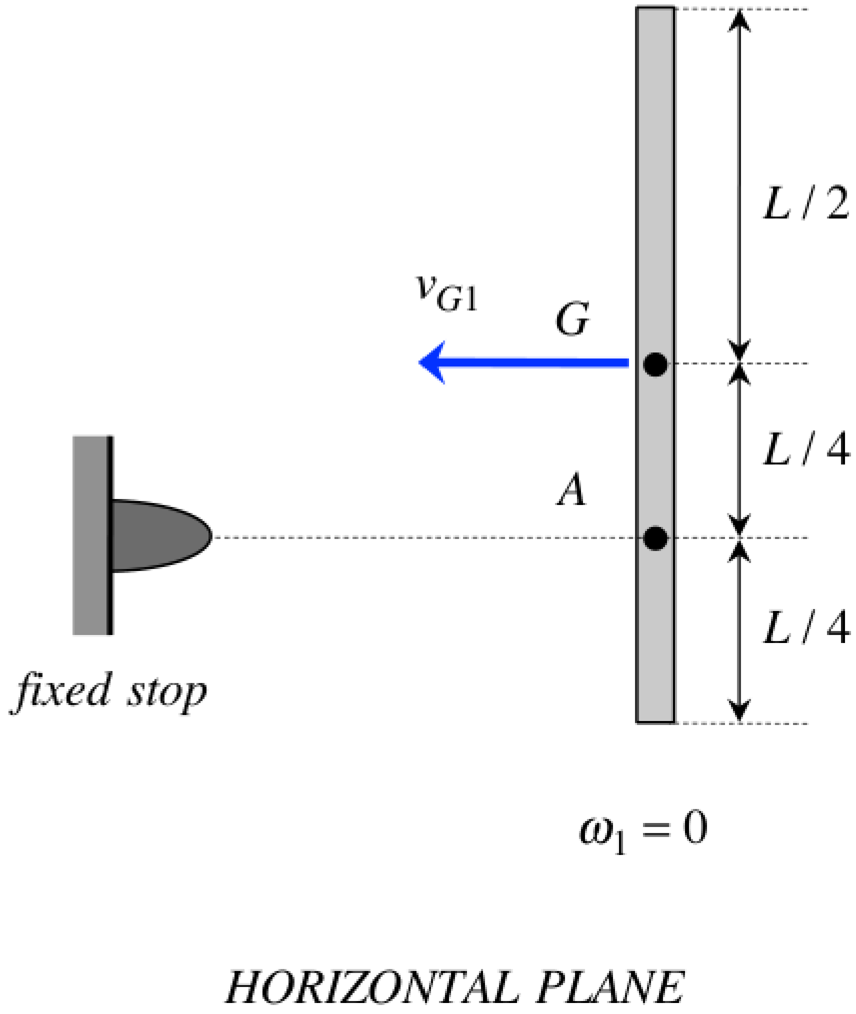
Example 5.C.7

Given: A thin homogeneous bar of mass m and length L translates along a smooth horizontal surface with a speed of v_{G1}. The bar strikes a fixed stop at location A on the bar, where the coefficient of restitution for this impact is e, where $0 < e < 1$.

Find: Determine:

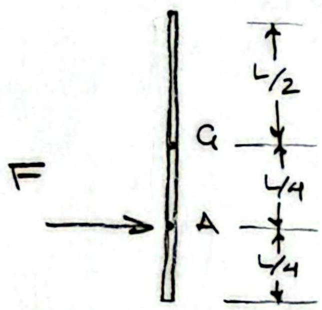
(a) The speed of the center of mass G for the bar after impacting the stop; and

(b) The angular velocity of the bar after impacting the stop.



example S.C.7

1. FBD : "unusual" problem, we want to account for the "internal" force at the fixed stop



2. Kinetics

Linear impulse-momentum:

In (x): $-m v_{G_1} + \int F dt = m v_{G_2}$ ... (1)

Angular impulse-momentum:

$\emptyset + \int F \left(\frac{L}{4}\right) dt = I_G \omega_2$ ... (2)

no rotation at instant ①

$\frac{L}{4} \int F dt = I_G \omega_2$

Coeff. of restitution: $e = -\dfrac{v_{Ax2} - \cancel{v_{wall_2}}}{v_{Ax1} - \cancel{v_{wall_1}}}$

0, wall!

0, wall!

3. Kinematics

$\vec{v}_{A_1} = \vec{v}_{G_1} + \cancel{\vec{\omega}_1} \times \vec{r}_{A/G,1} = \vec{v}_{G_1} = -v_{G_1}\hat{i} + \emptyset\hat{j}$ $\quad \omega_1 = \emptyset$

thus: $v_{Ax_1} = v_{Gx_1}$ ✓ known

And: $e = -\dfrac{v_{Ax_2}}{-v_{Gx_1}} = \dfrac{v_{Ax_2}}{v_{Gx_1}} \Longrightarrow v_{Ax_2} = e\, v_{G_1}$ ... (3)

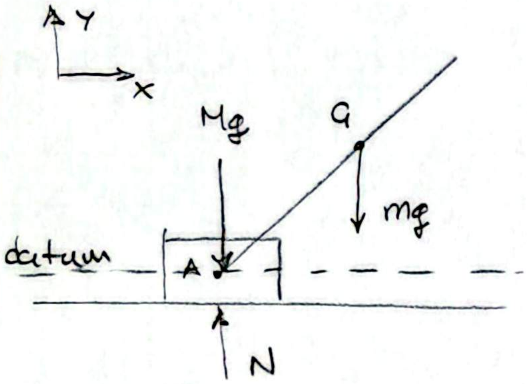
$\vec{v}_{A_2} = \vec{v}_{G_2} + \vec{\omega}_2 \times \cancel{\vec{r}_{A/G_2}} = v_{G_{2x}}\hat{i} + v_{G_{2y}}\hat{j} + \frac{\omega_2 L}{4}\hat{i}$ $\quad -\frac{L}{4}\hat{j}$

In $(\hat{i})$: $v_{Ax_2} = v_{G_2} + \dfrac{\omega_2 L}{4}$ ... (4)

4 unknowns ($\int F dt$, v_{G_2}, ω_2, v_{Ax_2}), 4 eqs.

Example 5.C.9

1. FBD



2. Kinetics

Linear Impulse - Momentum:

In (X): $M\cancel{v_{A_1}} + m\cancel{v_{Gx_1}} = M v_{A_2} + m v_{Gx_2}$

$\quad$ 0, rest $\quad$ 0, rest

$M v_{A_2} + m v_{Gx_2} = 0 \quad \cdots (1)$

Work-energy: $T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$

$T_1 = 0$: rest

... (2)

$V_1 = mg\,h_{G_1} = mg\,\frac{L}{2}\cos\theta_1$

$U_{1\to2}^{(nc)} = 0$: no nc forces

Note that $v_{G_2}^2 = v_{Gx_2}^2 + v_{Gy_2}^2$

$T_2 = \frac{1}{2} M v_{A_2}^2 + \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega_2^2$

for a rod: $I_G = \frac{1}{12} mL^2$

$V_2 = 0$

So: $mg\,\frac{L}{2}\cos\theta_1 = \frac{1}{2} M v_{A_2}^2 + \frac{1}{2} m v_{G_2}^2 + \frac{1}{2} I_G \omega_2^2 \quad \cdots (3)$

3 eqs., 5 uknowns

3. Kinematics

$\vec{v}_{G_2} = \vec{v}_{A_2} + \vec{\omega}_2 \times \vec{r}_{G/A_2}$

$v_{Gx_2}\,\hat{i} + v_{Gy_2}\,\hat{j} = v_{A_2}\,\hat{i} + (\omega_2\hat{k}) \times \left(\frac{L}{2}\hat{i}\right) = v_{A_2}\hat{i} + \frac{\omega_2 L}{2}\hat{j}$

In $(\hat{i})$: $v_{Gx_2} = v_{A_2} \quad \cdots (4)$

In $(\hat{j})$: $v_{Gy_2} = \frac{\omega_2 L}{2} \quad \cdots (5)$